CHARACTERISTICAL ANALYSIS OF PHYSICAL MAGNITUDES AT ELECTROMAGNETIC THEORY

H. Ergun BAYRAKÇI
Uludağ University Faculty of Engineering and Architecture
Department of Electronic Engineering
Bursa TURKEY

ABSTRACT

With the help of characteristical analysis, it can be possible to detect whether physical magnitudes are vectoral or scalar. Therefore expanding electromagnetic theory's axiom equations, physical magnitudes characterizing a surface have been added these equations. These physical magnitudes are electric and magnetic surface current densities and electric and magnetic surface charge densities.

In this paper, it has been proved that surface charge densities proposed by Gürmen [1, 2, 3, 4] are vectoral magnitudes and electric current density is equal to the tangent component of the magnetic field, magnetic current density is equal to the tangent component of the electric field.

INTRODUCTION

At first, an inverse vector was defined. Later, dot, cross and dyadic products of this inverse vector with another vector are defined. The curls of surface electric and magnetic current densities are added to Maxwell-Ampere and Maxwell-Faraday equations. Also, divergence of electric and magnetic charge densities that are considered as vectoral magnitudes are added to Maxwell Electric and Magnetic Gauss equations. In this case, in the lossless simple medium, it is obtained the vectoral Helmholtz equation of vector potential. It can be performed to integral solution of this equation with Green functions method. Finally, using of these solutions, it has been given some examples.

1. INVERSE VECTOR AND DOT, DYADIC, AND CROSS PRODUCT OF INVERSE WITH ANOTHER VECTOR

1.1 - Definitions

1. Inverse vector,

If $\vec{n}$ is the unit vector of vector $\vec{A}$, the ratio of $\vec{n}$ to magnitude $A = \|\vec{A}\|$ can be defined as inverse vector

$$\left(\vec{A}\right)^{-1} = \frac{\vec{n}}{A} = \frac{\vec{A}}{A^2} \qquad 1.1$$

2. Dot product of inverse vector of $\vec{A}$ with a $\vec{B}$ can be defined by

$$\vec{A}^{-1} \cdot \vec{B} = \frac{\vec{A} \cdot \vec{B}}{A^2} \qquad 1.2$$

3. Cross product of inverse vector of $\vec{A}$ with a vector $\vec{B}$ can be defined by

$$\vec{A}^{-1} \times \vec{B} = \frac{\vec{A} \times \vec{B}}{A^2} \qquad 1.3$$

4. Dyadic product of inverse vector of $\vec{A}$ with a vector $\vec{B}$ is a second order tensor and can be defined by.

$$\vec{A}^{-1}\vec{B} = \frac{1}{A^2}\vec{A}\vec{B} \qquad 1.4$$

1.2 - Examples

1. Pressure is the force affecting to the unit surface and a scalar magnitude. If the inverse of $\vec{s}$ surface vector is $\left(\vec{s}\right)^{-1} = \frac{\vec{n}}{s}$ pressure is defined by

$$p = \frac{\vec{F} \cdot \vec{n}}{s} \qquad 1.5$$

2. Surface element of volume current density is a vector and can be expressed as

$$\left(\Delta\vec{s}\right)^{-1} = \frac{1}{\Delta s}\vec{n} \qquad 1.6$$

so volume current density can be defined by.

$$\vec{J}_{ev} = \lim_{\Delta s \to 0} \Delta I \left(\Delta\vec{s}\right)^{-1} = \lim_{\Delta s \to 0} \frac{\Delta I}{\Delta s}\vec{n} \qquad 1.7$$

Since $\Delta\vec{s}$ is a vector it can be possible to express electric or magnetic surface charge densities as

$$\vec{\rho}_s = \lim_{\Delta s \to 0} \Delta Q \left(\Delta\vec{s}\right)^{-1} = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s}\vec{n} \qquad 1.8$$

2. AXIOM EQUATIONS OF ELECTROMAGNETIC FIELDS

A theory can be based on axioms, definitions and theorems. These axioms can be expanded to explain physical systems, if it is needed. This can be done in electromagnetic theory .

