Explicit Formulas for A Special Class of Two-Variable Resonant Ladder Networks with Simple Lumped Elements and Commensurate Stubs

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Abstract

For the restricted two-variable network topologies, the explicit characterization may be obtained using two-variable network functions. In this paper, explicit two-variable description of some classes of resonant ladder networks are presented. Up to a certain complexity, for the regular resonant ladder structure commonly used in practice, explicit expressions describing the two-variable scattering functions are obtained.

1. Introduction

As is well known, design of two-port lossless networks with mixed, lumped and distributed elements offer many practical advantages over those designed with lumped or distributed elements alone. Therefore, many researchers have examined the synthesis procedures to design two-variable networks with lumped and distributed elements[1-6]. In most of the existing studies, more interest has been devoted the ladder network structures consist of cascaded lossless lumped two-ports and commensurate transmission lines from the technical realization point of view. But, still exact solution of the synthesis problem of mixed structures has not been obtained analytically.

In recent literature[7-13], the design problem of lossless cascaded two-ports with mixed elements has been solved utilizing a semi-analytical approach. In that approach, by using the topological properties of ladder networks with two-kind element, contain the simple lumped elements and uniform transmission lines, it had been shown that two-variable network functions could be solved up to a certain complexity.

In this work, by using the semi-analytical approach, for the explicit description of a special class of resonant ladder networks, two-variable scattering functions are obtained.

2. Two-Variable Description of The Resonant Ladder with Mixed Elements

Consider the resonant ladder network with simple lumped element and equidelay short-circuited and open-circuited transmission lines (commensurate stubs) shown in Fig.1. The generic form of cascaded two-port composed of lumped elements and stubs can be described in terms of two-variable canonic polynomials as follows:

$$S = \frac{1}{g}\begin{pmatrix} h & \sigma f_* \\ f & -\sigma h_* \end{pmatrix}, \qquad (1)$$

where; (g, h and f) are real polynomials of the complex variables p and λ; $g(p, \lambda)$ is a Scattering Hurwitz polynomial; the lower asterisk ($_*$) denotes the paraconjugation i.e. $h_*=h(-p,-\lambda)$.

.The polynomials are related by the paraunitary condition:

$$g(p,\lambda)g(-p,-\lambda)=h(p,\lambda)h(-p,-\lambda)+f(p,\lambda)f(-p,-\lambda) \qquad (2)$$

where, $f(p,\lambda)=\prod_{i=1}^{n}[f_i(p)+f_i(\lambda)]$ or

$$f(p,\lambda)=[p^n+f_{n-1,1}\,p^{n-1}\lambda+......+f_{1,n-1}\,p\,\lambda^{n-1}+\lambda^n]$$

n represents the number of sections (the total number of lumped elements or stubs of the mixed structure).

As it can be easily shown that the two-variable real polynomials $g=g(p,\lambda)$ and $h=h(p,\lambda)$ can be expressed in coefficients form as follows:

$$g(p,\lambda)=\mathbf{p}^T\Lambda_g\boldsymbol{\lambda}, ; \quad h(p,\lambda)=\mathbf{p}^T\Lambda_h\boldsymbol{\lambda}, \qquad (3)$$

where, $\mathbf{p}^T=[1\ p\ p^2\ \cdots\ p^{n_p}]$, $\boldsymbol{\lambda}^T=[1\ \lambda\ \lambda^2\ \cdots\ \lambda^{n_\lambda}]$; n_p and n_λ are the total number of simple lumped elements and stubs in the mixed structure respectively.

In order to characterize the mixed elements network shown in Fig.1, by using the losslessness condition (2) of two-ports, the two-variable real canonic polynomials defining the scattering matrix of the resonant ladder networks are constructed. For the chosen restricted topologies, the canonic polynomials have to satisfy some additional conditions to ensure the realizability.

