The change of the active power transmitted through dc line during transient in the transient stability studies of ac/dc power systems

K. Ayan
Sakarya University
Department of Electrical and Electronics Engineering
54040 / Sakarya-Turkey

Z. Özdemir
Sakarya University
Department of Electrical and Electronics Engineering
54040 / Sakarya-Turkey

U. Arifoğlu
Sakarya University
Department of Electrical and Electronics Engineering
54040 / Sakarya-Turkey

M.Alcı
Sakarya University
Department of Electrical and Electronics Engineering
54040 / Sakarya-Turkey

Abstract - In this paper, the influence of the active power transmitted through the dc line to the system transient stability is studied via the controlling unstable equilibrium points approach based on direct method. It is well known that the transfer conductance's present in the internal bus description using the classical model pose a problem in constructing a valid Lyapunov Function, as well as in computing CCT in transient stability studies. On this occasion, loads and dc lines are expressed as current injections to the buses to which they are connected, without including them to the bus admittance matrix as the conventional constant admittance. In this study, a proportional controller is used for controlling of the active power transferred through the dc line and the ratio of reactive power absorbed the converters to the active power transmitted through the dc line is taken into account as a constant . This study is made for value of two different dc line power and the change of the active power transmitted through on dc line during transient is given. Above this study that is explained roughly is tested on a sample system with four generators and six buses.

Keywords: Lyapunov function, controlling unstable equilibrium points, integrated AC/DC power system, transient stability.

I. INTRODUCTION

An important characteristic that has a bearing on power system stability is the relationship between interchange power and angular positions of the rotors of synchronous machines. Transient stability is the ability of the power system to maintain synchronism when subjected to a severe transient disturbance as phase-ground, phase-phase-ground, and three phase faults. Stability depends on the initial conditions of the system together with the place, type, and intensity of the fault. HVDC systems have the ability to rapidly control the transmitted power. Therefore, they have a significant impact on the stability of the associated ac power systems[1,2].

II. POWER SYSTEM MODEL

For the power system which it has n generators, the swing equation of each one is given as follows

$$M_k \frac{d^2\delta_k}{dt^2} + D_k \frac{d\delta_k}{dt} = P_{mk} - P_{gk} \quad (k = 1,2,..,n) \qquad (1)$$

where M_k is the moment of inertia of the k^{th} machine, D_k is the damping coefficient of the k^{th} machine, P_{mk} is the mechanical input power of the k^{th}. machine and P_{gk}, is the electrical output power of the k^{th}. machine. P_{gk} in general is very complicated expression computed from the non-linear differential equations of the electrical part of the synchronous machine and the algebraic equations of the transmission network and the synchronous machine[3].

If we assume the transmission network to consist of n+m buses of which the n buses are buses where generators and loads are connected and at the m buses only loads are connected, generally the nodal admittance matrix $\underline{\tilde{Y}}_{BUS}$ to contain the admittance's to consist of the subtransient reactance's of the generators and the transmission lines of this network is given as follows:

$$\underline{\tilde{Y}}_{BUS} = \begin{bmatrix} \underline{y}_G & -\underline{y}_G & \underline{0} \\ -\underline{y}_G & \underline{Y}_1 + \underline{y}_G & \underline{Y}_2 \\ \underline{0} & \underline{Y}_3 & \underline{Y}_4 \end{bmatrix} \qquad (2)$$

In the expression $\underline{\tilde{Y}}_{BUS}$ in equation (2), the network equations can be expressed as follows by separating the internal generator buses from the other buses:

$$\begin{bmatrix} \underline{I}_G \\ \underline{0} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{GG} & \underline{Y}_{GL} \\ \underline{Y}_{LG} & \underline{Y}_{LL} \end{bmatrix} \begin{bmatrix} \underline{E}_G \\ \underline{V}_L \end{bmatrix} \qquad (3)$$

By eliminating all the nodes except for the internal generator buses and describing the reduced bus admittance matrix as $\underline{Y}_{red} = \lfloor \underline{Y}_{GG} - \underline{Y}_{GL}\underline{Y}_{LL}^{-1}\underline{Y}_{LG} \rfloor$, we have

$$\underline{I}_G = \lfloor \underline{Y}_{red} \rfloor \underline{E}_G \qquad (4)$$

