Low-pass Filter Approximation with Evolutionary Techniques

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Abstract

In this work, two evolutionary techniques, Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) algorithms are used to optimize the denominator coefficients of the low-pass filter transfer function. Optimum selection of the coefficients will approximate the transfer function to ideal characteristic. Two different order of transfer functions are taken into consideration. Compared to conventional methods, both PSO and ABC minimize the approximation error in a short computation time.

1. Introduction

The low-pass filter is used in virtually every communication, measurement, and control system. Its function is to suppress all high-frequency components of a signal beyond some cut off frequency ω_c, while allowing the lower frequencies to pass through the filter unattenuated. Hence the magnitude response of an ideal low-pass filter is defined by the "'brickwall" characteristic. However, this ideal response can not be expressed as a rational function of angular frequency. Since the ideal low-pass filter is not physically realizable, it is possible to design a physical circuit which approximates the ideal characteristics to within any prescribed [1]. Solution of the approximation problem is a major step in design procedure of a filter and is equally important in design of both analog and digital filters [2].

Approximation problem can be defined as a curve fitting optimization. Several methods are suggested to overcome this problem. For example, a Butterworth filter [3] meets magnitude specifications and employs no ripples in passband. However, it may have a long transition band. A Chebyshev filter [3] meets the hard specifications and owns lower complexity as well as ripples in the passband.

Before the era of fast and efficient computation, each method would be encapsulated as a table of transfer function coefficients and engineers would realize a filter with the table data. These tables supply order-based coefficients for a normalized low-pass filter. A vast space of filter coefficients remains unexplored by classical approximations [4]. The implementation of optimization techniques offers the opportunity to exploit a large solution space which is not covered by design considerations of conventional analog filters such as Butterworth and Chebyshev approximations and enables better suited filters for particular specifications.

The application of evolutionary techniques in filter approximation and thus optimization of coefficients of the transfer function is a promising area which is based on concepts of natural selection and survival of the fittest. In the literature, enhanced particle swarm optimization based Optim-filt system is developed which evolves filter approximations in the form of coefficients of a transfer function [4]. However approximation errors with respect to conventional methods are not specified. In [5], semidefinite programming (SDP) is used to optimize all-pole filters by compensating the classical designs in that overall performance in the passband or the stopband. In [6, 7], approximation problem is formulated as a sequential quadratic programming (SQP) problem and given filter specification is translated into a tolerance scheme which can be extended by constraints in the frequency and/or in the time domain.

In this work, Artificial Bee Colony (ABC) algorithm and Particle Swarm Optimization (PSO) which are swarm based evolutionary techniques are utilized for optimizing the coefficients of the transfer functions of a third order and a fifth order low-pass filter. The aim is to obtain the optimum coefficient value set which minimizes the error between the transfer function and ideal characteristic in a short computation time. A comparison of approximation error obtained with conventional methods which are reviewed in section 2 and evolutionary techniques which are reviewed in section 3 is provided in section 4. Finally, section 5 presents concluding remarks and suggestions for future work.

2. Conventional Filter Approximation Methods

In practical filter design, the amplitude response is more often specified than the phase response. The amplitude response of the ideal low-pass filter with normalized cutoff frequency at ω_c= 1 rad/s is shown in Fig 1. It has a gain of 1 (0 dB) in the passband and a gain of 0 in the stopband. This ideal amplitude response can not be expressed as a rational function of angular frequency (ω). Since the ideal low-pass filter is not physically realizable, there exists several approximation methods some of which are reviewed in the following.

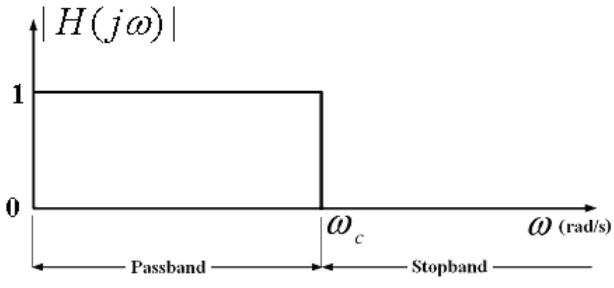

Fig. 1 Ideal low-pass characteristic

2.1. Butterworth Approximation

Butterworth approximation (1) is aimed at constructing maximally flat magnitude response filters, optimized for gain flatness in the pass-band.

