THE PROPAGATION CHARACTERISTICS OF THE SPECIFIC PHOTONIC CRYSTAL FIBER STRUCTURE: THE LOCALIZED BASIS FUNCTION METHOD

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ABSTRACT

In this study, propagation characteristics of a specific photonic crystal fiber (PCF) structure is investigated via Localized Basis Function (LBF) method. Simulation results are presented which demonstrate that modal field variations can be obtained in an accurate and numerically efficient manner utilizing the LBF approach proposed in this work.

I. INTRODUCTION

In recent years, there has been a significant interest in photonic crystal fibers (PCF's). PCF's are single material optical fibers with a periodic array of air holes made in their cross-sections running along the entire length of the fiber. The large and controllable periodic variations of transverse refractive index offered by these fibers opens up exciting new opportunities for the control and guidance of light [1, 2, 3, 4].

In a PCF, light can be guided using either one of two quite different mechanisms: Total Internal Reflection (TIR) mechanism and Photonic Band Gap (PBG) effect [1]. TIR occurs when the refractive index of the core is higher than that of the cladding surrounding it. TIR is a well known mechanism and has widely been used in describing propagation in optical waveguides. Within the last decade there has been an increasing interest in a physical mechanism known as PBG which provides some new opportunities in confining and controlling light in fibres. PBG can be obtained by introducing a periodic perturbation into the cross-section of the fiber. The main property characterizing the PBG structures is the occurrence of pass and stop bands in the frequency spectrum.

In this study, propagation characteristics of a specific photonic crystal fiber (PCF) structure depicted in Fig.1 are investigated via Localized Basis Function (LBF) method. The PCF structure considered here, consists of an unbounded cladding region formed by introducing circular perforations in a lossless dielectric conforming to a periodic pattern of hexagonal symmetry except for a "defect" region obtained by removing one of the holes. Guided mode energy is concentrated in the vicinity of this defect which acts as the (high-index) core region for the PCF. Hermite-Gaussian type Localized Basis Functions (LBF) [5, 6, 7] are utilized together with Fourier type expansions in representing the localized guided modal fields and the periodic variation of the refractive index in the transverse domain. Because the refractive index of the defect is higher than the "effective index" of the cladding, the guidance of the electromagnetic field in the PCF structure can also be thought to result due to TIR mechanism. Numerical results are given which demonstrate the applicability of the presented method.

II. METHOD

In the LBF method considered in this paper, expansions based on Hermite-Gaussian (HG) functions are used for representing the fields within the core (defect) region whereas a mixed representation utilizing HG functions and 2-D Fourier series are used to model the periodic refractive index variation together with the defect (core) region. These expansions reduce the problem of determining the propagation characteristics of PCF to a matrix eigenvalue problem. The solution of the latter contains spurious elements, which need to be identified and eliminated. In the present work, the concept of the " effective index" is utilized as a first approximation for rapidly discriminating between eigenvalues corresponding to the guided and radiation type solutions of the structure. Effective index is then utilized by arguing that the modal indices of the guided modes should fall within the interval bounded by the core (defect) refractive index and the effective index of the cladding. However, for verification purposes we have also plotted the variation of the fields corresponding to the calculated eigenvalues in order to that they actually correspond to the guided modes of the PCF.

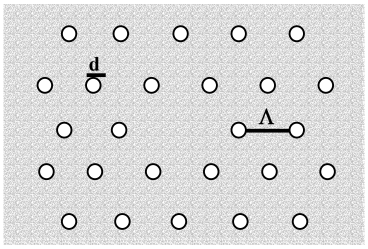


Figure 1. An unbounded periodic cladding structure and high-index core region (defect region); air holes are arranged in a hexagonal lattice in the cladding region, and the central air hole is removed.

In our work, scalar wave approximation is assumed because the ratio of the air hole diameter (d) to the hole separation (Λ) is small (d/Λ<0.35). A modal electric field component is expanded as

$$E(x,y)=\sum_{v}^{N^2} a_v F_v(x,y) \tag{1a}$$

Here a_v denote expansion coefficients and $v=(i,j)$ represents a double index. Thus written explicitly we have

$$F_v(x,y)=\sum_{i=1}^{N}\sum_{j=1}^{N}\Psi_i(x)\Psi_j(y) \tag{1b}$$

where N is the number of terms retained in the expansion and Ψ_i stands for orthonormal basis of Hermite-Gaussian functions:

$$\psi_i^m(x)=\frac{2^{-(i-1)}\pi^{-1/4}}{\sqrt{((2(i-1))!w}}\exp(\frac{-x^2}{2w^2})H_{2i}\left(\frac{x}{w}\right) \tag{2}$$

Here H_i is the ith-order Hermite polynomial. w in (2) which represents the characteristic width of the basis set is taken to be Λ/2 where Λ is the separation between the holes.