Thus the active power can be given as follows:

$$P_{gk} = \mathrm{Re}\{E_k I_k^*\} = \mathrm{Re}\left\{E_k \sum_{j=1}^{n} Y_{kj}^* E_j^*\right\} \quad k = (1,2,..,n) \qquad (5)$$

By describing $P_k = P_{mk} - |E_k|^2 G_{kk}$ and $P_{ek} = \sum_{\substack{j=1 \\ \neq k}}^{n} \left[C_{kj}\sin(\delta_k - \delta_j) + D_{kj}\cos(\delta_k - \delta_j)\right]$, equation (1) can be expressed as follows:

$$M_k \frac{d^2\delta_k}{dt^2} + D_k \frac{d\delta_k}{dt} = P_k - P_{ek} \quad (k = 1,2,..,n) \qquad (6)$$

Equation (6) is basic form for each generator. For faulted and postfault case, equation (6) is given in two forms as follows:

$$M_k \frac{d^2\delta_k}{dt^2} + D_k \frac{d\delta_k}{dt} = P_{mk} - P_{gk}^f = P_k^f - P_{ek}^f, \qquad (7)$$
$$0 < t \le t_{cl}, \delta_k(0) = \delta_k^0, \dot{\delta}_k(0) = 0 \; (k = 1,..,n)$$

$$M_k \frac{d^2\delta_k}{dt^2} + D_k \frac{d\delta_k}{dt} = P_{mk} - P_{gk}^{pf} = P_k^{pf} - P_{ek}^{pf}, \qquad (8)$$
$$t \ge t_{cl}, \; \delta_k(t_{cl}), \; \dot{\delta}_k(t_{cl}) \; (k = 1,..,n)$$

III. REPRESENTATION OF THE EFFECT OF LOADS

It is well known that the transfer conductance's present in the internal bus description using the classical model pose a problem in constructing a valid V-function, as well as in computing t_{cr}. These transfer conductance's are mostly due to the system loads being converted to constant impedance and subsequent elimination of the load buses. In the method proposed here, which also applies to the dc line element, the effect of loads is reflected at the internal buses in the form of additional bus power injections.

Consider a power system network consisting of n generators and m buses. The bus admittance matrix $\underline{Y}_{LL}$ for the transmission network alone, excluding the loads and DC line, is formulated and is then augmented with the network elements corresponding to direct axis reactance of the n machines. The resulting augmented matrix $\underline{\tilde{Y}}_{BUS}$ has (n+m) buses altogether, and is given as equation (3). $\underline{Y}_{GG}$, $\underline{Y}_{GL}$, $\underline{Y}_{LG}$, and $\underline{Y}_{LL}$ terms in equation (3) are submatrices of dimensions (nxn), (nxm), (mxn) and (mxm) respectively. The overall network representation is

$$\begin{bmatrix} \underline{I}_G \\ \underline{I}_L \end{bmatrix} = \left[\underline{\tilde{Y}}_{BUS}\right] \begin{bmatrix} \underline{E}_G \\ \underline{V}_L \end{bmatrix} \qquad (9)$$

where $\underline{I}_G^T = [I_{G1}, I_{G2}, \ldots, I_{Gn}]$ and $\underline{I}_L^T = [I_{L1}, I_{L2}, \ldots, I_{Lm}]$.

$\underline{I}_G$ and $\underline{I}_L$ are the current injections at the internal buses of the generators and the transmission network buses, respectively; $\underline{E}_G$ and $\underline{V}_L$ are the associated voltages. $\underline{\tilde{Y}}_{BUS}$ is computed for the faulted and postfault conditions by properly taking the corresponding network changes into account. The method of distribution factors as suggested in [4] is now used for reflecting loads at the internal buses. Eliminating $\underline{V}_L$ from equation (9), we get

$$\underline{V}_L = \underline{Y}_{LL}^{-1}\underline{I}_L - \underline{Y}_{LL}^{-1}\underline{Y}_{LG}\underline{E}_G \qquad (10)$$

and

$$\underline{I}_G = \left[\underline{Y}'_{red}\right]\underline{E}_G + \left[\underline{D}_L\right]\underline{I}_L \qquad (11)$$

where $\underline{Y}'_{red} = \left[\underline{Y}_{GG} - \underline{Y}_{GL}\underline{Y}_{LL}^{-1}\underline{Y}_{LG}\right]$ distribution factor matrix for loads is given by

$$\left[\underline{D}_L\right] = \underline{Y}_{GL}\underline{Y}_{LL}^{-1} \qquad (12)$$

