LYAPUNOV-BASED STATCOM CONTROL

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Abstract In this paper, we discuss the nonlinear controller design of STATic COMpensator (STATCOM) for voltage stability study. A state-space averaged model of the STATCOM is derived and is used for controller design. A Lyapunov-based nonlinear controller is proposed. The controller regulates the firing angle of the STATCOM to track a given reactive current reference. The controller achieves global asymptotic stability and is not affected by the operating point. Simulation results show that the performance of the nonlinear controller is much better than that of the conventional PI controller.

1. Introduction

Flexible AC Transmission System (FACTS) devices have received much attention in recent years. One important aspect of these devices is that they provide fast and reliable control over the three main transmission system parameters, namely, voltage magnitude, voltage phase angle and impedance [1]. This attractive feature enables these solid state controllers to be widely used in many fields of power systems, such as real and reactive power regulation, stability control and system damping [2].

The STATic COMpensdator (STATCOM), also referred to as a STATic CONdenser (STATCON) or Advanced Static VAR Compensator (ASVC) is a second generation FACTS device used for shunt reactive power compensation. It is based on solid state synchronous voltage source that is analogous to an ideal synchronous machine. The STATCOM can be controlled either to provide or absorb reactive power. The performance of reactive compensation of a STATCOM is better than the conventional compensators such as the Static Var Compensator (SVC) [3]. Consequently, the STATCOM offers potential benefits in applications such as increasing steady-state loadability, improving the security of systems with heavy motor loads, improving transient stability and adding damping to power swings.

One important application of STATCOM is to improve power system voltage stability [4]. Voltage stability is concerned with the ability of power systems to maintain acceptable voltages at all nodes in the systems under normal and contingent conditions. Voltage instability will occur when a disturbance causes a progressive and uncontrollable decline in voltage magnitude. It is always associated with inadequate local reactive power support. There is an increasing interest with this problem [5]. However most of the research activities is still concentrated on stability analysis. Relatively less attention has been paid to study voltage control and stabilization. In [6], the authors proposed a nonlinear controller for SVC and showed its effectiveness in voltage stability enhancement. Similarly, advanced controller can be designed for the STATCOM to improve voltage stability.

Because voltage instability evolves in a time-scale of minutes while the dynamics of the STATCOM is in a time-scale of milliseconds, the STATCOM can therefore be represented by its static characteristics in voltage stability study. The basic assumption is that the internal dynamics of the STATCOM is stable. It is not easy to guarantee this point. It is well-known that the STATCOM model is a nonlinear system. Because of the difficulties in controlling nonlinear systems, most of the previous works dealing with STATCOM control are confined to the application of the linear control theory to small-signal linearized models. These control laws only guarantee system stability against small perturbations from the nominal operating point. If a large-disturbance occur or the operating point deviates away from the nominal operating point too much, the performance of these controllers will be significantly deteriorated. To solve this problem, some nonlinear control strategies have been proposed, such as the fuzzy logic current and voltage controller in [7], the differential algebra based nonlinear controller in [8] and the nonlinear controller based on exact feedback linearization technique in [9].

In this paper, we propose a new control law for the STATCOM based on Lyapunov stability theory without linearization. In our approach, Lyapunov's direct method which is an important tool for nonlinear system control design is adopted. The basic idea of the approach is to construct a positive definite scalar Lyapunov function for the system and to examine the function's time derivative. The proposed control law guarantees global asymptotic stability of the closed-loop system.

The paper is organized as follows. In Section 2, the mathematical model of the STATCOM is studied. A state-space averaged model is derived. In Section 3, Lyapunove-based nonlinear controller design is discussed. Simulation results are presented in Section 4 to test the performance of the controller. We conclude the paper in Section 5.

2. System Model

A. Mathematical Model

A simplified STATCOM model is shown in Fig. 1. The voltage source converter is connected to the system bus through an inductance L and a resistor R. L represents the leakage of the coupling transformer, R_S represents the converter and the transformer conduction losses. On the DC side, a capacitor C is in parallel with a resistor R. The resistor R represents the converter switching losses.

In Fig. 1, E_a, E_b and E_c are the line voltages, V_a, V_b and V_c are the converter output phase voltages, V_{dc} and I_{dc} are the DC voltage and current, I_a, I_b and I_c are the AC line currents.