Taking into consideration that electric or magnetic volume current density and electric or magnetic surface charge densities are different physical magnitudes, axiom equations of electromagnetic fields can be expanded as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{J}_{mv} - \nabla \times \vec{J}_{ms}$$
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_{ev} + \nabla \times \vec{J}_{es} \qquad 2.1$$
$$\nabla \cdot \vec{B} = \rho_{mv} + \nabla \cdot \vec{\rho}_{ms}$$
$$\nabla \cdot \vec{D} = \rho_{ev} + \nabla \cdot \vec{\rho}_{es}$$

Here $\vec{J}_{ev}$, volume electric current density, is in $[A/m^2]$, $\vec{J}_{es}$ surface electric current density, is in $[A/m]$, ρ_v volume electric charge density, is in $[coulomb/m^3]$, and $\vec{\rho}_s$ surface electric charge density, is in $[coulomb/m^2]$, $\vec{J}_{mv}$ volume magnetical current density, is in $[V/m^2]$, $\vec{J}_{ms}$ surface magnetic current, is in $[V/m]$, ρ_{mv} volume magnetic charge density, is in $[weber/m^3]$, $\vec{\rho}_{ms}$ surface magnetic charge density, is in $[weber/m^2]$

From these axiom equations it can be seen that

$$\vec{J}_{ev} \neq \vec{J}_{es}\delta(s) \quad , \quad \rho_{ev} \neq \rho_{es}\delta(s)$$
$$2.2$$
$$\vec{J}_{mv} \neq \vec{J}_{ms}\delta(s) \quad , \quad \rho_{mv} \neq \rho_{ms}\delta(s)$$

where $\delta(s)$ is Delta-dirac distribution of the coordinate that is perpendicular to S surface.

3. BOUNDARY CONDITIONS THEOREM IN THE SIMPLE MEDIUM

The curl and divergense of a vector can be written in the meaning of distribution as

$$\nabla \times \vec{A} = \{\nabla \times \vec{A}\} + \Delta(n \times \vec{A})\delta(s)$$
$$3.1$$
$$\nabla \cdot \vec{A} = \{\nabla \cdot \vec{A}\} + \Delta(\vec{n} \cdot \vec{A})\delta(s)$$

where, $\Delta\vec{A} = \vec{A}_1 - \vec{A}_2$.

Using 3.1 in axiom equations one can write

$$\{\nabla \times \vec{E}\} + \Delta(\vec{n} \times \vec{E})\delta(s) = \left\{-\frac{\partial \vec{B}}{\partial t}\right\} - \{\vec{J}_{mv}\}$$
$$-(\nabla \times \vec{J}_{ms}) - \Delta(\vec{n} \times \vec{J}_{ms})\delta(s)$$
$$\{\nabla \times \vec{H}\} + \Delta(\vec{n} \times \vec{H})\delta(s) = \left\{\frac{\partial \vec{D}}{\partial t}\right\} + \{\vec{J}_{ev}\} \qquad 3.2$$
$$+(\nabla \times \vec{J}_{es}) + \Delta(\vec{n} \times \vec{J}_{es})\delta(s)$$
$$\{\nabla \cdot \vec{D}\} + \Delta(\vec{n} \cdot \vec{D})\delta(s) = \{\rho_{ev}\} + \{\nabla \cdot \vec{\rho}_{es}\}$$
$$+\Delta(\vec{n} \cdot \vec{\rho}_{es})\delta(s)$$
$$\{\nabla \cdot \vec{B}\} + \Delta(\vec{n} \cdot \vec{B})\delta(s) = \{\rho_{mv}\} + \{\nabla \cdot \vec{\rho}_{ms}\}$$
$$+\Delta(\vec{n} \cdot \vec{\rho}_{ms})\delta(s)$$

So boundary conditions in 3.2 can be dotained as

$$\Delta(\vec{n} \times \vec{E}) = \Delta(\vec{n} \times \vec{J}_{ms})$$
$$\Delta(\vec{n} \times \vec{H}) = \Delta(\vec{n} \times \vec{J}_{es})$$
$$3.3$$
$$\Delta(\vec{n} \cdot \vec{D}) = \Delta(\vec{n} \cdot \vec{\rho}_{es})$$
$$\Delta(\vec{n} \cdot \vec{B}) = \Delta(\vec{n} \cdot \vec{\rho}_{ms})$$

