H_∞ Observer Design for Linear Time-Delay Systems

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Abstract

The paper present the design of an observer for a general class of systems with delays in states. A state space model of observer with delays is proposed. The novelty of the study is to include the state derivatives in the design. The stability of the observer is proved by Lyapunov approach. Linear Matrix Inequality (LMI) approach is used in the analysis of the problem. Numerical examples are studied to see the validity of the approach.

1. Introduction

Time-delay system (TDS) is a system having delays in its states, inputs or outputs and occurs in many natural and engineering events. Time-delay is commonly encountered in chemical processes, biological systems, long transmission line of pneumatic, hydraulic system, steel rolling mills, space missions and usually a very common source of instability.Time-delay systems actually belongs to the class of functional differential equation (FDE),which has infinite dimensions making it more complex. Both to analysis and designs taking into the consideration of deviating arguments or differential difference term is necessary for engineers to make models to behave like more to real process.

H_∞ state observer problem has been studied for many years in order to improve satisfactory observer action under exogenous disturbance. Several methods for H_∞ observer design for time delay system such as Lyapunov–Krasovskii approach, algebraic Riccati Equation approach, Fattouh *et el* method has been discussed in [1].The delay-dependent design methods which are suitable for systems with time delay being of known size have been proposed in [2,3] based on Riccati Equation approach.A Lyapunov approach to design an observer for discrete time system in terms of Riccati Equation has been proposed in [4]. Based on Lyapunov stability theory, the design of observer with internal delay and unknown input is formulated in terms of Linear Matrix Inequality (LMI) in [5] and authors developed a delay independent matrix representation. A reset observer framework has been proposed in [6] for linear time-delay systems to improve settling time and overshoot performance. It is well known that H_∞ filtering problem is dual to the H_∞ control one for linear systems without uncertainty. H_∞ Controller (observer) design procedure has been proposed and developed in [7, 8, 9, 10, 11], which could be adopted for observer design too because of duality.

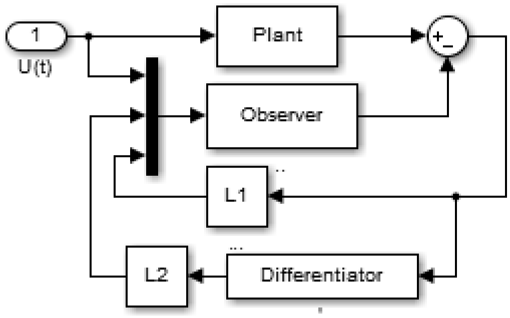

Fig.1. Block diagram of proposed observer

To design an observer for TDSS we use simple Luenberger approach, but we introduced here two feedback line instead of one. The first feedback line contains a proportional gain matrix and second feedback line has a gain matrix (given) followed by a differentiator block. So here we are considering not only the difference between real states and estimator states or error signals but also the rate of change of error signals. Taking into consideration both error and rate of change of error data would make the observer more reliable than simple Luenberger type one.

2. Problem Formulation

Consider the following linear time-delay system,

$$\dot{x}(t)=Ax(t)+A_dx(t-h)+Bu(t)+Nw(t)$$
$$y=Cx(t)$$
$$x(t+\theta)=\varphi(\theta) \quad \forall\, \theta\epsilon[-h,0] \tag{1}$$

where ,

$x(t)\epsilon\mathbf{R}^n$: The State vector.

$w(t)\epsilon\mathbf{R}^q$: The exogenous disturbance input which belongs to $L_2[0,\infty)$.

$y(t)\epsilon\mathbf{R}^p$: The output vector.

A, $A_d\,\epsilon\mathbf{R}^{n\times n}$, $B\,\epsilon\mathbf{R}^{n\times q}$, $N\,\epsilon\mathbf{R}^{n\times k}$, $C\,\epsilon\mathbf{R}^{p\times n}$.

The above matrices are constant and known system matrices.

