Chaotic Digital Image Watermarking Scheme Based on DWT and SVD

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Abstract

In this study chaos based digital watermarking scheme together with Discrete Wavelet Transform (DWT) and Singular Value Decomposition (SVD) is proposed. In the proposed watermarking scheme, the cover image is decomposed to its sub-bands (LL, LH, HL and HH) by a common used frequency domain transform: DWT. Then, the SVD is directly applied to the all sub-bands of the decomposed cover image. The watermark is shuffled with Arnold's Cat Map (ACM) to generate a chaotic watermark. By this way, the robustness and perceptual invisibility of the scheme is improved. In order to evaluate the robustness of the proposed scheme, several image processing and geometric attacks are applied to the scheme. The Normalized correlation (NC) and peak signal-to-noise ratio (PSNR) measures are used to show the performance of the proposed method in terms of robustness and perceptual invisibility. The proposed algorithm gives the promising results and meets the security requirements.

1. Introduction

In recent years digital watermarking has gained great deal of importance with the explosion of the digital media. Several digital watermarking techniques are developed to prevent the unpermitted data transmission and protect the data from the intentional and unintentional attacks. The goal of the digital image watermarking techniques is to embed a secret signal called as watermark to the cover image. The most common applications of the digital watermarking methods are copyright protection, broadcast monitoring, tamper detection, authentication and integrity verification, fingerprinting, content description and secure communication.

Digital watermarking schemes can be classified in different ways. According to the watermark embedding domain, digital watermarking methods can be categorized as spatial domain methods and transform domain methods. In the spatial domain watermarking methods, watermark is directly embedded into the cover image. After embedding process, only the pixel values of the cover image change. On the other hand, in the transform domain watermarking methods, embedding is performed in the transform domain by applying a technique such as Finite Ridgelet Transform (FRIT), Discrete Cosine Transform (DCT), Discrete Fourier Transform (DFT), Discrete Wavelet Transform (DWT), Redundant Discrete Wavelet Transform (RDWT) to the cover image. Then the obtained coefficients are modified according to the method. Consequently, the spatial domain methods are easier and less time consuming than the transform domain methods. However, they cannot ensure a good robustness against the common image processing and geometric attacks [1].

The watermarking applications have to meet some specific requirements. There are three main requirements common to most of the applications. They are perceptual invisibility, capacity and robustness. Perceptual invisibility or fidelity can be defined as the perceptual similarity between the original and watermarked data. The number of information bits (watermark) embedded into the original data denotes the capacity of the watermark scheme. Finally, robustness is the ability of detecting watermark that is subject to the common signal processing attacks [1, 2].

SVD is a complementary technique for transform domain techniques and used in the most popular studies about the watermarking. SVD can be applied directly to the image matrix and always shows a good performance. The transform domain techniques utilizing the properties of SVD are referred as the hybrid watermarking techniques in the literature and can be found in [3-5].

In this study, a chaos based hybrid digital image watermarking scheme based on DWT and SVD is proposed. ACM that transforms the original watermark image into the chaotic image pattern is combined to the watermarking scheme to improve the robustness and perceptual invisibility. Digital image watermarking schemes with chaos present in the literature are investigated to evaluate the performance of our method. There are several digital image watermarking schemes in the transform domain that utilizes the properties of the chaos [6-9]. The study is organized as follows. In Section 2, brief information about the ACM is given. To better understand the proposed algorithm, DWT and SVD techniques are introduced in Section 3 and 4, respectively. In Section 5, watermark embedding and extracting algorithms are discussed with all steps. Section 6 illustrates the simulation results to evaluate the performance of the algorithm. Finally, Section 7 concludes the paper.

2. Arnold's Cat Map

Chaotic signals are mainly used in secure communications, signal processing and cryptography because of their inherent properties that can be taken into account as complexity, orthogonality and having broad-band spectrum. As a result, many crucial chaos-based algorithms have been proposed for image processing applications to show whether the performance increase is possible compared to the other applications.