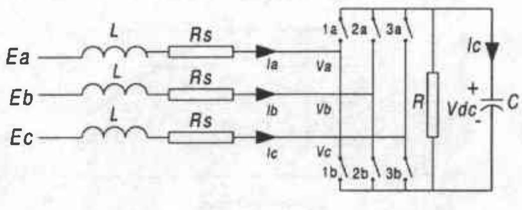

Fig. 1 STATCOM structure

The dynamical equations for the STATCOM are expressed by

$$\frac{dI_{abc}}{dt}=-\frac{R_s}{L}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}I_{abc}+\frac{1}{L}(E_{abc}-V_{abc}) \quad (1)$$

$$\frac{dV_{dc}}{dt}=\frac{I_{dc}}{C}-\frac{V_{dc}}{RC} \quad (2)$$

where $I_{abc}=[I_a \quad I_b \quad I_c]^T$, $V_{abc}=[V_a \quad V_b \quad V_c]^T$, $E_{abc}=[E_a \quad E_b \quad E_c]^T$. The line voltages are given by:

$$E_{abc}=\sqrt{\frac{2}{3}}E\begin{bmatrix}cos(\omega t+\theta)\\cos(\omega t+\theta-\frac{2\pi}{3})\\cos(\omega t+\theta-\frac{4\pi}{3})\end{bmatrix}$$

, where E is the line to line RMS voltage and θ is the voltage phase angle of the STATCOM bus.

The relationship between the AC side variables, i.g., V_{abc} and I_{abc}, and the DC side voltage V_{dc} and current I_{dc} is determined by the switching function S_{abc} of the converter. The switching function operates in discrete mode. This brings difficulties for system analysis and control. A generalized averaging method [10] is used to get a continuous time-invariant model of the converter. For a six-pulse converter, the averaged model is given by [7]

$$S_{abc}=\frac{2}{\pi}\begin{bmatrix}cos(\omega t+\alpha+\theta)\\cos(\omega t+\alpha+\theta-\frac{2\pi}{3})\\cos(\omega t+\alpha+\theta-\frac{4\pi}{3})\end{bmatrix}$$

, where α is the firing angle of the switches with reference to the phase angle of E_a.

If the converter is lossless, we will get the following equations

$$V_{abc}=V_{dc}S_{abc} \quad (3)$$

$$I_{dc}=I_{abc}^T S_{abc} \quad (4)$$

For modeling and control design, it is very convenient to transform three-phase variables into a rotating d-q frame using Park's transformation. Define the following transformation

$I_{abc}=P_k I_{dq0}$, $E_{abc}=P_k E_{dq0}$ and $V_{abc}=P_k V_{dq0}$

where P_k is defined as

$$P_k=\sqrt{\frac{2}{3}}\begin{bmatrix}cos(\omega t)&sin(\omega t)&\frac{1}{\sqrt{2}}\\cos(\omega t-\frac{2\pi}{3})&sin(\omega t-\frac{2\pi}{3})&\frac{1}{\sqrt{2}}\\cos(\omega t-\frac{4\pi}{3})&sin(\omega t-\frac{4\pi}{3})&\frac{1}{\sqrt{2}}\end{bmatrix}.$$

Note that the transformation P_k is power invariant.

Applying the Park's transformation to equation (1) and (2) and take into account the relations (3) and (4), we obtain the following time-invariant model

$$\frac{dI_d}{dt}=-\frac{Rs}{L}I_d-\omega I_q-\frac{K\,cos(\alpha+\theta)}{L}V_{dc}+\frac{E_d}{L} \quad (5)$$

$$\frac{dI_q}{dt}=-\frac{Rs}{L}I_q+\omega I_d+\frac{K\,sin(\alpha+\theta)}{L}V_{dc}+\frac{E_q}{L} \quad (6)$$

$$\frac{dV_{dc}}{dt}=-\frac{1}{RC}V_{dc}+\frac{K\,cos(\alpha+\theta)}{C}I_d-\frac{K\,sin(\alpha+\theta)}{C}I_q \quad (7)$$

where $K=\frac{\sqrt{6}}{\pi}$ and $E_d=E\,cos(\theta)$, $E_q=-E\,sin(\theta)$.

The power equations are expressed by

$$P=E_dI_d+E_qI_q \quad (8)$$

$$Q=E_dI_q-E_qI_d \quad (9)$$

The real and reactive power of the STATCOM is in fact not affected by the line phase angle θ [7]. Without loss of generality, we can set θ = 0 to simplify the model. Also the following per unit system on the STATCOM base is adopted:

$X_L=\frac{L\omega}{Z_{base}}$, $X_C=\frac{1}{C\omega Z_{base}}$, $R'=\frac{R}{Z_{base}}$,

$R'_S=\frac{R_S}{Z_{base}}$, $Z_{base}=\frac{V_{base}}{I_{base}}$.