$h>0$:a positive scalar denoting the time delay.

$\varphi(.)$: a continuously differentiable function on $[-h,0]$ representing the initial condition.

3. Main Results

Let us formulate an observer dynamics as follows,

$\dot{\hat{x}}(t)=F\hat{x}(t)+G\hat{x}(t-h)+Hu(t)+Mw(t)+L_1(y(t)-\hat{y}(t))+L_2(\dot{y}(t)-\dot{\hat{y}}(t))$
$\hat{y}(t)=C\hat{x}(t)$ (2)

Where,
$\hat{x}(t)\epsilon\mathbf{R}^n$: The estimator state vector

$L_1,L_2\epsilon\mathbf{R}^{n\times p}$:The constant observer gain matrix to be selected appropriately.
$\hat{y}(t)\epsilon\mathbf{R}^p$:The estimated output vector.

F, G $\epsilon\mathbf{R}^{n\times n}$, H $\epsilon\mathbf{R}^{n\times q}$, M $\epsilon\mathbf{R}^{n\times k}$, C $\epsilon\mathbf{R}^{p\times n}$.

Thorem: Observer in form of (2) can be constructed if there exists matrices $P=P^T>0$, $R_1=R_1^T>0$, $R_2=R_2^T>0$ and X for a given noise attenuation level γ, satisfying the following LMI

$$\begin{bmatrix} \Omega_2 & hZ^{-T}PA_dZ & hZ^{-T}PA_dZ & hA_d{}^T & h(A^TP-C^TX^T) & 0 \\ hZ^TA_d{}^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TA_d{}^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hA_d & 0 & 0 & h(R_2-2I) & 0 & 0 \\ h(PA-XC) & 0 & 0 & 0 & h(R_1-2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (3)$$

where $\Omega_2=(A^TPZ^{-1}-C^TX^TZ^{-1}+A_d^TPZ^{-1}+Z^{-T}PA-Z^{-T}XC+Z^{-T}PA_d+C^TC)$

3.1. Proof:

Subtracting equation (2) from equation (1) we get,

$\dot{x}(t)-\dot{\hat{x}}(t) = Ax(t)+A_dx(t-h)+Bu(t)+Nw(t)-F\hat{x}(t)-G\hat{x}(t-h)$
$-Hu(t)-Mw(t)-L_1(y(t)-\hat{y}(t))-L_2(\dot{y}(t)-\dot{\hat{y}}(t))$

$\dot{e}(t) =Ax(t)+A_dx(t-h)+Bu(t)+Nw(t)-F\hat{x}(t)-G\hat{x}(t-h)-Hu(t)$
$-Mw(t)-L_1(y(t)-\hat{y}(t))-L_2(\dot{y}(t)-\dot{\hat{y}}(t))+Fx(t)+Gx(t-h)$
$-Fx(t)-Gx(t-h)$

$\dot{e}(t) = (A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)+(N-M)w(t)$
$+F(x(t)-\hat{x}(t))+G(x(t-h)-\hat{x}(t-h))-L_1(Cx(t)-C\hat{x}(t))$
$-L_2(C\dot{x}(t)-C\dot{\hat{x}}(t))$

$\dot{e}(t) = (A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)+(N-M)w(t)$
$+Fe(t)+Ge(t-h)-L_1C(x(t)-\hat{x}(t))-L_2C(\dot{x}(t)-\dot{\hat{x}}(t))$

$\dot{e}(t)+L_2C\dot{e}(t)= (A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)+(N-M)w(t)$
$+Fe(t)+Ge(t-h)-L_1Ce(t)$

$(I+L_2C)\dot{e}(t) = (A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)$
$+(N-M)w(t)+(F-L_1C)e(t)+Ge(t-h)$

$\dot{e}(t) =(I+L_2C)^{-1}[(A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)$
$+(N-M)w(t)+(F-L_1C)e(t)+Ge(t-h)]$

$\dot{e}(t) = Z[(A-F)x(t)+(A_d-G)x(t-h)+(B-H)u(t)$
$+(N-M)w(t)+(F-L_1C)e(t)+Ge(t-h)]$

Where, $Z=(I+L_2C)^{-1}$
Here, we will choose L_2 arbitrarily and calculate the gain L_1 accordingly.