Since $\vec{E}_2 = 0$, $\vec{H}_2 = 0$ and $\vec{J}_{ms} = 0$, $\vec{\rho}_{ms} = 0$ in perfect conductor, from equations 3.3 on the surface of perfect conductor boundary conditions can be written as

$$\vec{n} \times \vec{E}_1\big|_s = 0 \qquad \vec{n} \times \vec{H}_1\big|_s = \vec{n} \times \vec{J}_{es}$$
$$3.4$$
$$\vec{n} \cdot \vec{D}_1\big|_s = \vec{n} \cdot \vec{\rho}_{es} \qquad \vec{n} \cdot \vec{B}_1\big|_s = 0$$

4. SOLUTIONS OF HELMHOLTZ EQUATIONS WITH GREEN FUNCTIONS METHODS

4.1- Integral Solution of the Helmholtz Equations

In the simple medium axiom equations of harmonic electromagnetic fields can be obtained as follows

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} - \vec{J}_{mv} - \nabla \times \vec{J}_{ms}$$
$$\nabla \times \vec{H} = (j\omega\varepsilon + \sigma)\vec{E} + \vec{J}_{ev} + \nabla \times \vec{J}_{es} \qquad 4.1$$
$$\nabla \cdot \vec{B} = \rho_{mv} + \nabla \cdot \vec{\rho}_{ms}$$
$$\nabla \cdot \vec{D} = \rho_{ev} + \nabla \cdot \vec{\rho}_{es}$$

Using $\vec{B} = \nabla \times \vec{A}$ definition equation of vector potential and

$$\nabla \cdot \vec{A} + j\omega\varepsilon\mu\varphi = 0 \quad 4.2$$

Lorentz condition in the lossless simple medium that does not include magnetic sources where $\rho_{mv} = 0$, $\vec{\rho}_{ms} = 0$, $\vec{J}_{mv} = 0$ and $\vec{J}_{ms} = 0$, vector Helmholtz equation can be found as

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J}_{ev} - \mu \nabla \times \vec{J}_{es} \quad 4.3$$

Considering second scalar Green theorem and scalar Helmholtz equation of Green function

$$\nabla^2 G + k^2 G = -4\pi\delta\left(\vec{r} - \vec{r}^1\right) \quad 4.4$$

the integral solution of 4.3 vector Helmholtz equation in the unbounded medium can be obtained as

$$\vec{A} = \frac{\mu}{4\pi}\iiint_{v'} \vec{J}_{ev}\frac{e^{-jkR}}{R}dv' + \frac{\mu}{4\pi}\iiint_{v'} \nabla \times \vec{J}_{es}\frac{e^{-jkR}}{R}dv' \quad 4.5$$

4.6 is the curl of surface current density in the meaning of distribution

$$\nabla \times \vec{J}_{es} = \left\{\nabla \times \vec{J}_{es}\right\} + \Delta\left(\vec{n} \times \vec{J}_{es}\right)\delta(s) \quad 4.6$$

Using this equations in 4.5 one can write

$$\vec{A} = \frac{\mu}{4\pi}\iiint_{v'} \vec{J}_{ev}(\vec{r}')\frac{e^{-jkR}}{R}dv' + \frac{\mu}{4\pi}\iiint_{v'} \left\{\nabla \times \vec{J}_{es}\right\}\frac{e^{-jkR}}{R}dv' + \frac{\mu}{4\pi}\iint_{s'} \Delta\left(\vec{n} \times \vec{J}_{es}\right)\frac{e^{-jkR}}{R}ds' \quad 4.7$$

for the integral solution of vector potential.

Electric vector potential can be defined as $\vec{D} = -\nabla \times \vec{F}$. In the lossless simple medium that does not include electrical sources using this definition similar to the equations 4.3 in the axiom equations, differential equation of electric vector potencial can be obtained as

$$\nabla^2 \vec{F} + k^2 \vec{F} = -\varepsilon \vec{J}_{mv} - \varepsilon \nabla \times \vec{J}_{ms} \quad 4.8$$

Similar to the equation 4.5 the integral solution of this equation with Green functions method will be

$$\vec{F} = \frac{\varepsilon}{4\pi}\iiint_{v'} \vec{J}_{mv}\frac{e^{-jkR}}{R}dv' + \frac{\varepsilon}{4\pi}\iiint_{v'} \nabla \times \vec{J}_{ms}\frac{e^{-jkR}}{R}dv' \quad 4.9$$

In the meaning of distribution potential magnetic surface current density can be expressed as

$$\nabla \times \vec{J}_{ms} = \left\{\nabla \times \vec{J}_{ms}\right\} + \Delta\left(\vec{n} \times \vec{J}_{es}\right)\delta(s) \quad 4.10$$

