A Complex-Valued Adaptive Filter Algorithm for System Identification Problem

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Abstract

In this study, a complex-valued adaptive filter algorithm based on Lyapunov stability theory is presented to solve a system identification problem in the complex domain. The performance of the proposed complex-valued Lyapunov adaptive filter (CLAF) algorithm is improved for the complex-valued system identification problem by integrating the LST into the filter optimization cost. The performance of the proposed algorithm is tested on a complex-valued moving average (MA) system identification problem and compared with the conventional complex-valued least mean square (CLMS) and complex-valued normalized least mean square (CNLMS) algorithms. The simulation results show that the proposed CLAF algorithm has achieved a faster convergence rate and a lower steady-state MSE performance when compared to the other algorithms.

1. Introduction

Nowadays, adaptive filters have been widely and successfully used as an engineering tool in prediction, noise cancellation, system identification problems etc. [1]. In adaptive filter applications, the signal magnitude is generally used as the main source of information [2]. Therefore, conventional adaptive filter algorithms are generally real-valued algorithms and provide a signal processing in the real domain. But, real word processes having the intensity and direction components (radar, sonar, vector fields, etc.) also require the phase information to be taken into account [2]. Complex-valued signals contain the phase information in their structure naturally. Also, the complex-valued signals may be defined as the reel and imaginer or the phase and magnitude components of them. The real-valued adaptive filter algorithms cannot be directly applied to the complex-valued signals. Therefore, adaptive filter algorithms must be developed in the complex domain [2].

In the literature, the CLMS and CNLMS algorithms are widely used for signal processing in the complex domain because of their low computational complexities [1, 3]. However, these algorithms highly depend on the statistical properties of the system input. Hence, the input signal directly affects the convergence dynamics of them [4, 5].

In order to overcome the mentioned problems above, the authors proposed the LST based algorithms [4-7]. However, these algorithms [4-7] were designed in the real domain and could be only used in real-valued signal tracking applications. Therefore, the performances of the aforesaid algorithms deteriorate in case of a measurement noise.

Mengüç and Acır [8] has also proposed a LST based adaptive filter algorithm for real-valued system identification problems by considering a measurement noise. In that study [8], the proposed algorithm having a fixed step size parameter achieved a better performance than the other LST based algorithms [4, 5].

Recently, the CLAF algorithm for the prediction of complex-valued signals was proposed in [9]. The CLAF algorithm in [9] has always guaranteed stability in the sense of Lyapunov. The CLAF aims to find the global minimum point along the energy surface [9].

In this study, the CLAF algorithm [9] is successfully applied to solve a system identification problem in the complex domain. The performance of the proposed algorithm is tested on a complex-valued MA system identification problem and compared with the CLMS and CNLMS algorithms. The simulation results shown that the proposed CLAF algorithm achieved a faster convergence rate and a lower steady-state MSE performance when compared to the other algorithms.

2. Complex-Valued LST based Adaptive Filter Algorithm

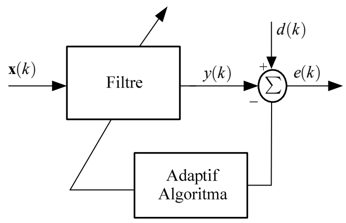

Fig. 1. Block diagram of an adaptive filter

The block diagram of an adaptive filter is shown in Fig. 1. Here, $\mathbf{x}(k)$ and $y(k)$ are the filter input signal and the finite impulse response (FIR) filter output signal, respectively. Also, $d(k)$ is the desired signal, and the complex-valued error signal of the FIR filter is given in Eq. (1).

$$e(k) = d(k) - y(k) = e^r(k) + je^i(k) \qquad (1)$$

where $e^r(k)$, and $e^i(k)$ represent the real and imaginary part of the error signal, respectively.

The complex-valued FIR filter output is given as follows:

$$y(k) = \mathbf{w}^T(k)\mathbf{x}(k) \qquad (2)$$

$$\mathbf{w}(k) = [w(k), w(k-1), ..., w(k-M)]^T \qquad (3)$$

$$\mathbf{x}(k) = [x(k), x(k-1), ..., x(k-M)]^T \qquad (4)$$

where the complex-valued weight vector $\mathbf{w}(k)$ and the complex-valued input vector $\mathbf{x}(k)$ are defined in Eq. (3) and (4). Also, M is the filter order.

After the error function $e(k)$ is defined in Eq. (1), a Lyapunov function $V(k)$ is primarily determined to design the CLAF algorithm [9]. In this study, the Lyapunov function $V(k)$ is selected as $V(k) = e(k)e^*(k) = |e(k)|^2$. To guarantee stability of the system, the selected Lyapunov function $V(k)$ must provide the negative definiteness condition $\Delta V(k) = V(k) - V(k-1) < 0$ for all k values. Therefore, this condition $\Delta V(k)$ is integrated into the constraint part of the proposed optimization problem to provide stability in the sense of Lyapunov in Eq. (5).