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}} \quad (1)$$

where, n represents both the order of the transfer function and the realized filter. The magnitude response of n[th] order Butterworth function is sketched in Fig. 1 considering $n = \{2, 3, 5, 10\}$ with respect to ideal characteristic of a low-pass filter. Observe that $|H(0)|=1$ and $|H(j1)|=0.707$. In terms of the decibel scale, the Butterworth magnitude response starts from 0 dB at DC and drops down monotonically by 3 dB (for all n values) at $\omega_c=1$ rad/s. Considering that $n\to\infty$, Butterworth approximation approaches to the ideal low-pass characteristic [1, 3].

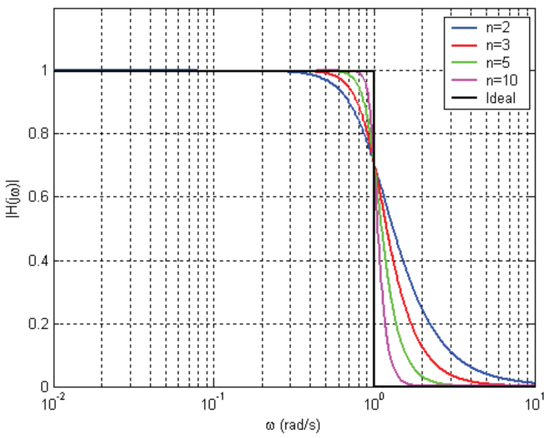

Fig. 2 Amplitude response of n[th] order Butterworth function

Before a circuit can be synthesized, it is necessary to derive the transfer function. Performing the mathematical operations for Butterworth approximation in [1-3], the coefficients of the general form of low-pass filter transfer function given in (2) is obtained. As an example, the second order low-pass filter transfer function is provided in (3). The denominator of (2) is known as a Butterworth polynomial. The first five Butterworth polynomials are tabulated in Table 1.

$$H(s) = \frac{b_0}{s^n + b_{n-1}s^n + \cdots + b_1 s + b_0} \quad (2)$$

$$H(s) = \frac{1}{(s-e^{j\frac{3\pi}{4}})(s-e^{+j\frac{3\pi}{4}})} = \frac{1}{s^2+\sqrt{2}s+1}, \quad (n=2) \quad (3)$$

Table 1 Butterworth Polynomials

n	Butterworth Polynomials
1	$s+1$
2	$s^2+1.414s+1$
3	s^3+2s^2+2s+1
4	$s^4+2.613s^3+3.414s^2+2.613s+1$
5	$s^5+3.236s^4+5.236s^3+5.236s^2+3.236s+1$

2.2. Chebyshev Approximation

The Chebyshev characteristic (4) has a steeper roll off near the cutoff frequency when compared to the Butterworth, but at the expense of monotonicity in the passband [2].

$$|H(j\omega)|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\omega)} \quad (4)$$

Where, ε is the ripple factor and its values are between 0 and 1, $C_n(\omega)$ is a Chebyshev polynomial of the n[th] order and it's terms are $C_0(\omega)=1$, $C_1(\omega)=\omega$, $C_{n+1}(\omega)=2\omega C_n(\omega)$- $C_{n-1}(\omega)$. The passband exhibits equiripple behavior, with the ripple determined by the ripple factor ε. In the passband, the Chebyshev polynomial alternates between 0 and 1 so the filter gain will alternate between maxima at $|H(j\omega)|=1$ and minima at $|H(j\omega)|=1/\sqrt{1+\varepsilon^2}$. The ripple is often given in dB (5).

$$Ripple\ in\ dB = 20\log\frac{1}{\sqrt{1+\varepsilon^2}} \quad (5)$$

As seen equation (5), the ripple amplitude is 3 dB for $\varepsilon=1$. Plot of the magnitude of Chebyshev approximation function for 1 dB ripple and n=3, 5, 9 are shown in Fig. 3.