The squared transverse refractive index profile is separated into two parts: corresponding to the periodic lattice of holes which may be described using a Fourier series, and that corresponding to the central defect (i.e., the core) which will again be described using localized orthonormal Hermite-Gaussian functions. We thus write,

$$n^2(x,y)=n_{defect}(x,y)+n_{periodic}(x,y)$$

$$=\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}C_{ij}\Psi_i(x)\Psi_j(y)+\sum_{i=1}^{N_2}\sum_{j=1}^{N_2}D_{ij}\cos\left(\frac{2\pi(i-1)x}{lx}\right)\cos\left(\frac{2\pi(j-1)y}{ly}\right) \tag{3}$$

where N_1 and N_2 terms are retained in expansions for the defect region and the periodic cladding region holes, respectively. lx and ly are the periods along the x and the y axis, which for the structure shown in Fig.2 are determined as lx=Λ and ly=1.732Λ, respectively.

ψ_i in (3) has the same functional form as given by (2) however, w is now replaced by $w=0.26d$ (d: the air hole diameter), i.e. the basis sets used in representing modal fields and $n^2(x,y)$ within the defect region are defined with different characteristic widths.

Expansion coefficients D_{ij} are evaluated via inner products defined over the unit cell (UC),

$$D_{ij}=\iint_{UC} n^2(x,y)\cos(\frac{2(i-1)\pi x}{lx})\cos(\frac{2(j-1)\pi y}{ly})d_x d_y \tag{4}$$

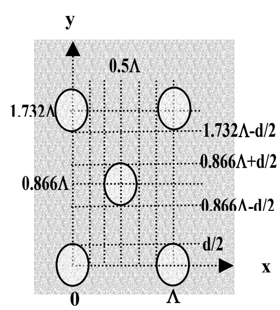


Figure 2. A periodic unit cell of the unbounded periodic cladding structure.

On the other hand C_{ij} coefficients in (3) are evaluated via inner products defined over the central defect (CD) region,

$$C_{ij}=\iint_{CD} n^2(x,y)\psi_i(x)\psi_j(y)d_x d_y \tag{5}$$

The representations for the scalar electric field component and the squared transverse refractive index given in (1) and (2), respectively, are substituted into the scalar wave equation.

$$\frac{1}{k^2}\nabla^2 E + (n^2 - \overline{n}^2)E = 0 \qquad (6)$$

where $\overline{n} = \beta / k_0$ denotes the modal index.

Next, each term of the resulting expression is multiplied by $F_\mu(x,y)$ and integrated over the entire transverse (x, y) plane. Making using of the orthonormality of the Hermite-Gaussian basis functions, the problem is then reduced to matrix form:

$$(M - \overline{n}^2 I)V = 0 \qquad (7)$$

where I stands for the unit matrix. Setting det[$M - \overline{n}^2 I$] = 0 yields the modal eigenvalues $\overline{n}^2$. Here V denotes the vector ($N^2 \times 1$) of expansion coefficients a, and the elements $M_{v,\mu}$ of $N^2 \times N^2$ matrix M are obtained as

$$M_{v,\mu} = \frac{1}{k^2} I^{(1)}_{v,\mu} + I^{(2)}_{v,\mu} \qquad (8a)$$

$$I^{(1)}_{v,\mu} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_v(x,y)\nabla^2\left[F_\mu(x,y)\right] d_x d_y \qquad (8b)$$

$$I^{(2)}_{v,\mu} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n^2(x,y) F_v(x,y) F_\mu(x,y) d_x d_y \qquad (8c)$$

The guided mode solutions can be distinguished from the radiation modes by extracting the eigenvalues $\overline{n}^2 = \beta^2 / k^2$, which fall within the following range

$$n_{eff}{}^2 \langle \frac{\beta^2}{k^2} \langle n_s^2 \qquad (9)$$

where n_s is the refractive index of substance and n_{eff} is the effective index of the periodic cladding region obtained by perforating the substrate. The determination of the effective index of the periodic cladding region involves the solution of an auxiliary problem defined on a unit cell [8]. On the other hand, some researches [8, 9, 10] have reported results by modeling the PCF as a step index fiber characterized by core and cladding regions by the refractive indices of the defect region and effective index, respectively. The problem in this approach resides in the ambiguity in defining the equivalent core radius. It would therefore be interesting to determine the extent to which the concept of effective index can be stretched in modeling PCF's.