The simplified STATCOM model is given by

$$\frac{dI_d}{dt}=-\frac{R'_S}{X_L}\omega I_d-\omega I_q-\frac{\omega K\,cos(\alpha)}{X_L}V_{dc}+\frac{\omega E}{X_L} \quad (10)$$

$$\frac{dI_q}{dt}=-\frac{R'_S}{X_L}\omega I_q+\omega I_d+\frac{\omega K\,sin(\alpha)}{X_L}V_{dc} \quad (11)$$

$$\frac{dV_{dc}}{dt}=-\frac{X_C\omega}{R'}V_{dc}+X_C\omega K\,cos(\alpha)I_d-X_C\omega K\,sin(\alpha)I_q \quad (12)$$

The firing angle α is the control input. The power equations (8) and (9) become

$$P = EI_d \tag{13}$$

$$Q = EI_q \tag{14}$$

As can be seen, the reactive power is proportional to EI_q while the real power is proportional to EI_d. To control the reactive power Q, it is sufficient to control the reactive current I_q.

B. *Steady-State Characteristics*

Setting the equations (10) to (12) equal to zero, we can get the steady-state values I_{d0}, I_{q0} and V_{dc0} corresponding to a given firing angle α_0. Figs. 2-4 show the steady-state characteristics of the STATCOM. The EPRI 80 MVAR STATCOM [11] is used to generate the characteristics. The values are given as X_L=0.15, R'_S = 0.01, X_C = 0.88 and R' =78, all in per unit. The bus voltage magnitude E is set to 1.0 per unit.

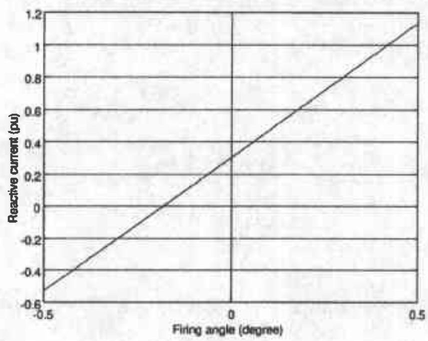

Fig. 2 Steady-state characteristics ($I_q - \alpha$)

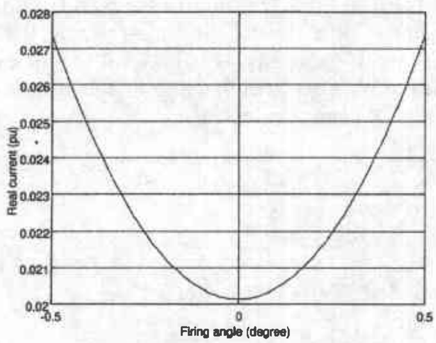

Fig. 3 Steady-state characteristics ($I_d - \alpha$)

As can be seen, when firing angle is zero, the real and reactive current are not zero because of the power losses on R_S and reactive power consumed by the leakage L. The real current of the STATCOM is almost constant and is very small compared to the reactive current. In steady state analysis, the real power losses can be neglected. The STATCOM becomes a pure reactive current source. Positive α means the STATCOM works in the inductive region while negative value implies that it is in the capacitive region. The reactive current I_{q0} is almost proportional to the firing angle α. Also note that the capacitor voltage is not constant during firing angle variation.

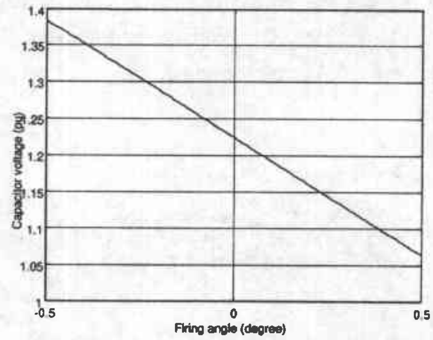

Fig. 4 Steady-state characteristics ($V_{dc} - \alpha$)

In voltage stability study, we can use a variable reactive current source to represent the STATCOM. This requires the internal dynamics of the STATCOM is fast enough and is stable. Although the open loop system is asymptotically stable, its dynamic behavior is not in general satisfactory. In fact, as R_S approaches zero, the system becomes increasingly oscillatory. As stated previously, the conventional PI controllers are tuned around a particular operating point. They are not robust against changing operation conditions. In the following Section, we will discuss the nonlinear controller design based on Lyapunov stability theory.

3. Lyapunov-Based Reactive Current Control

The control of switched converters is divided into two groups. In the first case, the control law commands the switch position directly, such as the sliding mode control. The switching frequency is usually high. The second approach relies on the state-space averaged model. The control variable is the firing angle of the converter. In this paper, the controller design is based on the averaged model (10) to (12). Since we are interested with controlling the internal dynamics of the STATCOM, we assume that the line voltage E does not change with time. The control objective is to regulate the firing angle α, so that the reactive current I_q tracks the given reference current I_{qref} quickly and asymptotically. At the same time, all other states are bounded. As can be seen from the steady-state characteristics, once the reference reactive current I_{qref} is given, I_{d0}, V_{dc0} and α0 are uniquely determined.