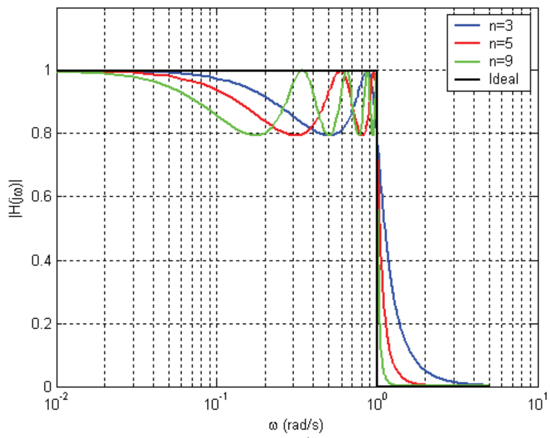

Fig. 3 Amplitude response of n[th] order Chebyshev function

The coefficients of the Chebyshev filter functions, as well as their poles, can be tabulated for various ripple dB values i.e {0.1, 0.5, 1, 3}. A sample of such tabulation for 1 dB ripple value is tabulated in Table 2. This table does not give the normalized Chebyshev filter functions. In all these cases, the upper edge of the passband ripple occurs at ω_c=1 rad/s.

Table 2 Chebyshev Polynomials

n	Chebyshev Polynomials
1	$s+1.965$
2	$s^2+1.097s+1.102$
3	$s^3+0.7378s^2+1.0222s+0.3269$
4	$s^4+0.952s^3+1.453s^2+0.742s+0.275$
5	$s^5+0.7064s^4+1.4995s^3+0.6935s^2+0.4594s+0.0817$

3. Evolutionary Algorithms

EA techniques differ in the implementation details and the nature of the particular applied problem. In this study, the performances of artificial bee colony optimization (ABC) and particle swarm optimization (PSO) which are nature inspired EA techniques are evaluated for active filter design. Details of those are introduced in the following.

3.1. Artificial Bee Colony Optimization

Artificial Bee Colony (ABC) algorithm [8] is a recently introduced optimization algorithm and simulates the foraging behavior of bee colony. In ABC algorithm, the position of a food source represents a possible solution to the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. First of all, the food source positions are randomly initialized as x_i (i=1,…,SN) where SN is the maximum number of the food sources. Each employed bee, whose total number equals to the the number of food sources, produces a new food source in her food source site as given in (6).

$$v_{ij} = x_{ij} + \varphi_{ij}(x_{ij} - x_{kj}) \tag{6}$$

where φ_{ij} is a uniformly distributed real random number within the range [-1, 1], k is the index of the solution chosen randomly from the colony and j is the index of the dimension of the problem. After producing v_{ij}, this new solution is compared to x_{ij} solution and the employed bee exploits a better source while each onlooker bee whose total number is equal to the number of employed bees selects a food source with the probability as given in (7).

$$p_i = \frac{fit_i}{\sum_{j=1}^{SN} fit_j} \tag{7}$$

where fit_i is the fitness of the solution x_{ij} and produces a new source in selected food source site by (4). After all onlookers are distributed to the sources, sources are checked whether they are to be abandoned. The employed bee associated with the abandoned source becomes a scout and makes random search in problem domain by (8). The best food source found so far has been memorized and the production steps are repeated until the stopping criterion is met [9].

$$x_{ij} = x_j^{\min} + (x_j^{\max} - x_j^{\min}) * rand \tag{8}$$

3.2. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is an evolutionary computation method based on the social behavior, movement and intelligence of swarms searching for an optimal location in a multidimensional search area which has been developed by Eberhart [10]. The approach uses the concept of population and a measure of performance similar to the fitness value used with evolutionary algorithms. Population consists of potential solutions called particles. Each particle is initialized with a random position value. In each iteration, the fitness function is evaluated by taking the current position of the particle in the solution space and two best values (p_{best}, g_{best}). Personal best value, p_{best}, is the location of the best fitness value obtained so far by the particle. Global best value, g_{best}, is the location of the best fitness value achieved so far considering all the particles in the swarm [10-12].