Obviously,

$e(t)\to 0$ as $t\to\infty$ if the following conditions are satisfied:

(1) The system is stable and observable.
(2) $(I+L_2C)$ is invertible.
(3) $A=F$,
$A_d=G$,
$B=H$,
$N=M$, then the error dynamics reduces to,

$\dot{e}(t) =(I+L_2C)^{-1}[(F-L_1C)e(t)+Ge(t-h)]$

$\dot{e}(t) = Z[(F-L_1C)e(t)+Ge(t-h)]$ (4)

We will utilize following the Leibniz rule

Lemma 1: $A(t-h) = A(t)-\int_{t-h}^{t}\dot{A}(\alpha)d\alpha$

We will also use the following lemma in our proof

Lemma 2: $-2U^TV\leq U^TRU+V^TR^{-1}V$

Then we have the error dynamics as follows,

$\dot{e}(t) = Z[(F-L_1C)e(t)+Ge(t-h)]$

Using Leibniz rule given in Lemma 1, we can write,

$e(t-h) = e(t)-\int_{t-h}^{t}\dot{e}(\alpha)d\alpha$

$= e(t)-\int_{t-h}^{t}Z[(F-L_1C)e(\alpha)+Ge(\alpha-h)]\,d\alpha$

$\dot{e}(t) =Z(F-L_1C)e(t)$
$+ZG\{e(t)-\int_{t-h}^{t}Z[(F-L_1C)e(\alpha)+Ge(\alpha-h)]\,d\alpha\}$

The error dynamics (4) is now transformed into the following equation.

$$\dot{e}(t) = Z[(F-L_1C)+G]e(t)$$

$$-ZGZ\int_{-h}^{0}[(F-L_1C)e(t+\alpha)+Ge(t+\alpha-h)]\,d\alpha \tag{5}$$

$\dot{e}(t)\rightarrow 0$ as $t\rightarrow \infty$ means error in (5) tends to '0' as time evolves.

Delay-dependent approach: Consider the following Lyapunov–Krasovskii functional

$$V(e,t)= e(t)^T Z^{-T}PZ^{-1} e(t)$$

$$+\int_{-h}^{0}\int_{t+\theta}^{t} e(\theta)^T(F-L_1C)^T R_1 (F-L_1C)e(\theta)\,d\theta.ds$$

$$+\int_{-h}^{0}\int_{t-h+\theta}^{t-h} e(\theta)^T G^T R_2 G\, e(\theta)\,d\theta.ds$$

$$\dot{V}(e,t) = \dot{e}(t)^T Z^{-T}PZ^{-1}e(t)+ e(t)^T Z^{-T}PZ^{-1}\dot{e}(t)$$

$$+h\, e(t)^T (F-L_1C)^T R_1 (F-L_1C)\, e(t)$$

$$-\int_{t-h}^{t} e(\theta)^T(F-L_1C)^T R_1 (F-L_1C)e(\theta)\,d\theta$$

$$+ h\, e(t)^T G^T R_2 G e(t)$$

$$-\int_{t-h-h}^{t-h} e(\theta)^T G^T R_2 G e(\theta)d\theta$$

$$= e(t)^T[(F-L_1C)+G]^T Z^T Z^{-T}PZ^{-1}e(t)$$

$$+ e(t)^T Z^{-T}PZ^{-1}Z[(F-L_1C)+G]\, e(t)$$

$$-2e(t)^T Z^{-T}PZ^{-1}ZGZ\int_{-h}^{0}[(F-L_1C)e(t+\theta)+Ge(t+\theta-h)]d\theta$$