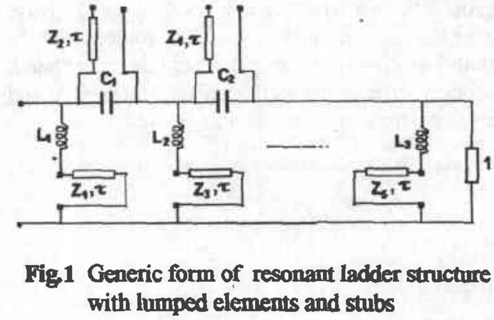

Fig.1 Generic form of resonant ladder structure with lumped elements and stubs

By using the one-variable boundary conditions and the topologic properties of the mixed element structures, some constraints for a realizable cascade structure with positive element values are obtained. Using these conditions in (2) properly, explicit expressions yielding the coefficients of the canonic polynomials ($h(p,\lambda)$ and $g(p,\lambda)$) are found. For low-order mixed structures, the obtained explicit solution to construct the lossless ladder networks is unique and constitutes the sufficient conditions. For high order ladders, the problem can be solved numerically.

In the following, the description of the resonant ladder structure with mixed elements is given.

The boundary conditions for the mixed structure shown in Fig.1 can be established by a direct way as following :

♣ Putting p=0 in (2), the obtained polynomials $g(0,\lambda)$, $h(0,\lambda)$ and $f(0,\lambda)$ define the cascaded of stubs

$$g(0,\lambda)g(0,-\lambda)=h(0,\lambda)h(0,-\lambda)+f(0,\lambda)f(0,-\lambda) \qquad (4)$$

♣ Putting $\lambda=0$ in (2), the obtained polynomials $g(p,0)$, $h(p,0)$ and $f(p,0)$ define the cascade of lumped section which has the transmission zeros at $p=\infty$ and

$$g(p,0)g(p,0)=h(p,0)h(p,0)+f(p,0)f(p,0) \qquad (5)$$

where $g(0,\lambda)$ and $g(p,0)$ are strictly Hurwitz polynomials ; $f(0,\lambda)=\lambda^n$ and $f(p,0)=p^n$.

Otherwise, by making the topological analysis of the mixed structure in Fig.1, some topologic constraints on the coefficients of the canonic polynomials besides the above boundary conditions (4-5), are founded. Thus, a direct solution up to a certain mixed element network complexity is obtained.

The founded **topologic properties:**

- The entries of Λ_g are nonnegative reel numbers,
- $g_{11}=\mu\, h_{11}+(1/g_{00}).(g_{01}g_{10}-h_{01}h_{10})$
- $h_{k,n-m}=0,$ ($k\geq m$; $k,m=0,1,..,n$)
- $g_{k,n-m}=0,$ ($k>m$; $k,m=0,1,..,n$)
- $g_{k,n-m}=f_{k,n-m}$ ($k=m$; $k,m=0,1,..,n$)
- $\mu=h_{00}/g_{00}=\pm 1.$ ($g_{00}=1$)

Squential cascade analysis of the generic resonant ladder structure under consideration exposes that the coefficient matrices Λ_h and Λ_g associated with the mixed structure have the following generic forms: .

$$\Lambda_h=\begin{pmatrix} h_{00} & h_{01} & h_{02} & \cdots & h_{0n-1} & 0 \\ h_{10} & h_{11} & h_{12} & \cdots & 0 & 0 \\ h_{20} & h_{21} & \cdots & .\cdot^{\cdot} & 0 & 0 \\ \vdots & \vdots & 0 & .\cdot^{\cdot} & .\cdot^{\cdot} & \vdots \\ h_{n-1,0} & 0 & 0 & 0 & .\cdot^{\cdot} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

$$\Lambda_g=\begin{pmatrix} g_{00} & g_{01} & g_{02} & \cdots & g_{0n-1} & 1 \\ g_{10} & g_{11} & g_{12} & \cdots & g_{1,n-1} & 0 \\ g_{20} & g_{21} & \cdots & .\cdot^{\cdot} & 0 & 0 \\ \vdots & \vdots & .\cdot^{\cdot} & .\cdot^{\cdot} & .\cdot^{\cdot} & \vdots \\ g_{n-1,0} & g_{n-1,1} & 0 & 0 & .\cdot^{\cdot} & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

Using the above coefficient properties and boundary conditions given in (4-5), the lossless equation (2) could be solved and obtained the explicit expressions up to 4 sections for the mixed structure and corresponding realizations characterized by the expressions are shown in Table · 1 and Fig.2 respectively.

Conclusions

In this study, two-variable description of a special class of simple lumped resonant ladders with commensurate stubs are investigated. By using the recently derived results for explicit formulas for two-variable scattering parameters, the explicit expressions for the resonant ladder network with mixed elements are obtained firstly in the literature.