In particle population matrix, containing N number of particles, i^{th} particle with a feature number of D is denoted as $x_i=[x_{i1}, x_{i2},…, x_{iD}]$. For each iteration, the velocity and the position vector of the i^{th} particle in NxD dimension of the search space are updated as follows:

$$v_{id}^{k+1} = w.v_{id}^{k} + c_1.rand_1^{k}.(pbest_{id}^{k} - x_{id}^{k}) + c_2.rand_2^{k}.(gbest_d^{k} - x_{id}^{k}) \tag{9}$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \tag{10}$$

Here, the range of i, d and k indices are defined as {1...N}, {1…D} and {1…max_iteration_number} respectively. The acceleration factors c_1 and c_2 indicates the relative attraction toward p_{best} and g_{best} respectively. Following $rand_1$ and $rand_2$ are random numbers uniformly distributed between zero and one. Inertia weight parameter, w, controls the tradeoff between the global and the local search capabilities of the swarm. Initially w should be chosen as less than one and should be decreased linearly in each iteration.

PSO algorithm used in this work has been built up for the global best (g_{best}) PSO model. The gbest model is chosen since it converges faster than local best (l_{best}) PSO [13]. This is due to the larger particle connectivity of g_{best} PSO. Each particle can interact with every other one in the swarm and can be attracted to the best position obtained by any other particle.

4. Simulation Results

In order to investigate the usage of evolutionary algorithms (EA) in filter approximation and to compare with conventional methods, optimization of transfer function coefficients of n^{th} order low-pass filter is carried out. By establishing design parameters to EA and satisfying desired constraints, the optimal coefficients were aimed to be determined by EA methods. Design problem has been introduced by composing an equation consists of design parameters as a cost function (CF). In the beginning of the algorithm a certain range was determined for design parameters by human designer. EA should minimize the given CF and obtain design parameter values for the given range which gives minimum CF value. In order to introduce the design problem to EA, a CF which includes values of coefficients as design parameters is constituted as given in (11).

$$Error = \left| 0.5\sum_{\omega=0.1}^{\omega_c}(1-|H(j\omega)|) + 0.5\sum_{\omega=\omega_c}^{2\omega_c}(0-|H(j\omega)|) \right| \tag{11}$$

where, ω_c=1 rad/s and ω is the discrete angular frequency and sampled as {0.1, 0.2, 0.3,…, 2}. $H(j\omega)$ is the transfer function of the low-pass filter. Considering ideal characteristic, the approximation error of pass-band and stop-band is determined by the first term and second term of (11), respectively.

The right side of (11) would constitute the CF which EA would minimize. It is desired to obtain the exact values of design parameters which equate CF to a very close value to zero. In this work, PSO and ABC algorithms are utilized for EA based filter approximation and performances of those are evaluated by means of computation time and accuracy. The aim is to estimate the coefficient values of transfer functions of a third order and a fifth order (n=3, n=5) low pass filter with minimum design error. Each coefficient value was designated to take value between 1 and 10.

Considering ABC, the number of colony size was set to 2000, the numbers of onlooker bees and employed bees are 50% of the colony size. Table 3 summarizes ABC parameters and simulation results for filter approximation design problem where n represents the order of the filter.

Table 3 ABC Parameters and Simulation Results

		ABC	
		n=3	n=5
Parameters	Search Limit	100	100
	Colony size	2000	2000
	Number of generations	500	500
Simulation Results	Computational Time (s)	12.5	30.3
	Design error	0.1246	0.0680
	Coefficient b_0	0.5003	2.2749
	Coefficient b_1	1.1405	5.5629
	Coefficient b_2	1.1836	8.8675
	Coefficient b_3	-	5.1283
	Coefficient b_4	-	9.4922

Considering PSO, initial population matrix size was *10xn* where row number of *10* indicates the number of particles in the population and column number of n is the dimension of particle vector where n also represents the order of the filter. Table 4 summarizes PSO parameters and simulation results for filter approximation design problem.

Error comparison of conventional approximation methods vs. EA based methods is tabulated in Table 5. Results demonstrate that EA methods outperform conventional ones by means of accuracy. ABC obtained smaller approximation error than PSO; however computational time of PSO is less than ABC.