There are several maps appropriate for image processing algorithms. ACM is one of the most famous chaotic maps used for randomizing the pixel locations in the image matrix. This randomizing provides security augmentation for the image watermarking schemes. 2D-ACM for NxN square image matrix can be expressed as

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & a \\ b & ab+1 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} mod\ N = \boldsymbol{A} \begin{bmatrix} x_n \\ y_n \end{bmatrix} mod\ N \qquad (1)$$

where (x_n, y_n) and (x_{n+1}, y_{n+1}) are the locations of pixels before and after iterations, respectively. In the above equation a and b are the positive integers provided that the det(**A**)=1 [7, 10].

After a few iterations locations of pixels will be scrambled but when the transformation is repeated enough we will attain the original image matrix again. ACM transformation for different iteration numbers are illustrated in Fig. 1. According to the figure, we can say that original watermark is obtained after 40^{th} iterations. Thus, the number of iterations in the watermark extracting process is determined by the number of iterations X in the embedding process as, 40-X. In this study the size of the binary watermark is 77x77.

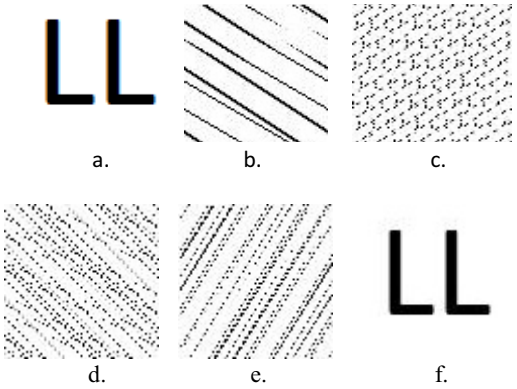

Fig. 1.(a) Original watermark and scrambled watermarks after related number of iterations in ACM: (b) 2, (c) 15, (d) 21, (e) 37 and (f) 40.

3. Discrete Wavelet Transform

As we discussed in Section 1, DWT is a common used method in watermarking schemes that transforms the image from the pixel domain to the frequency domain. It is proven that the DWT is superior the widely used DCT in watermarking schemes. DWT is a mathematical tool that splits a one dimensional signal into low-frequency and high-frequency parts. This process is called as decomposition. In DWT, high-pass and low-pass filter are used to analyze the high and low frequencies of the signal. The outputs of the filters are called as DWT coefficients and the original signal can be reconstructed by using them. This reconstruction is referred as inverse DWT (IDWT) [1].

By performing DWT one time, the cover image is divided into four sub-bands i.e.LL_1, LH_1, HL_1, HH_1. LL_1 is the low frequency component and has the maximum power of energy while the other sub-bands are middle and high frequency sub-bands. These sub-bands represent the edges, outline, texture and other detail information in an image. If we want to decompose the image to narrower frequency sub-bands, we should apply the DWT to the LL_1 sub-band, again. This process can be repeated until we get the desired composition level of wavelet transformation. In Fig. 2., three level DWT decomposition is illustrated.

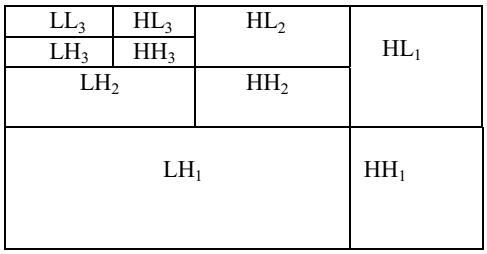

Fig. 2. The 3- level DWT decomposition scheme.

Watermark embedding is performed by changing the DWT coefficients according to the algorithm. We can embed the watermark either in low frequency sub-band or in high frequency sub-band. Embedding watermark to the high frequency sub-band provides high imperceptibility advantage but robustness and stability of the scheme are decreased. Robustness can be improved by embedding watermark to the middle and low frequency sub-bands but this causes the lack of imperceptibility.

4. Singular Value Decomposition

SVD is an efficiently used technique in image and signal processing applications such as image compression, data hiding, noise reduction and image watermarking. Singular value decomposition can be applied directly to the image matrix with any dimensions. Given the data matrix **A** which has the W linearly independent column (i.e. rank (**A**)=W), there are two unitary matrices **V** and **U** such that,

$$\boldsymbol{A = USV^T} \qquad (2)$$

where **S**= diag (σ_1, σ_2, …. , σ_W) is a diagonal matrix. Singular values (σ's) are ordered as $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_W > 0$. This equation is the mathematical statement of the SVD theorem and also referred as Autonne- Eckart- Young theorem [11].

