SIMULATION OF "DOUBLE SCROLLS" BY USING THE GENERALIZED SPICE MODEL FOR CELLULAR NEURAL NETWORKS

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ABSTRACT

The Cellular Neural Network (CNN) structure is first proposed by L.O. Chua in 1988. It finds very attractive applications in many fields – especially in image processing. In this paper, first a generalized model of CNN for PSpice is presented and then it is considered as a nonlinear system which can generate "double scrolls". Two sets of differential equation systems are used and a normalization process is carried out in order to find the practical values of the cell componants for the simulation of the above phenomenon. The results are verified with the ones that were found in the literature.

1. CELLULAR NEURAL NETWORKS (CNN)

CNN is a class of dynamic neural networks [1]. Unlike the Hopfield Network, in this structure a cell can only interact directly with its nearest cells for an $r=1$ neighbourhood. The neighbourhood in a CNN is defined as:

$$N_r(i,j) = \{C(k,l) \mid \max\{|k-i|,|l-j|\} \le r, 1 \le k \le M, 1 \le l \le N\} \qquad (1)$$

The equivalent circuit of a cell in a CNN is shown in Figure-1.

2. CIRCUIT EQUATIONS OF THE CNN

In order to derive the state equation of a cell, we apply the KCL to $v_{xi,j}$ node, therefore, we will find the equation below:

$$0 = -I + C\frac{dv_{xi,j}}{dt} + \frac{1}{R}v_{xi,j} - \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l)\cdot v_{yk,l} - \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l)\cdot v_{uk,l} \qquad (2)$$

Also the output equation will be as follows:

$$v_{yi,j} = f(v_{xi,j}) = \frac{1}{2}\left[\left|v_{xi,j}+1\right| - \left|v_{xi,j}-1\right|\right] \qquad (3)$$

If we have an *MxN* CNN structure including *NM* cells we can re-write the whole state equations in a matrix form:

$$\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{A} * \mathbf{y}(\mathbf{x}) + \mathbf{B} * \mathbf{u} + \mathbf{I} \qquad (4)$$

where (*) is the two-dimensional convolution operator. **A** is called *cloning template*, **B** is called *control template* and **I** is called *threshold*.

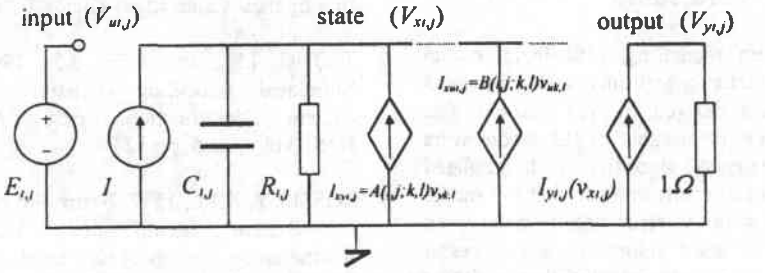

Figure – 1: *A generalized equivalent cell circuit of a CNN.*

The I_{xy} and I_{xu} dependent current sources are controlled by the outputs and inputs of the $C(k,l) \in N_r(i,j)$ neighbours of the cell respectively. The voltage controlled current source I_y – whose equation will be given in the next section – is a nonlinear (piecewise - linear) function of the $v_{xi,j}$ state and can be realized with a simple op-amp circuit [2].

Suppose we have a dynamical system whose state eqaution is:

$$\dot{\mathbf{X}} = -\mathbf{S}\cdot\mathbf{X} + \mathbf{A}\cdot\mathbf{Y} \qquad (5)$$

If we consider a CNN structure with three cells and replace (2) in (5), for each cell we will have the following matrix equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}\frac{1}{R_1C_1} & \frac{-s_{12}}{C_1} & \frac{-s_{13}}{C_1}\\ \frac{-s_{21}}{C_2} & \frac{1}{R_2C_2} & \frac{-s_{23}}{C_2}\\ \frac{-s_{31}}{C_3} & \frac{-s_{32}}{C_3} & \frac{1}{R_3C_3}\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}\frac{A_{11}}{C_1} & \frac{A_{12}}{C_1} & \frac{A_{13}}{C_1}\\ \frac{A_{21}}{C_2} & \frac{A_{22}}{C_2} & \frac{A_{23}}{C_2}\\ \frac{A_{31}}{C_3} & \frac{A_{32}}{C_3} & \frac{A_{33}}{C_3}\end{bmatrix}\cdot\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} \quad (6)$$

We can model any dynamical system with the state equations above by using the generalized equivalent of the CNN cell circuit. The design is the determination of the R, C passive componant values and the S, A voltage controlled current source values.