$$+ h\, e(t)^T(F-L_1C)^T R_1(F-L_1C)e(t)$$

$$-\int_{t-h}^{t} e(\theta)^T(F-L_1C)^T R_1(F-L_1C)e(\theta)d\theta$$

$$+ h\, e(t)^T G^T R_2 G e(t)- \int_{t-h-h}^{t-h} e(\theta)^T G^T R_2 G e(\theta)d\theta$$

$$\leq e(t)^T[(F-L_1C)+G]^T PZ^{-1}e(t)+ e(t)^T Z^{-T}P[(F-L_1C)+G]\, e(t)$$

$$+\int_{-h}^{0} e(t)^T Z^{-T}PGZR_1^{-1} Z^T G^T PZ^{-1}e(t)d\theta$$

$$+\int_{-h}^{0} e(t+\theta)^T(F-L_1C)^T R_1(F-L_1C)e(t+\theta)d\theta$$

$$+\int_{-h}^{0} e(t)^T Z^{-T}PGZR_2^{-1} Z^T G^T PZ^{-1}e(t)d\theta$$

$$+\int_{-h}^{0} e(t+\theta-h)^T G^T R_2 Ge(t+\theta-h)d\theta$$

$$+ h\, e(t)^T(F-L_1C)^T R_1(F-L_1C)e(t) + h\, e(t)^T G^T R_2 G e(t)$$

$$-\int_{-h}^{0} e(t+\theta)(F-L_1C)^T R_1(F-L_1C)e(t+\theta)d\theta$$

$$-\int_{-h}^{0} e(t+\theta-h)^T G^T R_2 G e(t+\theta-h)d\theta$$

$$\leq e(t)^T[F^T PZ^{-1} -C^T L_1^T PZ^{-1}+G^T PZ^{-1}+ Z^{-T}PF- Z^{-T}PL_1C+ Z^{-T}PG]e(t)$$

$$+ h\, e(t)^T Z^{-T}PGZR_1^{-1}Z^T G^T PZ^{-1}e(t)+ h\, e(t)^T Z^{-T}PGZR_2^{-1}Z^T G^T PZ^{-1}e(t)$$

$$+ h\, e(t)^T(F-L_1C)^T R_1(F-L_1C)e(t)+ h\, e(t)^T G^T R_2 G e(t)$$

Now applying Schur complement we get,

$$\leq e(t)^T \begin{bmatrix} \Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^T \\ hZ^T G^T PZ^{-1} & -hR_1 & 0 & 0 & 0 \\ hZ^T G^T PZ^{-1} & 0 & -hR_2 & 0 & 0 \\ hG & 0 & 0 & -hR_2^{-1} & 0 \\ h(F-L_1C) & 0 & 0 & 0 & -hR_1^{-1} \end{bmatrix} e(t)$$

Here, $\Omega= (F^T PZ^{-1} -C^T L_1^T PZ^{-1}+G^T PZ^{-1}+ Z^{-T}PF- Z^{-T}PL_1C+ Z^{-T}PG)$

If above matrix is less than 0,then $\dot{V}(e,t)$ is negative so $e(t)\rightarrow 0$ as $t\rightarrow \infty$.

$$\begin{bmatrix} \Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^T \\ hZ^T G^T PZ^{-1} & -hR_1 & 0 & 0 & 0 \\ hZ^T G^T PZ^{-1} & 0 & -hR_2 & 0 & 0 \\ hG & 0 & 0 & -hR_2^{-1} & 0 \\ h(F-L_1C) & 0 & 0 & 0 & -hR_1^{-1} \end{bmatrix} < 0 \tag{6}$$