Also, we have

$$I_{Li} = \frac{-P_{Li} + jQ_{Li}}{V_{Li}^*} \qquad (13)$$

where P_{Li} and Q_{Li} are the active and reactive power components of load at the i[th] bus. The additional bus power injections at the internal bus of the k[th] generator (k=1,2,....,n) due to the load i[th] bus (i=1,2,...,m) is obtained as follows

$$\Delta(P_{kLi} + jQ_{kLi}) = -\left(\frac{E_k}{V_{Li}}\right)d_{ki}^*(P_{Li} + jQ_{Li}) \underline{\underline{\Delta}} \; (a_{kLi} + jb_{kLi}).(P_{Li} + jQ_{Li}) \qquad (14)$$

where d_{ki} is the appropriate (k,i) element of $\left[\underline{D}_L\right]$. The complex ratio of voltages (E_k / V_{Li}) is assumed to be constant, corresponding to the prefault values. Thus the loads are represented truly during transient. Since only active power is of interest in the swing equation, we get

$$\Delta P_{kLi} = (a_{kLi}P_{Li} - b_{kLi}Q_{Li}) \qquad (15)$$

The effect of all the loads at the internal bus of the k[th] generator is then obtained as

$$\Delta P_{kL} = \sum_{i=1}^{m}(a_{kLi}P_{Li} - b_{kLi}Q_{Li}) \quad (k = 1,2,..,n) \qquad (16)$$

IV. REPRESENTATION OF THE EFFECT OF DC LINE

The effect of DC line is represented in a manner similar to that of the loads. For simplicity, we assume only one DC line to be present. The analysis, however, easily extends to cases of more than one DC line. In the $\tilde{\underline{Y}}_{BUS}$ of equation (9), all buses except the internal buses of the generators and the bus pair corresponding to the rectifier and inverter terminals of the DC line are eliminated. The reduced network may be represented as follows

$$\begin{bmatrix} \underline{I}_G \\ \underline{I}_D \end{bmatrix} = \begin{bmatrix} \underline{Y}'_{GG} & \underline{Y}_{GD} \\ \underline{Y}_{DG} & \underline{Y}_{DD} \end{bmatrix} \begin{bmatrix} \underline{E}_G \\ \underline{V}_D \end{bmatrix} \tag{17}$$

where $\underline{I}_D^T = [I_r, I_i]$ and $\underline{V}_D^T = [V_r, V_i]$. Subscripts r and i refer to the rectifier and inverter sides, respectively. $\underline{Y}'_{GG}$, $\underline{Y}_{GD}$, $\underline{Y}_{DG}$, and $\underline{Y}_{DD}$ are submatrices of dimensions (jxj), (jx2), (2xj) and (2x2) respectively. By pulling out $\underline{V}_D$ from equation (17), we get

$$\underline{V}_D = \underline{Y}'_{DD}\underline{I}_D - \underline{Y}'_{DD}\underline{Y}_{DG}\underline{E}_G \tag{18}$$

and

$$\underline{I}_G = [\underline{Y}'']\underline{E}_G + [\underline{D}_D]\underline{I}_D \tag{19}$$

where $\underline{Y}'' = [\underline{Y}'_{GG} - \underline{Y}_{GD}\underline{Y}_{DD}^{-1}\underline{Y}_{DG}]$ and distribution factor matrix for DC line is given by

$$[\underline{D}_D] = \underline{Y}_{GD}\underline{Y}_{DD}^{-1} \tag{20}$$

Now, we represent the effect of the DC line currents $\underline{I}_D$ as additional bus power injections at the internal buses of the generations. The rectifier side and inverter side currents of DC line are given in the following forms with respect to bus voltages and active power, as shown figure (1).

