# A NEW METHOD FOR OBTAINING THE SENTIVITIES OF PLANAR MICROSTRIP STRUCTURES BY A FULL-WAVE ANALYSIS

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Abstract - This paper, present the derivation of a new potential integral equation for the derivative of the surface current with respect to a geometrical parameter for microstrip structures embedded in multilayered substrate. This new equation integral is solved together with the original integral equation with the ameliorated moment method [9]. From the geometrical derivatives of the surfaces currents, geometrical derivatives of the S parameter are obtained.

#### Key words

Characterization , sensitivities, microstrip , Galerkin technique .

### I. INTRODUCTION

**A**CONSIDERABLE amount of work has been done in recent years on the full-wave analysis of planar microstrip structures as can be seen form the numerous publications[1-4] in this topic. Such a rigorous analysis is very often based on an integral equation formulation, typically solved with the method of the moments. In this work, we carry out a dynamic study of the analysis of the sensitivities of planar microstrip structure using the ameliorated moment method.

The shape sensitivities are obtained by the quasi-static method [5], or by using the method of the finite differences[2]. The first method provides only approached solution. The second method is not enough precise and the derivative or sensitivity, obtained with the new integral equation is superior with respect to a finite difference estimate [5]. The present paper presents the principles as well as a method for calculating the derivative of the S parameter with respect to an arbitrary geometrical parameter using the new potential integral equation as a full-wave analysis method. In this analysis, we have used the dynamic approach [6,8], based on the ameliorated moments method [9] and solution of Green functions for the dipole and miccrostrip antenna [7]. Expression for the matrix elements as a function of the basis and test functions is given.

### **II. THEORY**

The general geometry of planar microstrip circuit, embedded in multilayered substrate is depicted in Fig.1. The substrate consists of an arbitrary number of layers , stacked in the z – direction. Different types of geometrical

parameters appear in this multilayered planar microstrip structure: thicknesses of the substrate layers (such as  $h_1$ ,  $h_2$ ), distances between metalization surfaces (e.g.  $d_1$ ) and geometrical parameters pertaining to the surface itself like width or length (e.g.,  $L_1$ ,  $L_2$  or  $w_1$ ). We confine ourselves to the last types of geometrical parameters. Derivatives with respect to layer thicknesses will be the subject of forthcoming paper. The surface current is assumed to circulate only in the x direction in the strip [4,10] as shown in Fig.1.



Fig. 1. Example geometry of planar microstrip structure embedded in a substrate.

The plane-wave spectral representation of the grounded dielectric slab Green's function , representing the x-directed electrical field at(x, y, h) due to a an x-directed infinitesimal dipole of unit strength at  $(x_0, y_0, h)$ . The field element integral equation is expressed by [6,11]:

$$d\vec{E}(x, y / x_0, y_0) = \int_{-\infty}^{+\infty} Q(k_x, k_y) e^{jk_x(x - x_0)}$$

$$e^{jk_x(x - x_0)} \vec{J}(x_0, y_0) dk_x dk_y dx_0 dy_0$$
(1)

where  $Q(k_x, k_y)$  is given in [6] and J is given by:

$$J(x_0, y_0) = f(x_0)g(y_0)\vec{u}_x$$

where:

$$\begin{cases} f(x_0) = e^{-jk_e x_0} |x_0| < \infty \\ g(y_0) = \frac{1}{w} |y_0| < w/2 \end{cases}$$

The distribution of the transversal density current  $g(y_0)$  (in the y-direction ) is supposed uniform, it must verify the condition [6,15]:

$$\int_{-w/2}^{+w/2} g(y_0) dy_0 = 1 \Leftrightarrow g(y_0) = \frac{1}{w}$$

and it's Fourier transform F<sub>y</sub> is given by:

$$F_{y}(k_{y}) = \frac{2\sin(k_{y}w/2)}{k_{y}w}$$
(2)

### **II - A . INTEGRAL EQUATIONS**

From equation (1), the relation between the total incident tangential electrical field and the surface current is given by the potential equation integral defined by:

$$\vec{E}(x, y) = \iint_{x_0 y_0} G(x, y / x_0, y_0) \vec{J}(x_0, y_0) dx_0 dy_0 (3)$$

where G  $(x, y / x_0, y_0)$  is given by:

$$G(x, y/x_0, y_0) = - \int_{-\infty}^{+\infty} Q(k_x, k_y) e^{jk_x(x-x_0)} e^{jk_y(y-y_0)} dk_x dk_y$$
(4)

The total derivative of the surface current with respect to the geometrical parameter  $\xi$  ) is given by:

$$\vec{J}_{\xi}(x_0, y_0) = \left[\frac{\partial \vec{J}(x_0, y_0)}{\partial \xi} + (\vec{v} \cdot \vec{\nabla})\vec{J}(x_0, y_0)\right]$$
(5)

This derivative satisfies the following integral equation:

$$\frac{\delta E(x, y)}{\delta \xi} + (\vec{v}.\vec{\nabla})\vec{E}(x, y) = \int_{x_0 y_0} \int R(x, y / x_0, y_0)\vec{J}(x_0, y_0) dx_0 dy_0 \int_{x_0 y_0} \int G(x, y / x_0, y_0)\vec{J}_{\xi}(x_0, y_0) dx_0 dy_0$$
(6)

This new integral equation is obtained by applying the flux transport theorem [12,13] on the original equation integral (3).