SVD is an optimal decomposition method that can concentrates the maximum signal energy into as few coefficients as possible. From the image processing point of view, SVD has three main advantages important in image processing applications [3, 4]. These advantages can be summarized as follows:

1. The image matrix not has to be square matrix. It can be any dimension.
2. SVD puts forward the algebraic properties of an image where singular values correspond to the brightness of the image and **U** and **V** denotes the geometric properties of the image.
3. The slight variations of singular values of an image may not affect the human visual perception. This stability property of SVD is the main reason why it is preferred for watermarking applications.

4. Proposed Method

A general watermarking system consists of two main units: the watermark embedding unit and the watermark detection/ extraction unit. Both units can be considered as a separate

process. In the following subsections, watermark embedding and extracting processes for the proposed method are explained.

5.1. Watermark Embedding Algorithm

The steps of the proposed embedding algorithm are as follows:

Step 1: Apply 21 iteration ACM to the original watermark, **W**, in order to get chaotic watermark image, $\mathbf{W}_{ACM}$.

Step 2: Perform 3- level DWT to the cover image to decompose it into LL_3, LH_3, HL_3, HH_3 sub-bands.

Step3: Apply SVD to the chaotic watermark image, $\mathbf{W}_{ACM}$ as follows:

$$\boldsymbol{W}_{ACM} = \boldsymbol{U}_{ACM}\boldsymbol{S}_{ACM}V_{ACM}^{T} \qquad (3)$$

Step 4: Apply SVD to all sub-bands of the cover image (LL3, LH_3, HL_3, HH_3), as follows:

$$\boldsymbol{A}^{i} = \boldsymbol{U}^{i}\boldsymbol{S}^{i}\boldsymbol{V}^{i^{T}} \qquad (4)$$

where *i* indicates the sub-bands (i.e., LL_3, LH_3, HL_3, HH_3).

Step 5: Modify the singular values $\mathbf{S}^i$ with singular values $\mathbf{S}_{ACM}$ of chaotic watermark image

$$\boldsymbol{S}^{*i} = \boldsymbol{S}^{i} + \alpha\boldsymbol{S}_{ACM} \qquad (5)$$

where α is scaling factor fixed for all sub-bands and *i* denotes the sub-bands.

Step 6: Apply inverse SVD on the transformed cover image with modified singular values as follows:

$$\boldsymbol{A}^{*i} = \boldsymbol{U}^{i}\boldsymbol{S}^{*i}\boldsymbol{V}^{i^{T}} \qquad (6)$$

where *i* indicates the sub-bands (i.e., LL_3, LH_3, HL_3, HH_3), again.

Step 7: Finally, perform inverse 3-level DWT using the modified coefficients to construct the watermarked image.

In this study, watermark is embedded to the LL_3, LH_3, HL_3, HH_3 sub-bands, separately. Imperceptibility and robustness evaluated by the computer simulations under the most common attacks are very close for all sub-bands because of inserting ACM, SVD and DWT in the watermarking scheme.

5.2. Watermark Extraction Algorithm

The watermark extraction steps are given below:

Step 1: Perform 3- level DWT to the watermarked image $\boldsymbol{A}_{W}^{*}$, obtaining $LL3^*$, LH_3^*, HL_3^* and HH_3^*sub-bands.

Step 2: Apply SVD to all sub-bands, as follows:

$$\boldsymbol{A}^{*i} = \boldsymbol{U}^{*i}\boldsymbol{S}^{*i}\boldsymbol{V}^{*i^{T}} \qquad (7)$$

where *i* indicates the sub-bands (i.e., LL_3^*, LH_3^*, HL_3^*, HH_3^*).
Step 3: Calculate the singular values $\boldsymbol{S}^{W}$ as follows:

$$\boldsymbol{S}^{W} = (\boldsymbol{S}^{*i} - \boldsymbol{S}^{i})/\alpha \qquad (8)$$

where $\boldsymbol{S}^{i}$'s are the singular values of the cover image for all sub-bands, individually.

Step 4: Apply inverse SVD, in order to get extracted chaotic watermark image:

$$\boldsymbol{W}_{ACM}^{*} = \boldsymbol{U}^{i}\boldsymbol{S}^{w}\boldsymbol{V}^{i^{T}} \qquad (9)$$

Step 5: Apply (40 – 21= 19) iterations to $\boldsymbol{W}_{ACM}^{*}$in order to get finally extracted original watermark.