$$I_{dcr} = \frac{-P_{dcr} + jQ_{dcr}}{V^*_{dcr}} \tag{21}$$

and

$$I_{dci} = \frac{-P_{dci} + jQ_{dci}}{V^*_{dci}} \tag{22}$$

where $P_{dcr} = -P_{dci} = P_{dc}$ and $Q_{dcr} = Q_{dci} = Q_{dc}$. It is assumed here that the DC line is lossless and the power factors at the rectifier and the inverter stations are equal. The effect of the rectifier and inverter ends of the DC line as additional bus power injections at the internal bus of the k[th] generator is given by

$$\Delta(P_{kr} + jQ_{kr}) = -(\frac{E_k}{V_{dcr}})d^*_{kr}(P_{dc} + jQ_{dc}) \triangleq (a_{kr} + jb_{kr})(P_{dc} + jQ_{dc}) \tag{23}$$

$$\Delta(P_{ki} + jQ_{ki}) = -(\frac{E_k}{V_{dci}})d^*_{ki}(-P_{dc} + jQ_{dc}) \triangleq (a_{ki} + jb_{ki})(-P_{dc} + jQ_{dc}) \tag{24}$$

where d_{kr} and d_{ki} are the appropriate (k,1) and (k,2) elements of the distribution factor matrix $[D_D]$. From equations (23) and (24), we get

$$\Delta P_{kr} = (a_{kr}P_{dc} - b_{kr}Q_{dc}) \tag{25}$$

and

$$\Delta P_{ki} = (-a_{ki}P_{dc} - b_{ki}Q_{dc}) \tag{26}$$

As in the case of the load model representation, here also the ratios (E_k / V_{dcr}) and $E_k / V_{dci})$ in equations (23) and (24) are assumed to be constant, corresponding to their prefault values. Since a simple structure is assumed for the DC line controller, the output of which is P_{dc}, let $Q_{dc} = q_r P_{dc}$, where q_r is a constant. From equations (25) and (26), we get the total bus power injections at the k[th] generator due to the DC line as

$$\begin{aligned} &\Delta P_{kD} = \Delta P_{kr} + \Delta P_{ki} = \\ &\{(a_{kr} - a_{ki}) - q_r(b_{kr} + b_{ki})\}P_{dc} = \\ &c_{kD}P_{dc}\ (k = 1,..,j) \end{aligned} \tag{27}$$

where c_{kD} is the expression in brackets in equation (27). The parameters c_{kD}, a_{kLi}, and b_{kLi}, which reflect the effect of the DC line and the loads, are thus computed for both the faulted and postfault conditions.

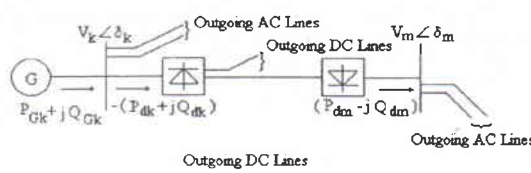

Figure 1. DC line scheme

V. INCLUSION OF DC LINE DYNAMICS

The structure for DC line controller is shown in figure 2 and the equality connected with it is as follows:

$$\dot{P}_{dc} = -(1/T_{dc})P_{dc} + (1/T_{dc})P_{ref} + (K_a/T_{dc})u \tag{28}$$

where u is the external control signal (ECS) obtained from the AC system quantities, such as the difference in rotor speeds of adjacent generators. P_{dc} is constrained to vary within the specified practical limits. While the faulted system equations are integrated, equation (28) also is solved for P_{dc}. At the end of each time step, the additional bus power injections at the internal buses of the generators are calculated using equation (27). The effect of DC line is thus represented as the term that modifies the power input of the generator.