Pre and post multiplying (6) by diag {I,I,I,I,P} and replacing $-hR_2^{-1}$ by $h(R_2-2I)$ as we know $-R_2^{-1}<(R_2-2I)$,we get

$$\begin{bmatrix} \Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^T P \\ hZ^T G^T PZ^{-1} & -hR_1 & 0 & 0 & 0 \\ hZ^T G^T PZ^{-1} & 0 & -hR_2 & 0 & 0 \\ hG & 0 & 0 & -hR_2^{-1} & 0 \\ hP(F-L_1C) & 0 & 0 & 0 & -hPR_1^{-1}P \end{bmatrix} < 0$$

We can replace $-hPR_1^{-1}P$ by $h(R_1-2P)$ as, $h(R_1-P)R_1^{-1}(R_1-P)>0$

$hR_1-hP-hP+ hPR_1^{-1}P>0$ then $-hPR_1^{-1}P<h(R_1-2P)$

$$\begin{bmatrix} \Omega & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F-L_1C)^T P \\ hZ^T G^T PZ^{-1} & -hR_1 & 0 & 0 & 0 \\ hZ^T G^T PZ^{-1} & 0 & -hR_2 & 0 & 0 \\ hG & 0 & 0 & h(R_2-2I) & 0 \\ hP(F-L_1C) & 0 & 0 & 0 & h(R_1-2P) \end{bmatrix} < 0$$

Now let PL_l=X and defining the matrix (right hand side of the equation) as Σ,

$$\Sigma = \begin{bmatrix} \Omega_1 & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F^TP - C^TX^T) \\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0 \\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0 \\ hG & 0 & 0 & h(R_2 - 2I) & 0 \\ h(PF - XC) & 0 & 0 & 0 & h(R_1 - 2P) \end{bmatrix} < 0$$

Here $\Omega_1=(F^TPZ^{-1} - C^TX^TZ^{-1}+G^TPZ^{-1}+ Z^{-T}PF- Z^{-T}XC+ Z^{-T}PG)$

For H_∞ observer,it has to satisfy the following equation,

$$\int_0^\infty [\dot{V}(e,t)+z(t)^Tz(t)-\gamma^2w(t)^Tw(t)]dt<0 \qquad (7)$$

If $\dot{V}(e,t)+z(t)^Tz(t)-\gamma^2w(t)^Tw(t)<0$ then (7) will be true also.

$$e(t)^T \Sigma e(t)+ e(t)^TC^TCe(t))-\gamma^2w(t)^Tw(t)<0 \qquad [\text{here, } z=y(t)- \hat{y}(t)= Cx(t)- C\hat{x}(t)= Ce(t)] \qquad (8)$$

if $\boldsymbol{\zeta}(t)=[e(t) ; w(t)]$ and applying Schur complement to (8) ,

$$\boldsymbol{\zeta}(t)^T \begin{bmatrix} \Omega_2 & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F^TP - C^TX^T) & 0 \\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hG & 0 & 0 & h(R_2 - 2I) & 0 & 0 \\ h(PF - XC) & 0 & 0 & 0 & h(R_1 - 2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix} \boldsymbol{\zeta}(t) < 0$$

where $\Omega_2=(F^TPZ^{-1} - C^TX^TZ^{-1}+G^TPZ^{-1}+ Z^{-T}PF- Z^{-T}XC+ Z^{-T}PG +C^TC)$

$$\begin{bmatrix} \Omega_2 & hZ^{-T}PGZ & hZ^{-T}PGZ & hG^T & h(F^TP - C^TX^T) & 0 \\ hZ^TG^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TG^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hG & 0 & 0 & h(R_2 - 2I) & 0 & 0 \\ h(PF - XC) & 0 & 0 & 0 & h(R_1 - 2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix} < 0$$