Thus, the proposed design can be constructed as the inequality optimization problem in Eq. (5) [9].

$$\begin{gathered}\mathbf{w}_o = Arg\min(\delta\mathbf{w}^H\delta\mathbf{w}) \\ \text{subject to} \\ \Delta V(k) = |e(k)|^2 - |e(k-1)|^2 < 0\end{gathered} \tag{5}$$

In Eq. (5), $\delta\mathbf{w}^H\delta\mathbf{w}$ and $\mathbf{w}_o$ represent the cost function and the optimum weight vector, respectively. Also, $\delta\mathbf{w} = \mathbf{w}(k) - \mathbf{w}(k-1)$ represents the difference between two consecutive weight vectors.

If this optimization problem is solved by using the method of Lagrange multipliers, the weight vector update law based on the LST can be obtained as follows:

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu\frac{\mathbf{x}^*(k)}{\mathbf{x}^T(k)\mathbf{x}^*(k)}\left(|\alpha(k)| - \kappa|e(k-1)|\right)sign\left(\alpha(k)\right) \tag{6}$$

where $\alpha(k)$, μ and κ represent the *a priori* estimation error, the step size and the adaptation gain rate, respectively.

$$\alpha(k) = d(k) - \mathbf{w}^T(k-1)\mathbf{x}(k) \tag{7}$$

As a result of the convergence (in the mean) analysis of the proposed algorithm, the range of the step size μ is obtained as $0 < \mu < 2$. The step size μ governs the steady-state convergence rate and the tracking capability of the proposed algorithm [8, 9].

The adaptation gain rate κ is selected as $0 \le \kappa < 1$ to satisfy stability in the sense of Lyapunov ($\Delta V(k) < 0$) [4, 5, 9]. It should be also noted that κ controls the convergence rate, and when κ is selected a small value, the proposed algorithm provides a faster convergence rate.

To avoid singularity in case of a vanishingly small $\mathbf{x}^T(k)\mathbf{x}^*(k)$, the weight vector update law in Eq. (6) can be modified by adding a small positive constant ε in Eq. (8).

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu\frac{\mathbf{x}^*(k)}{\left(\mathbf{x}^T(k)\mathbf{x}^*(k) + \varepsilon\right)}\left(|\alpha(k)| - \kappa|e(k-1)|\right)sign\left(\alpha(k)\right) \tag{8}$$

Finally, the proposed CLAF algorithm is presented step by step below.

Algorithm:

Parameters:

- $\varepsilon << 1$, $\varepsilon \in \Re^+$
- $0 \le \kappa < 1$
- $0 < \mu < 2$

Initialization: Primarily, the initial value of the weight vector $\mathbf{w}(0)$ and the filter order M must be determined.

Given Data:

- $\{x_k, d_k\}_{k=1}^K$

where $\mathbf{x}(k)$ is the filter input signal, and $d(k)$ is the desired signal.

Computation:

- $\alpha(k) = d(k) - \mathbf{w}^T(k-1)\mathbf{x}(k)$
- $\mathbf{w}(k) = \mathbf{w}(k-1) + \mu\frac{\mathbf{x}^*(k)}{\left(\mathbf{x}^T(k)\mathbf{x}^*(k) + \varepsilon\right)}\left(|\alpha(k)| - \kappa|e(k-1)|\right)sign\left(\alpha(k)\right)$
- $y(k) = \mathbf{w}^T(k)\mathbf{x}(k)$
- $e(k) = d(k) - y(k)$

Compute the above steps for $k = 1, 2, ..., K$.

2. Simulation Results

In this study, the performance of the proposed CLAF algorithm was tested on a complex-valued system identification problem.

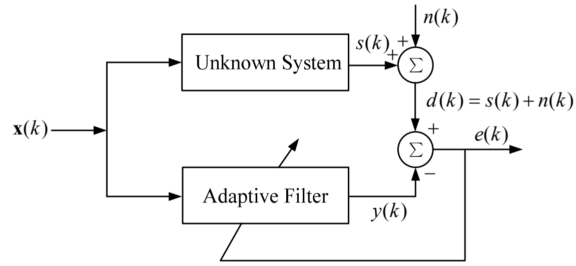

Fig. 2. Block diagram of a system identification problem

The classical structure of a system identification problem is shown in Fig. 2. As seen from Fig. 2, the desired signal $d(k)$ consists of following model:

$$d(k) = s(k) + n(k) = \mathbf{w}_o{}^T\mathbf{x}(k) + n(k) \tag{9}$$

$$\mathbf{w}_o = [w_0, w_1, ..., w_{M-1}]^T \tag{10}$$

where $s(k) = \mathbf{w}_o{}^T\mathbf{x}(k)$ represents the unknown system output, $\mathbf{w}_o$ is the optimal weight vector of the unknown system to be estimated, and $n(k)$ denotes the complex-valued measurement noise.