6. Experimental Results

In order to check our proposed algorithm we used gray scale Lena and Cameraman image as cover image of size 512 x 512 and the binary logo 'LL' as watermark image of size 77 x 77. All simulations are implemented by using MATLAB. Fig.(3). demonstrates the cover image, watermark, chaotic binary logo obtained by applying 21 iterations ACM, watermarked image, extracted chaotic watermark and the extracted original watermark, respectively.

After watermark extraction process, PSNR (Peak Signal to Noise Ratio) criterion that can be defined as the similarity between watermarked and cover image is calculated by using the following equation:

$$PSNR = 10log_{10}\left[\frac{\max\left((X(i,j))^2\right)}{MSE}\right] \qquad (10)$$

where MSE (Mean Square Error), which implies the noise energy, is defined as :

$$MSE = \frac{1}{mxn}\sum_{i=1}^{m}\sum_{j=1}^{n}[X(i,j) - Y(i,j)]^2 \qquad (11)$$

In Equation (11), *m* and *n* are dimensions of the image **X** and **Y**. In our simulations, we measure the PSNR between original watermark and extracted watermark. PSNR values are usually given in (dB) and larger values of PSNR such as 30 dB and higher indicate better watermark concealment.

NC (Normalized Correlation) is a parameter used to measure robustness that is another important requirement for any watermarking scheme.NC indicates the similarity between binary logo and logo extracted from the extraction process after attack. Mathematically it can be expressed as:

$$NC = \frac{\sum_{k=1}^{m}\sum_{j=1}^{n}[W(k,j)W'(k,j)]}{\sqrt{\sum_{k=1}^{m}\sum_{j=1}^{n}[W(k,j)]^2}\sqrt{\sum_{k=1}^{m}\sum_{j=1}^{n}[W'(k,j)]^2}} \qquad (12)$$

where **W** and **W'** represent the original and extracted watermark, respectively. The correlation coefficient can take values from the interval [-1, 1]. If it is near 0, the extracted watermark is completely uncorrelated. Generally, the NC is considered acceptable if it is0.75 or above [5, 12].

In order to investigate the robustness and imperceptibility of our proposed scheme, the watermarked image was attacked by applying salt & pepper noise, Gaussian noise, Poisson noise, histogram equalization, contrast adjustment, wiener filter [3x3],

median filter [3x3], speckle noise, JPEG compression and rotation. DWT based digital image watermarking scheme that uses the features of the logistic map in the embedding and extraction steps is proposed in [8]. To show the advantages of our method, comparative results with [8] are given in Table 1. In Table 1, NC values under different attacks are given to show the robustness of our method for all bands. The higher values are denoted with bold numbers. As can be seen from the Table 1, the proposed chaos based digital watermarking algorithm with SVD and DWT is providing considerably high robustness against the most common attacks.

In Table 2, NC and PSNR values of the proposed method are given in all bands for Lena and Cameraman cover images. The values calculated for Cameraman cover image are denoted with italic numbers in the table. Seventeen attacks are applied to the watermarking scheme in this experiment. The results show that the proposed digital image watermarking scheme is robust against to the most common attacks and meets the perceptual invisibility requirement of the watermarking schemes.

In Fig.4, watermarked image under a few attacks are illustrated. The PSNR values between the cover image and watermarked image is given in parenthesis. Because of the limitations on the number of pages, the watermarked image is given just for some of the attacks.

7. Conclusion

In this study, digital watermarking algorithm based on DWT, SVD and ACM is presented. As discussed in the sections of the paper, SVD is a very convenient tool for watermarking schemes performing in the DWT domain. Therefore, in our hybrid algorithm we used SVD together with DWT. Because of the known specific properties of the chaos especially in signal processing applications, we combined ACM to the proposed scheme to meet the security requirements. PSNR and NC measures are calculated for different cover images in all sub-bands to evaluate the performance of the proposed method. The proposed method is also compared with a current chaos based digital image watermarking scheme. Experimental results show that the proposed method is robust against the most common attacks and meets the security requirement of the watermarking schemes.

