FINITE-DIFFERENCE MODELING OF THE TRANSIENT RADIATION FROM COAXIAL WAVEGUIDE

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ABSTRACT

The coaxial waveguide transient radiation is analyzed by the finite-difference method. Accurate Absorbing Boundary Conditions (ABC) are used on virtual boundaries for restriction of the computation domain. The numerical example of the near-zone field transformation makes better physical insight into radiation process.

To compute non-stationary radiation in electrodynamic structures, as a rule, the finite-difference method is used. But for all that the restriction of the computational domain problem is arised in unbounded regions. For solving this problem the computational domain is usually restricted by virtual boundaries. The approximate or accurate Absorbing Boundary Conditions (ABCs) are applied on these boundaries.

In 1990 the Malony and Smith work [1] was devoted to the numerical modeling of the radiation of coaxial waveguide with the endless flange. The problem geometry is shown in Fig. 1.

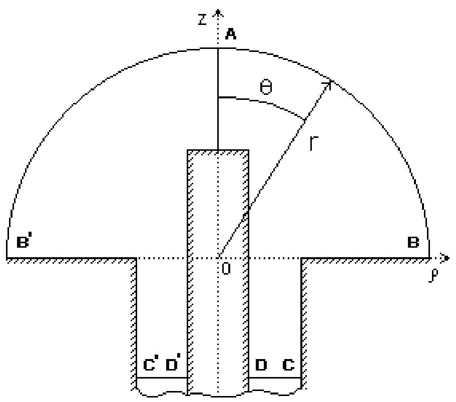

Fig. 1 Geometry for the boundary value problem – coaxial waveguide with infinite image plane. Axial symmetry is assumed.

Axial symmetry is supposed. All conductors are assumed perfect. The electromagnetic field in this system is described by the following initial-boundary problem in cylindrical coordinates:

$$\begin{cases} \left[\frac{\partial}{\partial \rho}\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}\right] U(\rho,z,t)=0 \\ U(\rho,z,t)\equiv H_\varphi;\ \frac{\partial E_\rho}{\partial t}=-\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\partial H_\varphi}{\partial z};\ \frac{\partial E_z}{\partial t}=\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho H_\varphi\right] \\ \textit{boundary condition } E_{tg}=0 \textit{ on the metal surface} \\ \textit{initial condition } H_\varphi^0=U^0(\rho,z,t)=\frac{A}{\rho}\cdot\exp\left\{-a\frac{(z-t+t_0)^2}{t_0^2}\right\} \end{cases}$$

There is H - polarization is considered in our work. That is the magnetic field has only azimuth component H_φ. The initial disturbance is the TEM pulse of the coaxial waveguide with time duration $\tau=2t_0$. In [1] the approximate absorbing boundary conditions were used. The essential drawback of these conditions is the great error of the field definition under the small grazing angels on the virtual boundaries. Because of it time growth the integral error growth in the entire domain. To get rid of this drawback the accurate ABC must be applied. In our work we have gotten such accurate ABC in the waveguide cross section CD and on the hemisphere AB in the top half-space (Fig. 1).

<u>ABC in the coaxial waveguide cross section</u>

$$U(\rho,-L_0,t)=\sum_{n=0}^{\infty} w_n(\rho)\cdot U_n(-L_0,t)$$

$$U_n(-L_0,t)=\int_0^t\int_a^b \frac{\partial U}{\partial z}(\rho_1,-L_0,t)w_n(\rho_1)\rho_1 d\rho_1\ J_0(\beta_n(t-\tau))d\tau$$

Here w_n are the field distribution in guide cross section for TM_n mode:

$$w_n(\rho)=\begin{cases} \frac{1}{\rho}\frac{1}{\sqrt{\ln b/a}} & if \quad n=0 \\ \frac{\varphi(\rho,b)}{\sqrt{\frac{2}{\pi^2\beta_n^2}-\frac{a^2}{2}\varphi(a,b)}} & if \quad n\neq 0 \end{cases},$$

where $\varphi(\rho,b)=J_1(\beta_n\rho)N_0(\beta_n b)-J_0(\beta_n b)N_1(\beta_n\rho)$

and β_n is the root of equation:

$$J_0(\beta_n b)N_0(\beta_n a)-J_0(\beta_n a)N_0(\beta_n b)=0$$

ABC on the AB arc

$$U(r,\theta,t)=\int_0^{\pi/2} w_{2n+1}(r,t)\mu_{2n+1}(\cos\theta)\sin\theta\, d\theta$$

$$w_{2n+1}(r,t)=$$

$$\left[(a/r)^{2n+2}+S_{2n+3/2}(r,a,r-a)\right]w_{2n+1}\left(a,t-(r-a)\sqrt{\varepsilon}\right)-$$

$$-\int_0^{t-(r-a)\sqrt{\varepsilon}} w_{2n+1}(a,\tau)\frac{\partial}{\partial\tau}S_{2n+3/2}\left(r,a,(t-\tau)/\sqrt{\varepsilon}\right)d\tau$$