Using 4.10 in 4.9 integral expression of electric vector potential can be written as

$$\vec{F} = \frac{\varepsilon}{4\pi}\iiint_{v'} \vec{J}_{mv}(\vec{r}')\frac{e^{-jkR}}{R}dv' + \frac{\varepsilon}{4\pi}\iiint_{v'} \left\{\nabla \times \vec{J}_{ms}\right\}\frac{e^{-jkR}}{R}dv' + \frac{\varepsilon}{4\pi}\iint_{s'} \Delta\left(\vec{n} \times \vec{J}_{ms}\right)\frac{e^{-jkR}}{R}ds' \quad 4.11$$

4.2- Examples

1. Volume electric current density of electric line source that coincides z axis at the origin can be expressed as

$$\vec{J}_{ev} = \vec{e}_z I_0 \delta(x)\delta(y) \quad 4.12$$

where, $\vec{J}_{es} = 0$

2. Volume current density of electric Hertz vector that coincides z axis at the origin can be written as

$$\vec{J}_{ev} = \vec{e}_z I_0 \delta(x)\delta(y)\left[u(z + h/2) - u(z - h/2)\right] \cong \vec{e}_z h I_0 \delta(x)\delta(y)\delta(z) \quad 4.13$$

where, $\vec{J}_{es} = 0$

3. If there is a magnetic field at the aperture surface, first two volume integrals of vector potential given by 4.7 are equal to zero. The third term which is surface integral, can be written as 4.13 if boundary equivalent source theorem are considered

$$\Delta\left(\vec{n} \times \vec{J}_{es}\right) = 2\vec{n} \times \vec{H}_1 \quad 4.14$$

4. If there is only an electric field an a surface, that first two volume integral of electrical vector potential given by 4.11 will be zero. In the third term of 4.11, which is surface integral, considering equivalent source theorem and boundary conditions, it can be written

$$\Delta\left(\vec{n} \times \vec{J}_{ms}\right) = -2\vec{n} \times \vec{E}_1 \quad 4.15$$

5. If there are and tangent component of both electric and magnetic fields, the first two volume integrals of $\vec{A}$ and $\vec{F}$ will be zero and considering equivalence source theorem, can be written as, in surface integral of 4.7

$$\Delta\left(\vec{n} \times \vec{J}_{es}\right) = \vec{n} \times \vec{H}_1 \qquad 4.16$$

and the surface integral of 4.11

$$\Delta\left(\vec{n} \times \vec{J}_{ms}\right) = -\vec{n} \times \vec{E}_1 \qquad 4.17$$

6. In *quasioptic* scattering from a perfect conductor for reflected waves $\vec{F}=0$ and the first two volume integrals of 4.7 are zero, in surface integral of 4.7 it also can be written as

$$\Delta\left(\vec{n} \times \vec{J}_{es}\right) = \vec{n} \times \vec{H}_1 \qquad 4.18$$

CONCLUSION

In this paper, using the definition of inverse vector, it can be shown that surface charge densities are vectoral magnitudes, and also these magnitudes are added to axiom equations. That is, Maxwell equations are expanded by being added the divergences of surface charge densities and the curls of surface current densities to these equations.

As a result, electric and magnetic surface charge densities are vectoral magnitudes. Moreover, at the boundary of two simple medium, electric surface current density is equal to tangent component of magnetic field and also magnetic surface current density is equal to tangent component of electric field .

REFERENCES

[1] Gürmen, H., "Ters vektör", Boğaziçi University periodical vol. 1-1973.

[2] Gürmen, H.,"Inverse vector", Bulletin of Boğaziçi University, May 1973

[3] Gürmen H., "Characterial Analysis of Physical Magnitudes", IV. Scientific Meeting of Turkish Scientific and Technical Research Organization, June 1973

[4] Gürmen H., "Fiziksel büyüklüklerin Karaktersel Analizi", Bursa IV.Elektromekanik Sempozyumu, 1997

[5] İdemen, M., "The Maxwell's Equations in the Sense of Distributions", IEEE Transactions on Antennas and Propagation, Sept. 1973.

[6] Üçoluk, M.," Methode der character analysis der Physikalishen Grössen und ihre Beziehung mit der Tensor rechnung" Bulletion of Technical University of İstanbul, July 1978