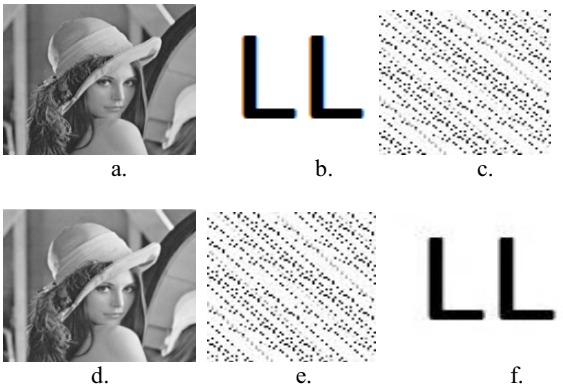

Fig. 3. (a) Cover Image (512 x 512 Lena), (b) Binary logo-watermark (77 x 77), (c) Chaotic binary logo, (d) watermarked image, (e) extracted chaotic watermark, (f) extracted original watermark.

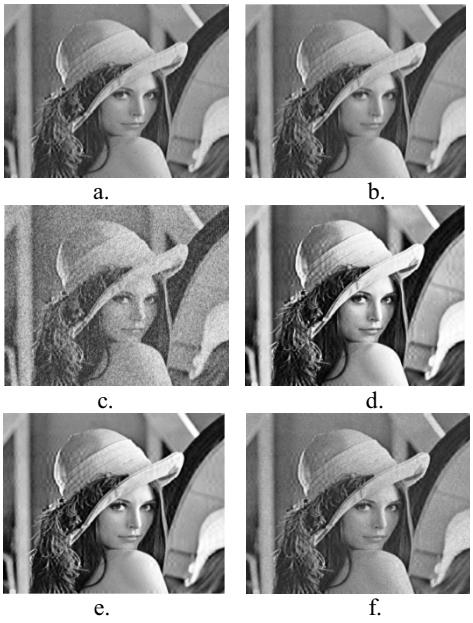

Fig. 4. Watermarked image under several attacks. (a) Salt & Pepper (var= 0.005), (56.38 (dB)),(b) Gaussian Noise (var= 0.001), (49.36 (dB)), (c) Gaussian Noise (var= 0.05), (35.75 (dB)), (d) histogram equalization (33.8525 (dB)), (e) contrast adjustment (35.12 (dB)), (f) speckle noise (var= 0.01), (42.26 (dB))

Table 1. Comparison results of NC values between proposed scheme and Khare et al[8].

	LL		LH		HL		HH	
Attacks	[8]	Ours	[8]	Ours	[8]	Ours	[8]	Ours
Gaussian Noise (0.01)	**0.979**	0.851	**0.974**	0.960	**0.951**	0.913	**0.985**	0.965
Contrast Enhancement	0.970	**0.977**	0.981	**0.993**	0.986	**0.994**	0.987	**0.997**
Average Filtering	0.983	**0.992**	0.955	**0.981**	0.987	**0.997**	**0.976**	0.972
Median Filtering	0.961	**0.993**	0.953	**0.981**	0.974	**0.997**	0.931	**0.973**
Gamma Correction	0.973	**0.999**	0.991	**0.999**	0.991	**0.997**	0.998	**0.999**
Histogram Equalization	**0.975**	0.974	0.978	**0.991**	0.981	**0.994**	0.980	**0.996**
Wiener Filtering	0.979	**0.988**	**0.998**	0.993	**0.995**	0.992	0.962	**0.983**
JPEG (50)	-	**0.977**	-	**0.993**	-	**0.854**	-	**0.975**

("-" denotes the value does not exist in corresponding study.)

Table 2. The NC and PSNR values of the proposed algorithm in all sub-bands for Lena and Cameraman cover images.