Where S_γ are expressed through Hankel function:

$$S_\gamma\left(r,a,(t-\tau)/\sqrt{\varepsilon}\right)=$$

$$\sum_{s=1}^{n}2\,\mathrm{Re}\left(\frac{H_\gamma^{(1)}(rz_s)}{H_{\gamma-1}^{(1)}(az_s)}\frac{1}{az_s}\exp\left\{-iz_s(t-\tau)/\sqrt{\varepsilon}\right\}\right)+$$

$$+\frac{H_\gamma^{(1)}(rz_{n+1})}{H_{\gamma-1}^{(1)}(az_{n+1})}\frac{1}{az_{n+1}}\exp\left\{-iz_{n+1}(t-\tau)/\sqrt{\varepsilon}\right\}$$

and $\mu_m(x)$ is Legendre functions, z_s is the roots of the equation

$$H_\gamma^{(1)}(Lz_s)=0\,, \qquad \gamma=2n+3/2$$

in fourth quadrant.

These absorbing boundary conditions are applicable for arbitrary three-dimension systems with axial symmetry. They are nonlocal in space and in time. The main advantage of the ABC in hemisphere is the absence of the normal derivative. Therefore it can be realized in the Cartesian grid effectively.

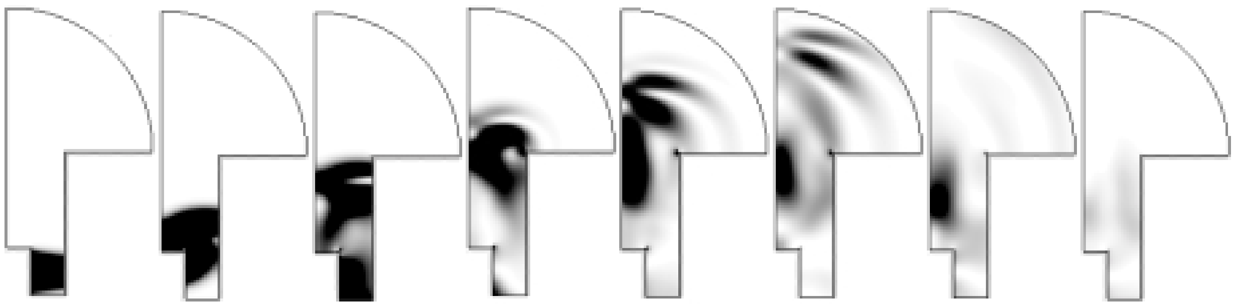

Fig. 2.1. T = 115 ps.
Fig. 2.2. T = 185 ps.
Fig. 2.3. T = 255 ps.
Fig. 2.4. T = 325 ps.
Fig. 2.5. T = 395 ps.
Fig. 2.6. T = 455 ps.
Fig. 2.7. T = 555 ps.
Fig. 2.8. Time is 785 ps.

Numerical realization and testing of these conditions were carried out on the same pulse antenna model as the fig. 1. For the ABC error estimation the field calculation was carried out at presence of the virtual boundaries and in their absence. The average fractional error is about 10^{-4} for any moments of time. That is it is letter than the finite-difference discretization error. So our accurate ABC is not distort of the simulated process.

Our computer program allows to study coaxial waveguide transient radiation and to optimize the antenna geometry for its efficiency maximization.

Finally we want to demonstrate the numerical example of the coaxial waveguide transient radiation. On figures 2 we can see the TEM pulse transformation at the consecutive moments of time. The energy density gray scale is used. The half of computational domain is shown only. Time is specified in picoseconds. The inner and the outer radii of the coaxial waveguide are 0.01m and 0.023m respectively. The distance from the inner conductor tip to image plane level is 0.04 m. The pulse duration is $\tau=2t_0=1/3\,[\mu s]$. The calculated efficiency of this antenna geometry is 61.5%.

REFERENCES

[1] Maloney J.G., Smith G.S. and Scott W.R., Accurate Computation of Radiation from Simple Antennas Using the Finite-Difference Time-Domain Method // IEEE Trans. Antennas Propagat., - 1990. – V. 38, № 2, - P. 1059-1068.