

# UNCONDITIONAL MAXIMUM LIKELIHOOD APPROACH FOR CHANNEL ESTIMATION IN OFDM SYSTEMS

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## ABSTRACT

In this paper, unconditional maximum likelihood approach for channel estimation in OFDM systems is proposed. This method is based on the idea of fixed point algorithm that is using the property of digital modulated signals getting values from finite alphabet. This algorithm jointly estimates channel parameters by unconditional maximum likelihood estimation and gives maximum a posteriori (MAP) estimates of the digital modulated signals. Performance analysis of the proposed algorithm is then carried out through the evaluation of Cramer-Rao bounds. Finally, simulation results are presented to demonstrate the applicability and effectiveness of the proposed unconditional maximum likelihood method.

## 1. INTRODUCTION

It is widely accepted that the need for high bit rate wireless communication systems will reach its peak in the years to come. However, despite of the huge progress achieved, the peak nominal rates will be confined factors such as multipath propagation and unwanted inter and intracell interference. OFDM is a strong candidate to mitigate the effects of these confined factors in wireless environment. OFDM, sometimes referred to as multi-carrier or discrete multi-tone modulation, utilizes multiple subcarriers to transport information in from one particular user to another. An OFDM-based system divides a high-speed serial information signal into

multiple lower-speed sub-signals that the system transmits simultaneously at different frequencies in parallel. The orthogonal nature of OFDM allows subchannels to overlap, having a positive effect on spectral efficiency. Each one of the subcarriers transporting information are just far enough apart from each other to theoretically avoid interference. The benefits of OFDM are high spectral efficiency, resiliency to RF interference, and lower multi-path distortion. To address frequency selectivity, we propose to use OFDM.

Blind estimation of channel parameters along with the transmitted signals in OFDM systems is an important problem. When developing a ML estimator, there are two approaches in the literature: a deterministic or conditional maximum likelihood estimator and unconditional maximum likelihood estimator. The first estimate assumes the source signals are unknown but nonrandom signals. This estimate is referred to as the conditional (or deterministic) maximum likelihood (CML) estimate. The second estimate assumes the source signals are sample functions from Gaussian random processes. This estimate is referred to as the unconditional (or stochastic) maximum likelihood (UML) estimate [1],[2]. Blind estimation algorithms based on the conditional signal model (i.e., treating the digital signals as deterministic sequences) have been proposed in [3], [4], [5] for different types of estimation problems. In this paper, a maximum likelihood approach that is treating the transmitted signals as stochastic IID sequences against the estimation method based on the conditional signal model is proposed. The effective solution of the unconditional maximum likelihood function appears is also given by fixed point iteration algorithm. Moreover, this method gives maximum like-

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THIS WORK WAS SUPPORTED BY THE RESEARCH FUND OF THE UNIVERSITY OF ISTANBUL, PROJECT NUMBER: T-130/11112002.

likelihood estimation of channel parameters and the maximum a posteriori (MAP) estimates of the modulated signals.

## 2. OFDM SIGNAL MODEL

An OFDM system decomposes the available bandwidth into  $N$  overlapping narrow frequency bands. The effective symbol length is  $T = KT_s$ , where  $T_s$  is the sampling period of the system. The channel  $g(\tau; t)$  is assumed to be slowly fading and we consider it to be constant during one OFDM symbol. Furthermore, we assume that the use of a cyclic prefix (CP) of length  $T_{cp}$  both preserves the orthogonality of the subchannels and eliminates intersymbol interference (ISI) between consecutive OFDM symbols. In this case we can describe the system as a set of parallel Gaussian channels. The received signal on subchannel  $k$  can thus be described as

$$y_k = h_k x_k + v_k, \quad k = 0 \dots K - 1, \quad (1)$$

where  $x_k$  is the transmitted symbol,  $v_k$  is additive channel noise and

$$h_k = G\left(\frac{k}{KT_s}; t\right), \quad k = 0 \dots K - 1, \quad (2)$$

is the attenuation at subcarrier  $k$  and  $G(f; t)$  is the frequency response of the channel  $g(\tau; t)$  during the OFDM symbol at time  $t$ . The signal model in (1) can be written in matrix form as follows:

$$\mathbf{y}(n) = \mathbf{H} \cdot \mathbf{x}(n) + \mathbf{v}(n) \quad 0 < n < N - 1 \quad (3)$$

where

$$\begin{aligned} \mathbf{y}(n) &= [y_0(n), \dots, y_{K-1}(n)]^T, \\ \mathbf{x}(n) &= [x_0(n), \dots, x_{K-1}(n)]^T \text{ and} \end{aligned}$$

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & \dots & 0 \\ 0 & h_1 & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & h_{K-1} \end{bmatrix} \quad (4)$$

The main problem in this paper is to estimate channel parameters  $\boldsymbol{\theta} = [h(0) \ h(1) \dots h(N-1)]^T$  from the signal  $\mathbf{y} = [\mathbf{y}(0) \ \mathbf{y}(1) \dots \mathbf{y}(N-1)]^T$  that is distorted by additive Gaussian noise. Unconditional maximum likelihood estimation method which is asymptotically efficient will be used for the solution of the problem. We therefore develop the signal model of the problem first in the following subsection.

### 2.1. Unconditional Signal Model

The only difference between the conditional and unconditional models is the assumption regarding the signal vectors  $\mathbf{x}(n)$ s. In the conditional signal model the signal vectors are treated as unknown but deterministic quantities and are part of the set of unknown parameters. Hence, the number of unknown parameters increases linearly with the increase in the number of data vectors which results in inconsistent estimates. In contrast, under the unconditional signal model the signal vectors are treated as random quantities and are not included in the parameter set. As a result, the number of unknown parameters is fixed and it is possible to obtain consistent estimates. For the problem at hand, the signal vectors are assumed to be temporally independent vectors of  $\pm 1$  (BPSK). Let  $S = \{s_m\}$ ,  $m = 1, \dots, 2^K$ , be the set of all possible  $K$  vectors of  $\pm 1$ , i.e., the set  $\mathbf{S}$  represents the  $K$  dimensional binary constellation. Using the temporal whiteness of the signal and noise vectors, the probability of the data matrix  $\mathbf{A}$  can be written in terms of the constellation vectors as

$$f_{\mathbf{H}}(\mathbf{A}) = \frac{1}{2^{KN} (\pi\sigma^2)^{KN}} \prod_{n=1}^N \sum_{m=1}^{2^K} \exp\left\{-\frac{\|\mathbf{y}(n) - \mathbf{H}\mathbf{s}_m\|^2}{\sigma^2}\right\} \quad (5)$$

Note that the density of  $\mathbf{A}$  is a finite mixture of complex normal densities where the family of distributions  $f_{\mathbf{H}}(\cdot)$  is indexed by the unknown parameter matrix  $\mathbf{H} \in \mathcal{C}^{K \times 1}$ .

For the unconditional signal model (5), the negative log-likelihood function is given by

$$\mathcal{L}(\mathbf{H}) = - \prod_{n=1}^N \log \sum_{m=1}^{2^K} \exp\left\{-\frac{\|\mathbf{y}(n) - \mathbf{H}\mathbf{s}_m\|^2}{\sigma^2}\right\} + const. \quad (6)$$

and the UMLE of  $\mathbf{H}$  is the global minimizer of  $\mathcal{L}(\mathbf{H})$ . It is unlikely that a globally convergent algorithm for this highly nonlinear cost function (6) exists. However, a locally convergent algorithm can be formulated based on the first order likelihood equations

$$\frac{\partial \mathcal{L}(\mathbf{H})}{\partial \mathbf{H}} = 0 \quad (7)$$

where the  $i, j$ th element of  $\partial \mathcal{L}(\mathbf{H}) / \partial \mathbf{H}$  is  $\partial \mathcal{L}(\mathbf{H}) / \partial \mathbf{H}_{i,j}$ .