In our simulation, the unknown system [10] is the complex-valued moving average (MA) model as follows:

$$s(k) = w_0 x(k) + w_1 x(k-1) + w_2 x(k-2) + w_3 x(k-3) \quad (11)$$

where the optimal weight coefficients are $w_0 = 6 - 6j$, $w_1 = 0.5 + j$, $w_2 = -2 + j$ and $w_3 = 2 + 3j$.

Also, the statistics of the input signal $\mathbf{x}(k)$ and measurement noise signal $n(k)$ are given in Eq. (12).

$$x \sim N(0,1) + jN(0,1), \; n \sim N(0,0.1) + jN(0,0.1) \quad (12)$$

The FIR adaptive filter structure was used to identify the optimal weight vector of the unknown complex-valued MA (4) system. Also, we assume that the adaptive FIR filter and the unknown system had the same number of weight coefficients. The performance of the proposed algorithm was compared with the CLMS and CNLMS algorithms. While the step sizes of the CLAF and CNLMS algorithms were selected as $\mu = 0.1$, the step size of the CLMS algorithm was chosen as $\mu = 0.01$. Also, the adaptation gain rate of the proposed CLAF algorithm was selected as $\kappa = 0.001$.

In order to evaluate the performance of the algorithms, we used the mean square error (MSE). Also, all the simulation results were obtained by ensemble averaging over 100 independent trials.

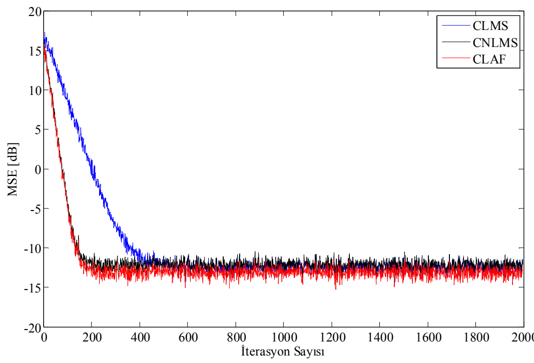

Fig. 3. The MSE performance of all the adaptive filter algorithms for the complex-valued MA(4) system problem

The MSE performance of all the adaptive filter algorithms for the complex-valued MA(4) system is shown in Fig. (3). In order to better observe, the detail of MSE performance of all the adaptive filter algorithms for the complex-valued MA(4) system is also shown in Fig. (4).

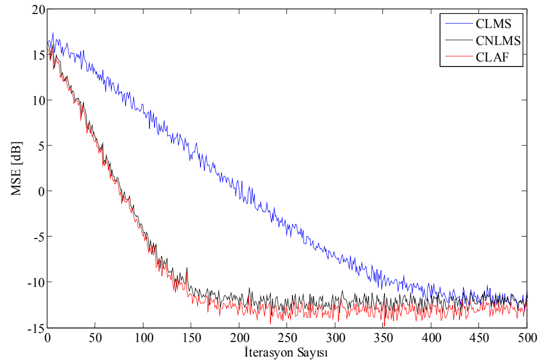

Fig. 4. Zoom of the MSE performance of all the adaptive filter algorithms for the complex-valued MA(4) system problem

As seen from Fig. (3) and (4), the proposed CLAF algorithm shows a fast convergence rate when compared to the other algorithms. After the 450th iteration, all the adaptive filter algorithms exhibit approximately the same MSE performance.

Table 1. MSE of the CLAF, CLMS and CNLMS algorithms for the known MA(4) system identification problem

Algorithms	**MSE** $J_{\min} = \sigma_n^2 = 0.1$
CLMS	0.2119
CNLMS	0.1126
CLAF	**0.1058**

Table 1 compares the steady state MSE values of all the algorithms. Here, after all the adaptive filters converge to the optimal weight vector coefficients, we expect that the variance of the error signal is theoretically equal to the variance of the measurement noise signal. Table 1 shows that the proposed CLAF algorithm achieves a lower MSE performance than the other algorithms. Moreover, its MSE value is almost close to the variance of the measurement signal. As a results, the performance of the proposed CLAF algorithm is improved for the complex valued system identification problem by integrating the LST into the filter optimization cost.

6. Conclusions

In this paper, we have presented a complex-valued adaptive filter algorithm based on the LST to solve a system identification problem in the complex domain. The performance of the proposed CLAF algorithm has been developed for the complex valued system identification problem by integrating the LST into the filter optimization cost. The performance of the proposed algorithm has been tested on the complex-valued MA system identification problem and compared with the conventional CLMS and CNLMS algorithms. The simulation results show that the proposed CLAF algorithm achieved a faster convergence rate and a lower steady state MSE performance when compared to the other algorithms. In our future study, complex-valued circular and noncircular signals will be examined, and novel complex-valued adaptive filtering algorithms by using the LST will be conducted for the filtering of general complex valued signals.

7. References

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