Where  $R(x,y/x_0,y_0)$  is given by:

$$R(x,y/x_0,y_0) = \vec{\nabla}_0 [(\vec{v}_0 - \vec{v})G(x,y/x_0,y_0]$$
(7)

 $\vec{v}$  and  $\vec{v}_0$  are the velocity vectors given in [12].

### II-B. SOLUTION OF THE INTEGRAL EQUATIONSWHITH THE AMELIORATED MOMENTS METHOD

The two integral (3) and (6) will be solved with the method of moments. The integral equation (3) is written by imposing the boundary condition such as the total electrical field ,due to all the currents in the line , is null. This equation leads to :

$$\int_{x_{0}y_{0}} (I_{i} + I_{r} + I_{t} + \sum_{n=1}^{2N} I_{n}f_{n}) \int_{-\infty}^{+\infty} Q(k_{x}, k_{y})e^{jk_{x}(x-x_{0})}$$

$$e^{jk_{y}(y-y_{0})}dk_{x}dk_{y}dx_{0}dy_{0} = 0$$

$$for x < \infty \quad and \quad |y| < w / 2$$

$$(8)$$

Where  $f_n(x,y)$  is given in [6] and  $I_i$ ,  $I_r$  and  $I_t$  are the incident, the reflective and the transmitted current components given by:

$$I_i(x, y) = e^{-jkx}$$
(9)

$$I_i(x, y) = -Re^{jkx}$$
(10)

$$I_i(x,y) = Te^{-jkx}$$
(11)

R and T are the reflection and transmission coefficients. To analyze this sensitivity, three dominant modes are used in the representation of the incident, reflective and transmitted currents. The transmitted current  $I_t$  is introduced in the equation (3) with additional PWS modes that have to exist in  $x_n = -nd$  for n = 1, 2,...N and in  $x_n = nd$  for n=N+1, N+2,...2N. It results therefore 2N PWS modes. Equation (3) is thus modified knowing that R, T,  $I_n$  become 2N+2 unknowns in order that one can solve the equation (3) by using 2N+2 test functions[6]

$$\int_{-\infty}^{+\infty} Q(k,k) \sum_{m=1}^{2N+2} F_{xm}(k_x) \left[ (1-R) F_{xc}^*(k_x) - j(1+R) F_{xs}^*(k_x) + TF_{xct}^*(k_x) - jTF_{xst}^*(k_x) + \sum_{n=1}^{N} I_n F_{xn}^*(k_x) \right] F_y^2(k_y) dk_x dk_y = 0$$
(12)

Where  $F_{xm}$ ,  $F_{xn}$ ,  $F_{xm c}$ ,  $F_{xms}$ ,  $F_{xmet}$ ,  $F_{xmm}$  and  $F_{xmst}$  are defined in [6,14]. The substitution of the double summation in the resulting integral equation, allow us to define the impedance matrix by:

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & -(Z_{k}+jZ_{k}) & Z_{2kt}-jZ_{2kt} \\ Z_{21} & Z_{22} & \dots & -(Z_{k}+jZ_{k}) & Z_{2kt}-jZ_{kt} \\ \vdots & \dots & \vdots & \dots & \dots \\ Z_{2N+1}IZ_{2N+12} \dots -(Z_{2N+k}+jZ_{2N+k})Z_{2N+kt} & JZ_{2N+kt} - JZ_{2N+kt} \\ Z_{2N+2}Z_{1}Z_{N+22} \dots -(Z_{2N+2k}+jZ_{2N+2k})Z_{2N+2kt} - JZ_{2N+2kt} \\ \end{bmatrix} \begin{bmatrix} -Z_{k}+jZ_{k} \\ -Z_{2k}+jZ_{k} \\ \vdots \\ -Z_{2N+k}+jZ_{2N+k} \\ -Z_{2N+k}+jZ_{2N+k} \\ \end{bmatrix} = \begin{bmatrix} -Z_{k}+jZ_{k} \\ -Z_{2k}+jZ_{k} \\ -Z_{2N+k}+jZ_{2N+k} \\ -Z_{2N+k}+jZ_{2N+k} \\ \end{bmatrix}$$

$$(13)$$

Where the impedance matrix elements  $Z_{mn}$ ,  $Z_{ms}$   $Z_{mc}$ ,  $Z_{mst}$ ,  $Z_{mst}$  are given in [6].Using the lower upper decomposition technique for inversing the resulting impedance matrix, we can resolve the matrix equation (13). This expression allows obtaining the reflection coefficient R, the transmission coefficient T and the I<sub>n</sub> coefficients.