III. RESULTS

The specific PCF structure of hexagonal symmetry considered in this work is shown in Fig.1. In all numerical calculations presented here we assumed d/Λ=0.2 and n_s=1.45, corresponding to a silica substance.

Fig.3 demonstrates that the convergence properties of the representation given in (3) are fairly good and the refractive index profile can be represented quite accurately by using reasonable number of terms in the expansions.

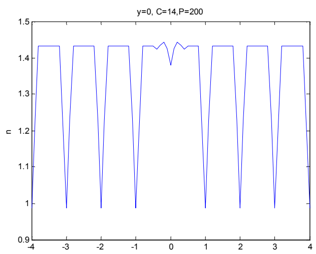


Figure 3. Variation of index profile along x profile obtained from (3) using N_1=14, N_2=200.

It is reported in the literature [8] that the PCF considered here supports only one guided mode. The calculated variation of the normalized propagation factor defined as

$$b = \frac{\overline{n} - n_{ef}}{n_s - n_{ef}} \qquad (10)$$

is shown in Fig.4. It can be seen from this figure that a single mode starts propagating in the PCF for d/λ > 0.65 and no other mode propagates in the range of d/λ values shown in the figure.

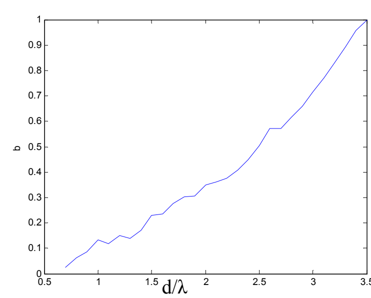
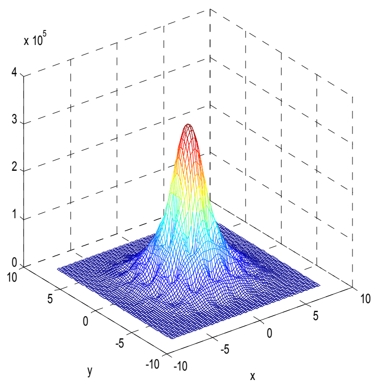
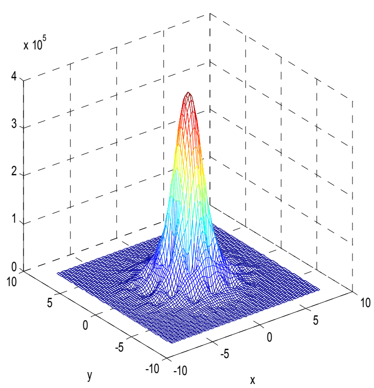


Figure 4. Variation of b with d/λ.

The variation of calculated transverse profile of the dominant mode is given in Figs. 5a and 5b for d/λ=0.4 and d/λ=0.9 respectively. It is clearly seen from a comparison of Fig. 5a with Fig. 5b that one obtains a radiation type field behaviour and a guided mode behaviour when d/λ is chosen below or above the modal cut-off value d/λ=0.65, respectively.



(a)



(b)

Figure 5. The radiation and guided modes below and above the modal cut-off value d/λ=0.65. (a) The radiation mode, d/λ=0.4. (b) The guided mode, d/λ=0.9. N=10, N_1=10, N_2=100.

IV. CONCLUDING REMARKS

In this study, propagation characteristics of the specific photonic crystal fiber (PCF) structure are investigated via Localized Basis Function (LBF) method. Hermite-Gaussian type Localized Basis Functions (LBF's) [5, 6, 7] are utilized together with Fourier type expansions in representing the localized guided modal fields and the periodic variation of the refractive index in the transverse domain. We have presented numerical results, which compare very well with those reported in the literature [6]. This scaler work demonstrates that the fields in the PCF can be obtained in a straight forward manner using LBF approach. Moreover, the numerical efficiency of this approach is quite good. Although the scalar case is considered in this paper, the method can also be used to obtain full wave solutions for the vector fields.

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