Define the following variables:

$x_1 = I_d - I_{d0}$, $x_2 = I_q - I_{qref}$, $x_3 = V_{dc} - V_{dc0}$,
$P_d = \cos\alpha$, $P_{d0} = \cos\alpha_0$, $P_q = \sin\alpha$, $P_{q0} = \sin\alpha_0$,
the incremental STATCOM model are thus obtained:

$$\frac{X_L}{\omega}\frac{dx_1}{dt} = -R'_S x_1 - X_L x_2 - KP_d V_{dc} + KP_{d0} V_{dc0} \quad (15)$$

$$\frac{X_L}{\omega}\frac{dx_2}{dt} = -R'_S x_2 + X_L x_1 + KP_q V_{dc} - KP_{q0} V_{dc0} \quad (16)$$

$$\frac{1}{X_C\omega}\frac{dx_3}{dt} = -\frac{1}{R'}x_3 + KP_d I_d - KP_{d0} I_{d0} - KP_q I_q + KP_{q0} I_{q0} \quad (17)$$

Consider the positive definite Lyapunov function candidature

$$V(x) = \frac{X_L}{2\omega}x_1^2 + \frac{X_L}{2\omega}x_2^2 + \frac{1}{2X_C\omega}x_3^2 \quad (18)$$

Taking the derivative of (18) along any system trajectory gives

$$\begin{aligned}\dot V(x) &= -R'_S x_1^2 - R'_S x_2^2 - \frac{1}{R'}x_3^2 \\ &+ KP_d(V_{dc}I_{d0} - V_{dc0}I_d) + KP_q(V_{dc0}I_q - V_{dc}I_{q0}) \\ &+ KP_{d0}(V_{dc0}I_d - V_{dc}I_{d0}) + KP_{q0}(V_{dc}I_{q0} - V_{dc0}I_q)\end{aligned} \quad (19)$$

Note that P_{d0}, P_{q0} satisfy the steady-state equations of (10) – (12). The derivative $\dot V(x)$ becomes

$$\begin{aligned}\dot V(x) &= -R'_S x_1^2 - R'_S x_2^2 - \frac{1}{R'}x_3^2 \\ &+ KP_d(V_{dc}I_{d0} - V_{dc0}I_d) + KP_q(V_{dc0}I_q - V_{dc}I_{q0}) \\ &+ (E - R'_S I_{d0} - X_L I_{q0})I_d - (R'_S I_{q0} - X_L I_{d0})I_q - \frac{V_{dc}V_{dc0}}{R'}\end{aligned} \quad (20)$$

It is obvious that $\dot V(x)$ along any system trajectory is negative definite if

$$P_d\lambda_d + P_q\lambda_q + \lambda_c = 0 \quad (21)$$

where

$\lambda_d = K(V_{dc}I_{d0} - V_{dc0}I_d)$,
$\lambda_q = K(V_{dc0}I_q - V_{dc}I_{q0})$ and
$\lambda_c = (E - R'_S I_{d0} - X_L I_{q0})I_d - (R'_S I_{q0} - X_L I_{d0})I_q - \frac{V_{dc}V_{dc0}}{R'}$

Note that

$$P_d^2 + P_q^2 = 1 \quad (22)$$

Solving (21) and (22) we get the following control law

$$P_d = \frac{-\lambda_c\lambda_d + |\lambda_q|\sqrt{\lambda_d^2 + \lambda_q^2 - \lambda_c^2}}{\lambda_d^2 + \lambda_q^2} \quad (23)$$

$$P_q = \frac{-\lambda_c\lambda_q^2 - \lambda_d|\lambda_q|\sqrt{\lambda_d^2 + \lambda_q^2 - \lambda_c^2}}{(\lambda_d^2 + \lambda_q^2)\lambda_q} \quad (24)$$

4. Simulation

The performance of the proposed controller is tested through simulation study. For comparison, a PI controller which is given by

$$\Delta\alpha = -K_P(I_q - I_{qref}) - K_I\int_0^t (I_q - I_{qref})$$

where $K_P = K_I = 500$ is also studied. The transient responses of the real current I_d, reactive current I_q and the capacitor voltage V_{dc} to positive and negative step references I_{qref} are studied. The results are shown in Fig. 5 through Fig. 10. In these figures, the dynamic behaviors under the proposed controller are identified by solid curves while those under the conventional PI controllers are shown by dashed curves. In Figs. 5 to 7 the reference I_{qref} jumps from 1 per unit to -1 per unit at 0.2 s. In Figs. 8 to 10 I_{qref} jumps from -1 per unit to 1 per unit at 0.2 s.