Table 4 PSO Parameters and Simulation Results

		PSO	
		n=3	n=5
Parameters	c_1,c_2	1.7	1.7
	w	0.99	0.99
	Swarm size	10	10
	Number of generations	1000	1000
Simulation Results	Computational time (s)	1.965	8.42
	Design error	0.1633	0.0885
	Coefficient b_0	0.9339	1.4413
	Coefficient b_1	1.8823	3.0078
	Coefficient b_2	1.9169	2.9807
	Coefficient b_3	-	3.2120
	Coefficient b_4	-	3.4273

Table 5 Error Comparison of EA vs. Conventional methods

Approximation Method	Error (n=3)	Error (n=5)
Chebyshev1 (Ripple=1 dB)	0.4023	0.4971
Butterworth	0.3211	0.1866
Chebyshev1 (Ripple=0.1 dB)	0.3022	0.1206
PSO	**0.1633**	**0.0885**
ABC	**0.1246**	**0.0680**

The graphs in Fig. 4 and Fig. 5 are illustrating the results of the proposed evolutionary techniques with respect to the conventional methods for a third order and fifth order approximation problem, respectively. It can be demonstrated that PSO and ABC both minimizes the approximation error in a short computation time. The sharpest descent in transition band is obtained with ABC; however, PSO is much approximated to ideal characteristic in passband.

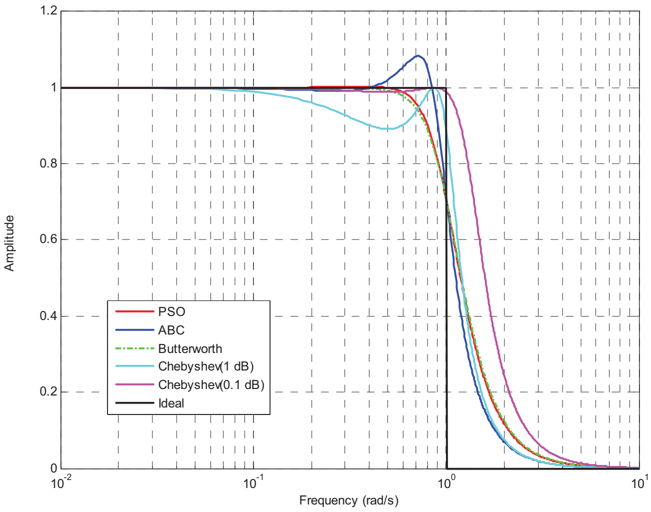

Fig. 4 Amplitude responses of 3rd order approximation functions

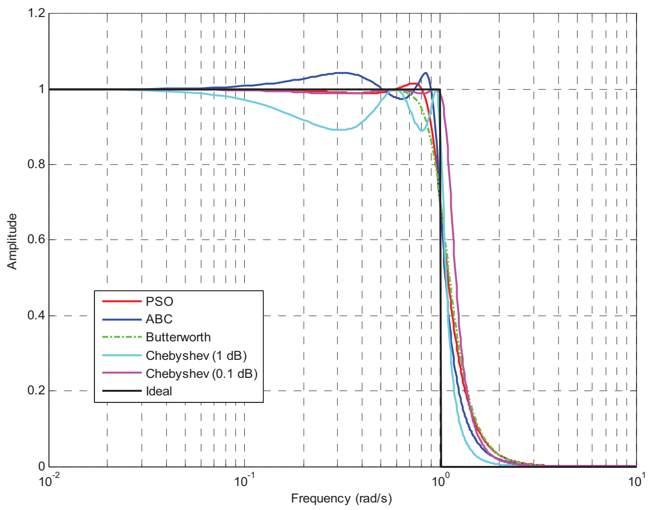

Fig. 5 Amplitude responses of 5th order approximation functions

5. Conclusion

The performances of evolutionary algorithms on n^{th} order low-pass filter approximation have been investigated. ABC and PSO algorithms were utilized for optimization of both 3^{rd} and 5^{th} order low-pass transfer function and selection of optimum denominator coefficients which will approximate the relevant transfer function to ideal characteristic by means of accuracy and computation time.

Simulation results demonstrate that both PSO and ABC minimize the approximation error compared to the conventional methods in a short computation time. The sharpest descent in transition band is obtained with ABC; however, PSO is much approximated to ideal characteristic in passband. Consequently, evolutionary techniques have effectively explored the search space in order to obtain denominator coefficients of a low-pass transfer function.

As a further work, studies will be carried out for improving the evolutionary algorithm based approximation method. An extension of this research might be either phase approximation or optimization of step response in time domain.

6. References

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