Alternatively, by investigating topologic properties of another resonant simple lumped ladders with commensurate stubs,the explicit characterization of some practical resonant networks can be obtained.

It is hoped that the explicit formulas presented in this study will provide an insight into the analytic approximation problem of two variable networks, especially in microwave circuit design.

Table 1 **Explicit Formulas and Coefficient Matrices for The Mixed Structure with Low-Sections**

2 Section

$g_{00}=|h_{00}|$, $g_{10}=\{(h_{10}^2+2\alpha_{20}g_{00}\}^{1/2}$,

$g_{01}=\{(h_{01}^2+2\alpha_{02}g_{00}\}^{1/2}$,

$g_{11}=\mu h_{11}+(1/g_{00})(g_{10}g_{01}-h_{01}h_{10})$, $\mu=\mathrm{Sgn}(h_{00})$

$$\Lambda_h=\begin{bmatrix} h_{00} & h_{01} & 0 \\ h_{10} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \Lambda_g=\begin{bmatrix} g_{00} & g_{01} & 1 \\ g_{10} & g_{11} & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

3 Section

$g_{00}=|h_{00}|$,

$g_{10}=\{(h_{10}^2+2\alpha_{20}g_{00}\}^{1/2}$, $g_{01}=\{(h_{01}^2+2\alpha_{02}g_{00}\}^{1/2}$,

$g_{20}=\{(h_{20}^2+2g_{10}g_{30}\}^{1/2}$, $g_{02}=\{(h_{02}^2+2g_{01}g_{03}\}^{1/2}$,

$g_{12}=f_{12}$ $g_{21}=f_{21}$

$g_{11}=\mu h_{11}+(1/g_{00})(g_{10}g_{01}-h_{01}h_{10})$, $\mu=\mathrm{Sgn}(h_{00})$

$$h_{11}=\frac{g_{20}\,(g_{01}g_{12}+g_{10}g_{03})-g_{02}\,(g_{10}g_{21}+g_{01}g_{30})}{h_{20}\,g_{02}-h_{02}\,g_{20}}$$

$$\Lambda_h=\begin{bmatrix} h_{00} & h_{01} & h_{02} & 0 \\ h_{10} & h_{11} & 0 & 0 \\ h_{20} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \Lambda_g=\begin{bmatrix} g_{00} & g_{01} & g_{02} & 1 \\ g_{10} & g_{11} & g_{12} & 0 \\ g_{20} & g_{21} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$\alpha_{02}=g_{02}-\mu h_{02}$, $\alpha_{20}=g_{20}-\mu h_{20}$,

4 Section

$g_{00}=|h_{00}|$

$g_{10}=\{(h_{10}^2+2\alpha_{20}g_{00}\}^{1/2}$, $g_{30}=\{(h_{30}^2+2g_{20}g_{40}\}^{1/2}$,

$g_{20}=\{(h_{20}^2+2(g_{10}g_{30}-h_{10}h_{30})-2g_{00}g_{40}\}^{1/2}$,

$g_{01}=\{(h_{01}^2+2\alpha_{02}g_{00}\}^{1/2}$, $g_{03}=\{(h_{03}^2+2g_{02}g_{04}\}^{1/2}$,

$g_{02}=\{(h_{02}^2+2(g_{01}g_{03}-h_{01}h_{03})-2g_{00}g_{04}\}^{1/2}$

$g_{11}=\mu h_{11}+(1/g_{00})(g_{10}g_{01}-h_{01}h_{10})$, $\mu=\mathrm{Sgn}(h_{00})$

$g_{31}=f_{31}$, $g_{22}=f_{22}$, $g_{13}=f_{13}$,

$h_{12}=(1/h_{03})(g_{03}g_{12}-g_{04}g_{11}-g_{13}g_{02})$

$h_{21}=(1/h_{30})(g_{30}g_{21}-g_{40}g_{11}-g_{31}g_{20})$

$g_{21}=(1/\beta_{13})(\beta_{24}g_{11}-h_{20}h_{11}+\beta_{02}g_{31}-g_{01}g_{30}+h_{01}h_{30})$