3. NUMERICAL EXAMPLES

Consider the following astable biased CNN structure [3]:

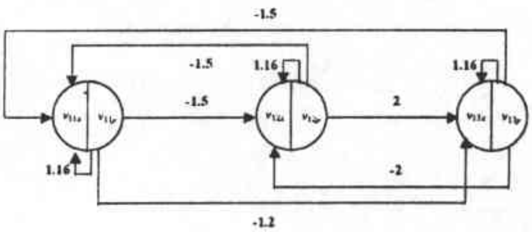

Figure – 2: *A CNN structure for the simulation of "double scrolls".*

The matrix form of the state equations for the system above will be

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1.16&-1.5&-1.5\\-1.5&1.16&-2.0\\-1.2&2.0&1.16\end{bmatrix}\cdot\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} \quad (7)$$

If we choose the capacitances of C_1, C_2 and C_3 all equal to 100µF and the time – constant for each cell τ= 1 s., then the normalized values of the componants will be found as:

$R_1=R_2=R_3=10\text{k}\Omega$,
$A_{11}=A_{22}=A_{33}=116\mu S$
$A_{12}=A_{21}=A_{13}=-150\mu S$
$A_{23}=-A_{32}=200\mu S$
$A_{31}=-120\mu S$

If we sketch the (v_{x11}, v_{x12}) plane in PSpice we will observe the double scroll seen in Figure - 3.

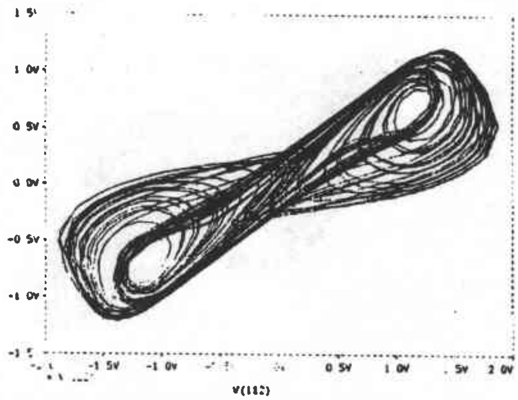

Figure – 3: *The double scroll generated by the differential equation in (7). Projection onto the (v_{x11}, v_{x12}) plane.*

Another system of equations that generates "double scrolls" can be derived from the structure below [4]:

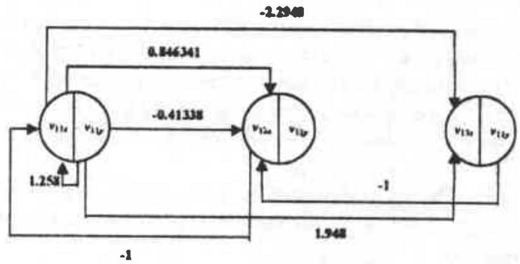

Figure – 4: *Another system that can be used for the generation of "double scrolls".*

By arranging the differential equations that represent the system we will have

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=-\begin{bmatrix}1.168&1&0\\-0.846341&0&1\\1.2948&0&0\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1.258&0&0\\-0.41338&0&0\\1.948&0&0\end{bmatrix}\cdot\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix} \quad (8)$$

In this case, it is clear that we will need an extra dependent current source for each cell which are controlled by the states of the neighbour cells. This specific structure is called *State Controlled CNN* (SC–CNN). By applying the same procedure for the componant values,

$C_1=C_2=C_3=100\mu F$
τ=1 s., $R_3=10\text{k}\Omega$, $R_1=8561\Omega$
$S_{12}=S_{23}=-100\mu S$
$S_{21}=84.6341\mu S$
$S_{31}=-129.48\mu S$
$A_{11}=125.8\mu S$
$A_{21}=-41.338\mu S$
$A_{31}=194.8\mu S$

will be found. The (v_{x11}, v_{x12}) plane is shown in Figure - 5.

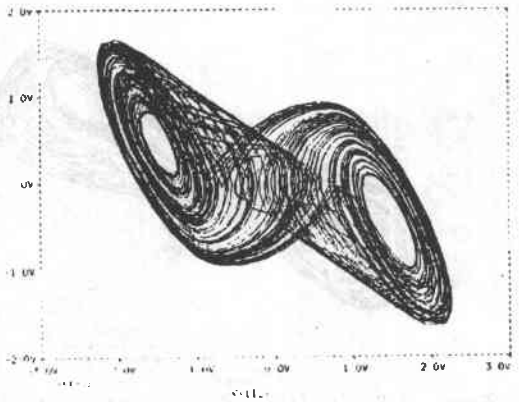

Figure – 5: *The double scroll generated by the differential equation in (8). Projection onto the (v_{x11}, v_{x12}) plane.*

4. CONCLUSION

In this paper, a topology for the simulation of "double scrolls" in nonlinear dynamic systems is proposed. This topolgy basicly depends on an equivalent cell circuit in a CNN structure. Two different sets of state equations that generate "double scrolls" are used as the numerical examples. The normalized componant values are also computed for PSpice simulations.

These PSpice models can be used in the design of chaotic systems as well as in analyzing the stability of Cellular Neural Networks.

5. REFERANCES

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