By the help of the fixed point iteration algorithm the UMLE expression for  $\mathbf{H}$  can be written as

$$\mathbf{H}_u \left( \sum_{n=1}^N \sum_{m=1}^{2^K} p_m(n) \mathbf{s}_m \mathbf{s}_m^T \right) = \sum_{n=1}^N \sum_{m=1}^{2^K} p_m(n) \mathbf{y}(n) \mathbf{s}_m^T \quad (8)$$

where

$$p_m(n) = \frac{\exp\{-\frac{1}{\sigma^2} \|\mathbf{y}(n) - \mathbf{H}\mathbf{s}_m\|^2\}}{\sum_{l=1}^{2^K} \exp\{-\frac{1}{\sigma^2} \|\mathbf{y}(n) - \mathbf{H}\mathbf{s}_l\|^2\}} \quad (9)$$

is the a-posteriori probability that  $\mathbf{x}(n) = \mathbf{s}_k$  given  $\mathbf{x}(n)$ .

The solution steps of nonlinear set of equations in (7) by using fixed point iteration is listed below.

## 2.2. Proposed Algorithm

Fixed Point technique (FPT)

1. The algorithm starts with an initial estimate  $\mathbf{H}^{(0)}$
2. For  $i = 1, 2, \dots$ , compute

$$\mathbf{H}_u^{(i+1)} \left( \sum_{n=1}^N \sum_{m=1}^{2^K} p_m^{(i)}(n) \mathbf{s}_m \mathbf{s}_m^T \right) = \sum_{n=1}^N \sum_{m=1}^{2^K} p_m^{(i)}(n) \mathbf{y}(n) \mathbf{s}_m^T \quad (10)$$

where

$$p_m^{(i)}(n) = \frac{\exp\{-\frac{1}{\sigma^2} \|\mathbf{y}(n) - \mathbf{H}^i \mathbf{s}_m\|^2\}}{\sum_{l=1}^{2^K} \exp\{-\frac{1}{\sigma^2} \|\mathbf{y}(n) - \mathbf{H}^i \mathbf{s}_l\|^2\}} \quad (11)$$

Evaluate  $\mathbf{H}_u^{(i+1)}$  from set of equations in (10).

3. Repeat step 2 until  $|\mathcal{L}(\mathbf{H}^{(i+1)}) - \mathcal{L}(\mathbf{H}^{(i)})| < \epsilon$  where  $\epsilon$  is a pre-specified tolerance parameter.
4. For  $n = 1, \dots, N$ , find  $m_n = \operatorname{argmax}_m p_m^f(n)$ , where  $p_m^f(n)$  denotes the final a posteriori probability. The MAP estimates of the signal vectors are  $\mathbf{x}(n) = \mathbf{s}_{m_n}$ .

## 3. PERFORMANCE ANALYSIS

The Cramer-Rao bound (CRB) is a lower bound on the covariance matrix of any unbiased estimator. Suppose,  $\hat{\boldsymbol{\theta}}$  is an unbiased estimator of a vector of deterministic unknown parameters  $\boldsymbol{\theta}$  (i.e.,  $E\{\hat{\boldsymbol{\theta}}\} = \boldsymbol{\theta}$ ) then the estimator's covariance matrix satisfies

$$\mathbf{J}^{-1}(\boldsymbol{\theta}) \leq E\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T\} \quad (12)$$

where  $\mathbf{J}_{k,l}(\boldsymbol{\theta}) = -E\{\partial^2 \log f_{\boldsymbol{\theta}}(\mathbf{y}) / \partial \theta_k \partial \theta_l\}$  is the (k,l)th entry of the Fisher information matrix (FIM) and  $f_{\boldsymbol{\theta}}(\mathbf{y})$  stands for the likelihood function of  $\mathbf{y}$ .