$g_{12}=(1/\gamma_{13})(\gamma_{24}g_{11}-h_{02}h_{11}+\gamma_{02}g_{13}-g_{10}g_{03}+h_{10}h_{03})$

$$h_{11}=\mu\left[\frac{-\Delta_1(l_{13}/\beta_{13})-\Delta_2(m_{13}/\gamma_{13})+\Delta_3}{(\alpha_{20}-\delta_1 g_{40})(l_{13}/\beta_{13})+(\alpha_{02}-\delta_2 g_{04})(m_{13}/\gamma_{13})+\Delta_4}\right]$$

Where;

$\beta_{13}=g_{10}-\delta_1 g_{30}$, $\beta_{24}=g_{20}-\delta_1 g_{40}$, $\beta_{02}=g_{00}-\delta_1 g_{20}$,

$\gamma_{13}=g_{01}-\delta_2 g_{03}$, $\gamma_{24}=g_{02}-\delta_2 g_{04}$, $\gamma_{02}=g_{00}-\delta_2 g_{02}$,

$\delta_1=(h_{10}/h_{30})$; $\delta_2=(h_{01}/h_{03})$, $\delta_3=(h_{01}/h_{30})$, $\delta_4=(h_{10}/h_{03})$

$l_{13}=g_{01}-\delta_3 g_{30}$, $m_{13}=g_{10}-\delta_4 g_{03}$,

$\Delta_1=\beta_{02}g_{31}-g_{01}g_{30}+h_{01}h_{30}+\beta_{24}\Delta_5$,

$\Delta_2=\gamma_{02}g_{13}-g_{10}g_{03}+h_{10}h_{03}+\gamma_{24}\Delta_5$,

$\Delta_3=[g_{00}g_{22}+g_{20}g_{02}-h_{20}h_{02}+0.5\Delta_5^2-\delta_3(g_{40}\Delta_5+g_{31}g_{20})$
$-\delta_4(g_{04}\Delta_5+g_{13}g_{02})$,

$\Delta_4=[\delta_4 g_{04}+\delta_3 g_{40}-\Delta_5]$, $\Delta_5=(1/g_{00})(g_{10}g_{01}-h_{01}h_{10})$

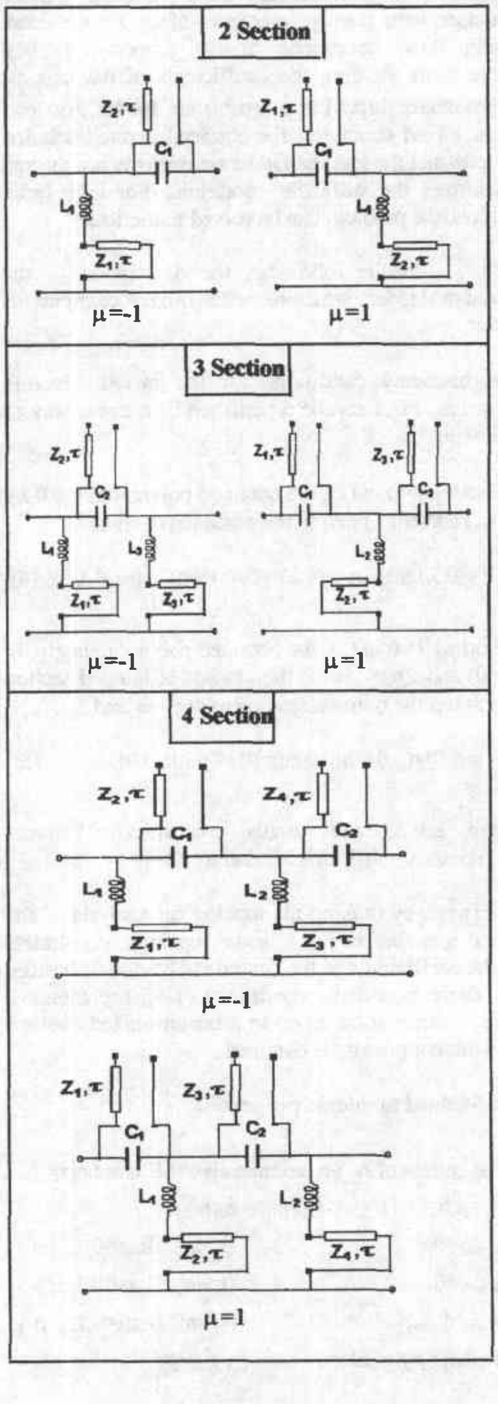

Fig. 2. Low-section realizations of the resonant ladder with mixed elements.

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