	LL		HL		LH		HH	
Attacks	NC	PSNR	NC	PSNR	NC	PSNR	NC	PSNR
Salt&Pepper(var 0.001)	0.9990 *0.9936*	74.9372 *65.5565*	0.9966 *0.9947*	67.2580 *57.8288*	0.9982 *0.9979*	69.9036 *52.0944*	0.9989 *0.9974*	72.1557 *58.8756*
Salt&Pepper(var 0.005)	0.9938 *0.9906*	66.9966 *65.1130*	0.9831 *0.9905*	62.0005 *58.0767*	0.9955 *0.9912*	66.2591 *51.6129*	0.9931 *0.9877*	63.4295 *55.2503*
Salt&Pepper(var 0.01)	0.9885 *0.9665*	64.2959 *59.9923*	0.9855 *0.9784*	58.9737 *56.3609*	0.9893 *0.9718*	63.6980 *51.0023*	0.9871 *0.9867*	59.8822 *55.3219*
Salt&Pepper(var 0.05)	0.8256 *0.7265*	53.0939 *49.6725*	0.9585 *0.9424*	47.9444 *47.8014*	0.9602 *0.9052*	51.9295 *47.5211*	0.9518 *0.9623*	47.2206 *46.8338*
Gaussian Noise (var= 0.001)	0.9952 *0.9914*	66.2858 *64.9980*	0.9934 *0.9931*	65.5344 *57.1841*	0.9960 *0.9905*	68.8256 *50.9433*	0.9952 *0.9937*	66.2592 *54.8286*
Gaussian Noise (var= 0.005)	0.9827 *0.9280*	56.2390 *53.5154*	0.9792 *0.9663*	55.6396 *54.3311*	0.9854 *0.9483*	57.8249 *49.1942*	0.9792 *0.9718*	54.0747 *51.0585*
Gaussian Noise (var= 0.01)	0.9510 *0.8516*	51.1713 *48.8959*	0.9709 *0.9607*	50.7160 *49.7779*	0.9782 *0.9137*	52.5021 *47.9220*	0.9700 *0.9655*	50.1253 *49.6661*
Gaussian Noise (var= 0.05)	0.6528 *0.6806*	38.5996 *38.7769*	0.9644 *0.9653*	39.6235 *41.0077*	0.9496 *0.8846*	42.6086 *42.1383*	0.9685 *0.9674*	39.5609 *39.8649*
Poisson Noise	0.9909 *0.9974*	64.6738 *61.6287*	0.9916 *0.9777*	63.2975 *56.2685*	0.9948 *0.9753*	65.8062 *51.2034*	0.9912 *0.9777*	61.5429 *53.9909*
Histogram Equalization	0.9745 *0.9762*	32.5982 *43.9370*	0.9949 *0.9864*	42.6029 *43.5188*	0.9916 *0.9862*	40.2104 *46.8931*	0.9967 *0.9862*	41.5807 *46.3495*
Contrast Adjustment	0.9771 *0.9707*	34.0830 *31.9373*	0.9945 *0.9952*	44.8962 *51.0624*	0.9931 *0.9915*	42.6766 *48.6690*	0.9979 *0.9959*	44.7096 *52.1178*
Wiener Filter [3x3]	0.9888 *0.9906*	62.8137 *61.4984*	0.9927 *0.9887*	52.6239 *50.3623*	0.9934 *0.9914*	54.7920 *47.8345*	0.9831 *0.9810*	50.3758 *48.1274*
Median Filter [3x3]	0.9937 *0.9901*	64.1797 *62.4765*	0.9977 *0.9860*	52.7629 *51.3897*	0.9813 *0.9757*	53.6070 *47.7819*	0.9735 *0.9776*	47.3619 *45.9702*
Speckle Noise (var=0.01)	0.9876 *0.9664*	63.3280 *60.0561*	0.9871 *0.9731*	61.5260 *55.9425*	0.9906 *0.9594*	62.0874 *50.3564*	0.9855 *0.9817*	59.2274 *54.0012*
JPEG Q=25	0.9725 *0.9202*	51.3872 *52.3483*	0.9963 *0.9897*	64.9863 *54.4521*	0.9908 *0.8387*	59.5707 *45.2749*	0.9915 *0.9744*	59.1090 *43.8541*
JPEG Q=50	0.9725 *0.9771*	51.3875 *71.9262*	0.9965 *0.9930*	65.0541 *54.8623*	0.9932 *0.8540*	62.5538 *45.4421*	0.9979 *0.9757*	63.5272 *44.3329*
Rotation 2	0.9463 *0.8287*	35.4852 *33.4206*	0.9480 *0.9378*	41.4470 *42.9997*	0.9782 *0.9493*	41.8661 *40.8659*	0.8720 *0.9258*	40.7455 *41.6365*

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