Under the assumption that  $v_k$  is zero-mean white Gaussian noise with covariance matrix  $\sigma^2 \mathbf{I}$ , the likelihood function  $f_{\boldsymbol{\theta}}(\mathbf{y})$  can be obtained after averaging  $f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})$  over  $\mathbf{x}$ :  $f_{\boldsymbol{\theta}}(\mathbf{y}) = \mathbf{E}_{\mathbf{x}}[f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})]$  The exact CRB is then given by  $\mathbf{J}_{k,l}(\boldsymbol{\theta}) = -E\{\partial^2 \log E_{\mathbf{x}}[f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})] / \partial \theta_k \partial \theta_l\}$ . The evaluation of the exact CRB requires

the Hessian matrix for  $\log E_{\mathbf{x}}[f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})]$ , which is analytically intractable due to the nature of  $f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})$ . However, it is common to adopt (see, e.g., [6]) an approximate bound which is not as tight as the exact CRB, but computationally easier to evaluate. Due to the concavity of the log function and Jensen's inequality, we obtain the following valid CRB:

$$\mathbf{J}_{k,l}(\boldsymbol{\theta}) \leq -E_{\mathbf{x}} E\{\partial^2 \log[f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})] / \partial \theta_k \partial \theta_l\} \quad (13)$$

where

$$\log[f_{\boldsymbol{\theta}}(\mathbf{y} | \mathbf{x})] = -KN \log(\pi \sigma^2) - \frac{1}{\sigma^2} \sum_{n=0}^{N-1} \|\mathbf{y}(n) - \mathbf{H}\mathbf{x}(n)\|^2 \quad (14)$$

The entries of the FIM are obtained from (13) and (14) and  $\mathbf{J}_{\mathbf{H},\mathbf{H}} = \frac{1}{\sigma^2} \sum_{n=0}^{N-1} E_{\mathbf{x}}[\mathbf{x}(n)\mathbf{x}^H(n)]$  can be obtained.

## 4. SIMULATION RESULTS

In this section a simulation example is presented to demonstrate the improved performance of the UML approach. A BPSK OFDM system with 12 subchannels and 2 cyclic prefixes is considered. The observation interval is chosen as  $N = 8$ . The root mean square (rms) width is assumed to be  $\tau_{rms} = 0.2\mu s$  for the power-delay profile of the channel and the channel correlation between the attenuations  $h_m$  ve  $h_n$  is given as

$$r_{m,n} = \frac{1 - e^{-L((1/\tau_{rms}) + 2\pi j(m-n)/N)}}{\tau_{rms}(1 - e^{-(L/\tau_{rms})})(\frac{1}{\tau_{rms}} + j2\pi \frac{m-n}{N})} \quad (15)$$

in [7].

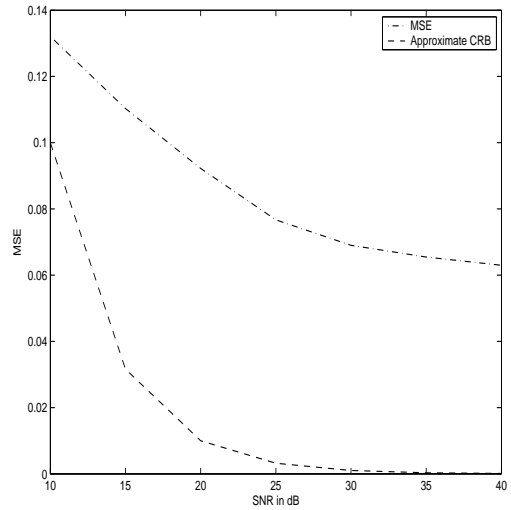


Figure 1: Performance of the proposed algorithm

The proposed fixed point iteration channel estimation algorithm is tested for the signal to noise ratios between the 10dB-40dB range. The algorithm is tested for each SNR with 50 Monte Carlo realizations and mean square error is recorded for each realization of the algorithm. The results are presented in Figure 1. It is observed from this figure that mean square error decreases with high signal to noise ratios. At the same time it is observed that the CRB plot which is derived from approximate logarithmic likelihood formula is loosely coupled for higher SNR values.

## 5. REFERENCES

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