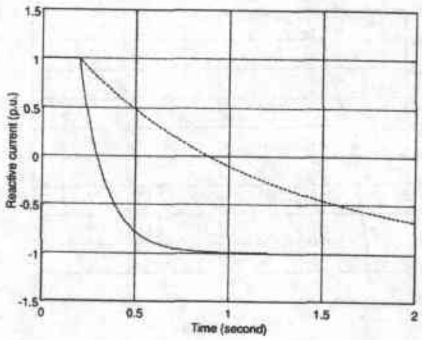

Fig. 5 I_q response (negative transition)

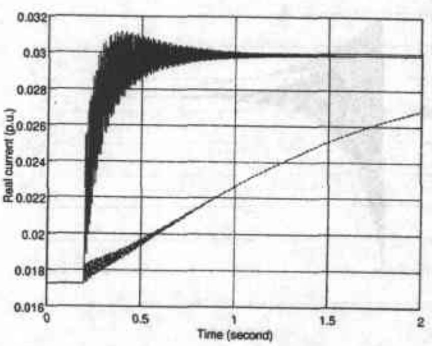

Fig. 6 I_d response (negative transition)

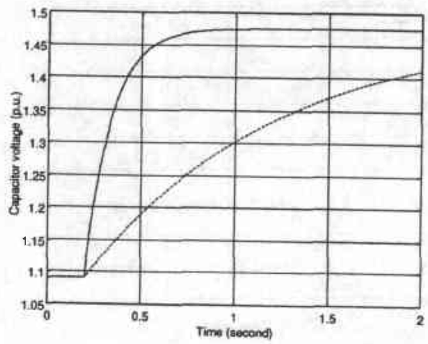

Fig. 7 V_{dc} response (negative transition)

It is clearly shown that the transient responses follow the step change of references quickly under the proposed nonlinear controller. All transients reach steady state within 0.5 second. On the other hand, the responses are rather slow under the PI controller, although they approach steady-state finally. From Figs. 6 and 9, we see that the real current exhibits high frequency oscillation before it reaches steady-state. Also note that the dynamic behavior of positive transition is different from negative transition. The system is less damped during positive transition and the real power oscillation is severer. In both cases, the reactive current and capacitor voltage are asymptotically stable.

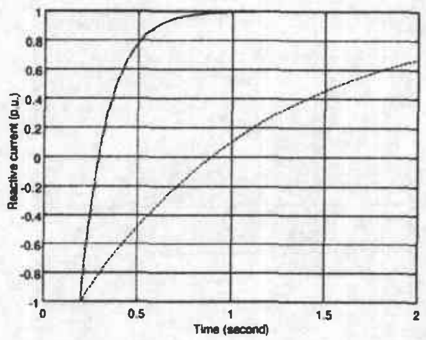

Fig. 8 I_q response (positive transition)

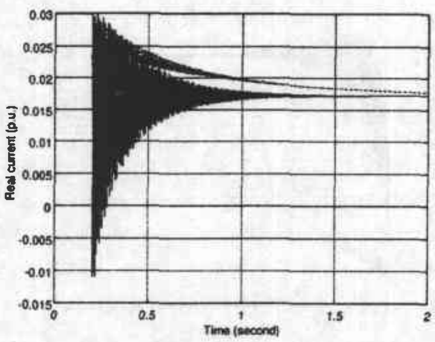

Fig. 9 I_d response (positive transition)

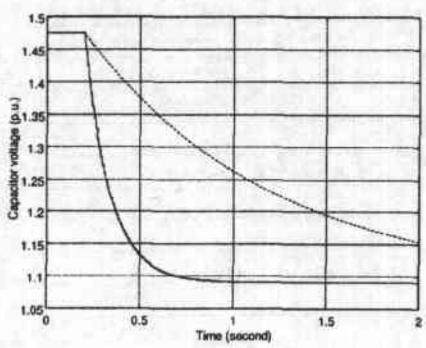

Fig. 10 V_{dc} response (positive transition)

5. Conclusion

In this paper, a reactive current model of the STATCOM is studied. A Lyapunov-based reactive current controller is designed. Global asymptotic stability is guaranteed by the controller. Simulation results show that the reactive current tracks the reference value quickly and asymptotically. The performance of the proposed nonlinear controller is better than the conventional PI controller.

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