According to necessary condition, replacing F=A and G=A_d we get the following final LMI

$$\begin{bmatrix} \Omega_2 & hZ^{-T}PA_dZ & hZ^{-T}PA_dZ & hA_d{}^T & h(A^TP - C^TX^T) & 0 \\ hZ^TA_d{}^TPZ^{-1} & -hR_1 & 0 & 0 & 0 & 0 \\ hZ^TA_d{}^TPZ^{-1} & 0 & -hR_2 & 0 & 0 & 0 \\ hA_d & 0 & 0 & h(R_2 - 2I) & 0 & 0 \\ h(PA - XC) & 0 & 0 & 0 & h(R_1 - 2P) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix} < 0$$

where $\Omega_2=(A^TPZ^{-1} - C^TX^TZ^{-1}+A_d{}^TPZ^{-1}+ Z^{-T}PA- Z^{-T}XC+ Z^{-T}PA_d +C^TC)$

Solving the LMI for P and X we can get $L_l=P^{-1}X$.

4. Numerical Example

In this section, we will demonstrate the theory developed in this paper by means of simple examples. Here to solve problem we have used Matlab software,Yalmip Optimization Toolbox and Sedumi solver. Consider the linear continuous time-delay system (9) and (10) with parameters given by

Example (1): A= $\begin{bmatrix} -2 & -0.5 \\ 0.5 & -3 \end{bmatrix}$ A_d= $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ (9)

C= [1 0] L_2=[0.5 0.4]^T (chosen) (10)

Where 0 < h≤ 0.77 is an unknown positive scalar.

The purpose is to design H_∞observer using equation (3) according to the block diagram. The transfer function from exogenous disturbances to error state outputs meets the prescribed H_∞ norm upper bound constraint $||H_{yw}(s)||_\infty \leq 0.8$

Here, we take the value γ=0.3

Solving the LMI, we get P= $\begin{bmatrix} 1.0374 & -0.6950 \\ -0.6950 & 0.9064 \end{bmatrix}$

R_1= $\begin{bmatrix} 0.3822 & -0.2531 \\ -0.2531 & 0.2936 \end{bmatrix}$ R_2= $\begin{bmatrix} 0.9766 & -0.3675 \\ -0.3675 & 0.8124 \end{bmatrix}$

X= $\begin{bmatrix} 0.0918 \\ 0.2156 \end{bmatrix}$ L_1 = $\begin{bmatrix} 0.5095 \\ 0.6285 \end{bmatrix}$

Here in the example plant initial state is [5;-2] and estimator initial state is [0;0].

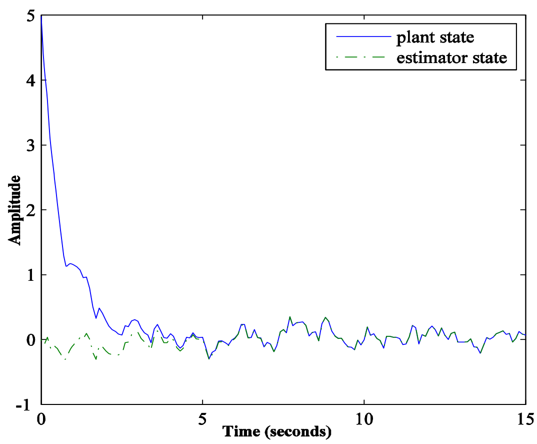

Fig.2. Trajectories of state x_1(t) and $\hat{x}_1$(t)

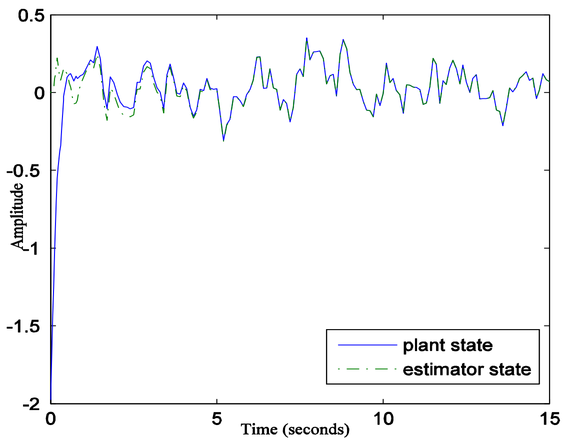

Fig.3. Trajectories of state x_2(t) and $\hat{x}_2$(t)

Example (2):

A= $\begin{bmatrix} -2.5 & 1.2 \\ -1.25 & -4.3 \end{bmatrix}$ Ad= $\begin{bmatrix} -2.3 & 1.5 \\ -1.4 & -3.2 \end{bmatrix}$ (11)

C= [0 1] L_2=[0.5 0.4]^T (chosen) (12)

Where 0 < h≤ 0.32 is an unknown positive scalar.