If the total electrical field ,due to all the currents in the line , is null. Equation (6) leads to :

$$\int_{x_{0}y_{0}} (I_{i} + I_{r} + I_{t} + \sum_{n=1}^{2N} I_{n}f_{n})R(x, y/x_{0}, y_{0})dk_{x}dk_{x}dx_{0}dy_{0}$$

$$+ \int_{x_{0}y_{0}} (I_{i}^{\xi} + I_{r}^{\xi} + I_{t}^{\xi} + \sum_{n=1}^{2N} I_{n}^{\xi}f_{n})C(x, y/x_{0}, y_{0})dk_{x}dk_{y}dx_{0}dy_{0} = 0$$
(14)

To solve the second integral equation (6), we enforced it with 2N+2 weighting or test functions .Then following the same method for calculating the Z matrix from (3), we will obtained W-matrix elements. The impedance matrix of (6) is given by:

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & -(Z_{1c} + jZ_{1s}) & Z_{2ct} - jZ_{2st} \\ Z_{21} & Z_{22} & \cdots & -(Z_{2c} + jZ_{2s}) & Z_{2ct} - jZ_{2st} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{2N+11} Z_{2N+12} \cdots -(Z_{2N+1c} + jZ_{2N+1S}) Z_{2N+1ct} - jZ_{2N+1st} \\ Z_{2N+21} Z_{12N+22} \cdots -(Z_{2N+2c} + jZ_{2N+2s}) Z_{2N+2ct} - jZ_{2N+2st} \end{bmatrix} \begin{bmatrix} I_{1}^{\xi} \\ I_{2}^{\xi} \\ \vdots \\ I_{2}^{\xi} \\ I_{2}^{\xi} \end{bmatrix} + \\ \begin{bmatrix} W_{11} & W_{12} & \cdots & -(W_{1c} + jZ_{1s}) & W_{2ct} - jW_{2st} \\ W_{21} & W_{22} & \cdots & -(W_{2c} + jW_{2s}) & W_{2ct} - jW_{2st} \\ \vdots & \vdots & \ddots & \vdots \\ W_{2N+11} W_{2N+12} \cdots -(W_{2N+1c} + jW_{2N+1S}) W_{2N+1ct} - jW_{2N+1st} \\ W_{2N+21} W_{12N+22} \cdots -(W_{2N+2c} + jW_{2N+2s}) W_{2N+2ct} - jW_{2N+2st} \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ \vdots \\ R \\ T \end{bmatrix} = \\ \begin{bmatrix} (-Z_{4c} + jZ_{1s}) + (-W_{1c} + jW_{1s}) \\ (-Z_{2c} + jZ_{2s}) + (-W_{2c} + jW_{2N}) \\ \vdots \\ (Z_{2N+4c} + jZ_{2N+1c}) + (-W_{2N+2c} + jW_{2N+2s}) \end{bmatrix}$$
(15)

Where the impedance matrix elements  $Z_{mn}$ ,  $Z_{ms}$ ,  $Z_{mst}$ ,  $Z_{mst}$ ,  $Z_{mst}$ , are calculated during the solution of the equation integral (3) and the impedance matrix elements  $W_{mn}$ ,  $W_{ms}$ ,  $W_{mc}$ ,  $W_{mst}$ ,  $W_{mst}$  are given in[12]. Using the lower upper decomposition technique for inversing the resulting impedance matrix, we can resolve the matrix equation (15). This expression allows obtaining the derivative, of reflection coefficient  $R^{\xi}$ , the transmission coefficient  $T^{\xi}$  and the  $I^{\xi}_{n}$ .

### **III. EXAMPLE**

We consider a transmission line with two parallel stubs on an alumina substrate ( $\epsilon_r = 9.6$  and h=0.635mm)(see fig.2). The stubs have the same length L and change simultaneously. The width w of the transmission line and stubs is 0.635mm. We vary the length L from 0.3175 mm to 3.4925 mm in step of 0. 3175 mm. The operating frequency is 10 GHz and the electrical wavelength in the substrate 11.7 mm. There are two surfaces of which the shape is modified. The different velocity vectors for the second stub are defined in terms of the same geometrical parameter L in [12].



Fig. 2. Geometry and meshing of double stub example.



Fig. 3. Real and imaginary part of the transmission coefficient and of the geometrical derivative of the transmission coefficient at 10 GHz for the double stub case.

The theoretical results for the transmission real and imaginary part for the double stub, as shown in Fig. 2, are compared with the theoretical results in [12], the difference between our theory computations and those of Urrel & zutter in [12] are less than 0.02 over the frequency range, and the calculation time decrease more, which present a good agreement.

#### **IV. CONCLUSION**

The underlying principles and derivation of a new integral equation for the total derivative of the surface current with respect to a geometrical parameter were presented. By expanding the unknown total derivative of the current over the same set of basis and test functions as the current, numerically efficient computation of the geometrical derivative becomes possible as a byproduct of the electromagnetic simulation. S-parameter with respect to a geometrical parameter are obtained. Calculation of the matrix elements, filling and inversion of the impedance matrix is performed once. The impedance matrix can be reused for each geometrical parameter.

The approach treats all possible geometrical parameters in the plane of the circuit in a uniform way. Through the examples we saw that the integral equation calculated derivatives coincides well with those obtained in [12].

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