The transfer function from exogenous disturbances to error state outputs meets the prescribed H_∞ norm upper bound constraint $||H_{yw}(s)||_\infty \leq 0.8$

Here, we chose γ=0.3.

Solving the LMI, we can get the values as follows,

P= $\begin{bmatrix} 0.9438 & -0.0715 \\ -0.0715 & 0.9943 \end{bmatrix}$ R_1= $\begin{bmatrix} 0.9319 & -0.2139 \\ -0.2139 & 0.9992 \end{bmatrix}$

R_2= $\begin{bmatrix} 0.8275 & -0.0966 \\ -0.0966 & 1.1380 \end{bmatrix}$ X= $\begin{bmatrix} 2.5462 \\ 0.2880 \end{bmatrix}$

L_1 = $\begin{bmatrix} 2.7347 \\ 0.4862 \end{bmatrix}$

Here in the example plant initial state is [4;-3] and estimator initial state is [0;0].

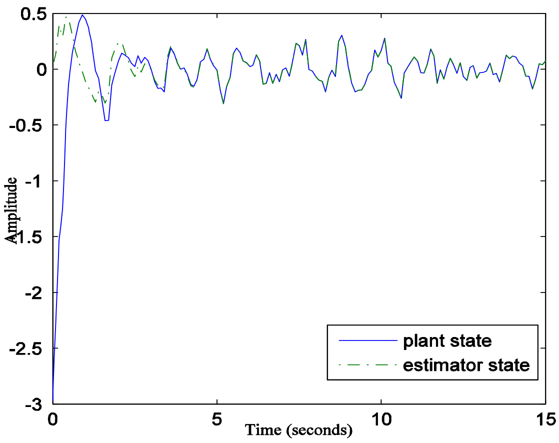

Fig.4. Trajectories of state $x_1(t)$ and $\hat{x}_1(t)$

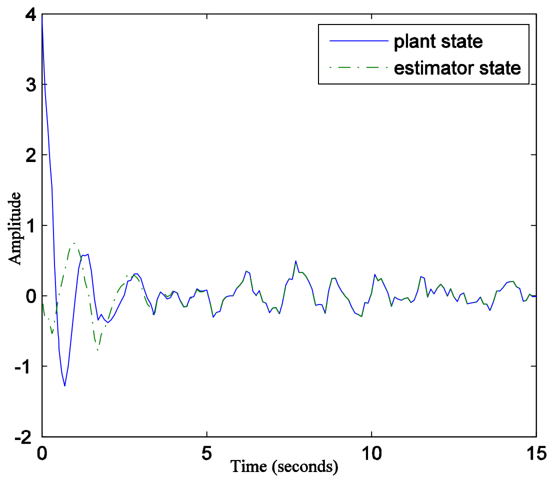

Fig.5. Trajectories of state $x_2(t)$ and $\hat{x}_2(t)$

From the simulation result shown on graphs,we can see that the trajectories of plant states and observer states converge within few seconds,which is pretty good performance by the observer designed using the method developed in this paper.

5. Conclusions

In this paper an observer design procedure for systems with delays in states has been studied. An appropriate gain matrix for observer is calculated while the gain matrix for differentiator block has been predetermined. Necessary and sufficient conditions have also been derived. Numerical examples provided here described the effectiveness of this method.

Future research will be focused on upgrading this observer dynamics into a delay free observer design.

6. References

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