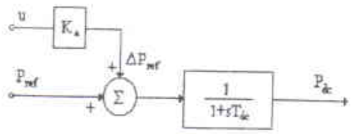

Figure 2. Simplified dynamical for P_{dc}

VI. SYSTEM EQUATIONS

The system equations containing the loads and the dynamics of DC line can be given with respect to the reference of the machine angle.

$$\begin{aligned} M_k\dot{\omega}_k + D_k\omega_k &= P_k - P_{ek} \quad (k=1,2,..,n) \\ \dot{\delta}_{kn} &= \omega_{kn} \quad (k=1,2,..,n-1) \end{aligned} \tag{29}$$

The expression P_{ek} in equation (29) was identified in earlier sections, and the expression P_k is modified as follows[5].

$$P_k = P_{mk} - \Delta P_{kL} - \Delta P_{kD} - |E_k|^2 G_{kk} \tag{30}$$

ΔP_{kL} and ΔP_{kD} were identified in earlier sections. Because the loads are not expressed as constant admittance, it is possible to ignore the parameters D_{kj} in P_{ek}. Thus, postfault stable equilibrium points are obtained by solving the non-linear equation sets in the following.

$$P_k = P_{ek} \quad (k=1,..,n-1) \tag{31}$$

VII. LYAPUNOV FUNCTION

Lyapunov function used to determine the stability region of the system is given in the following[6].

$$V(x) = x^T P x + \sum_{i=1}^{L} 2q_i \int_0^{c_i x} f_i(\sigma_i)\,d\sigma_i \tag{32}$$

$$\begin{aligned} V(x) = \frac{1}{2}\sum_{k=1}^{n} M_k x_k^2 + \sum_{j=2}^{n} C_{1j}\Big\{\cos(\delta_1^s - \delta_j^s) - \\ \cos(x_{j+n-1} + \delta_1^s - \delta_j^s) - x_{j+n-1}\sin(\delta_1^s - \delta_j^s)\Big\} + \\ \sum_{k=2}^{n-1}\sum_{j=i+1}^{n} C_{kj}\Big\{\cos(\delta_k^s - \delta_j^s) - \cos(x_{j+n-1} - \\ x_{k+n-1} + \delta_k^s - \delta_j^s) - (x_{j+n-1} - x_{k+n-1})\sin(\delta_k^s - \delta_j^s)\Big\} \end{aligned} \tag{33}$$

The terms as q_i, c_i, and P in equation (32) is given in ref.[7] and if these terms are substituted in equation (32), for the transient system energy the expression in equation (33) is obtained:

VIII. SAMPLE SYSTEM

A 4-machine 6-bus system whose single-line diagram is shown in figure 3 is considered. For details of the AC system data refer to [8]. We assume that the fault occurs on ac line of the number 3 and near bus of the number 4. A DC line is added to the system across buses 2 and 5, buses 3 and 4. The following parameters are chosen for the DC line:

K_a=1.0 pu/rad Per sec, T_{dc}=0.1 sec, P_{ref}=0.0

For both prefault and postfault conditions, P_{ref} is assumed to be zero for the sake of convenience. In general, P_{ref} will have different values in the prefault and postfault states. The external control signal (ECS) is chosen to be the difference between the rotor speeds of the generators nearest to the rectifier and inverter bus terminals, i.e., $u=\omega_8-\omega_9$. q_r is taken into account as 0.5 during the transient and the AC/DC system is analysed both $P_{dc} = \mp 0.6$ pu and $P_{dc} = \mp 0.8$ pu

The resistance of DC line is taken consideration as the value of the resistance of ac line which it is parallel.

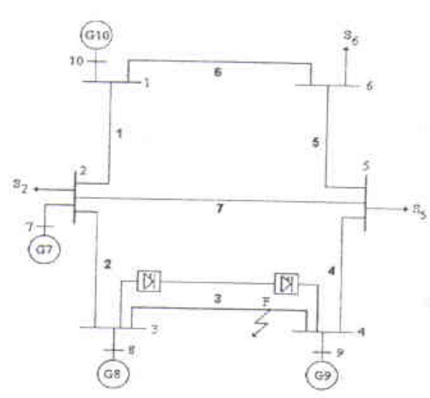

Figure 3. Single-line diagram of the studied sample system

IX. RESULTS

TABLE I - COMPARISON OF RESULTS OBTAINED FOR DIFFERENT CASES IN A MULTIMACHINE AC/DC POWER SYSTEM

DC Line Power	Buses of DC line	Faulted Bus	Cleared Line	CCT (sec)
±0.6	2-5	4	3	0.68
±0.8	2-5	4	3	0.69
±0.6	3-4	4	3	0.70
±0.8	3-4	4	3	o.73

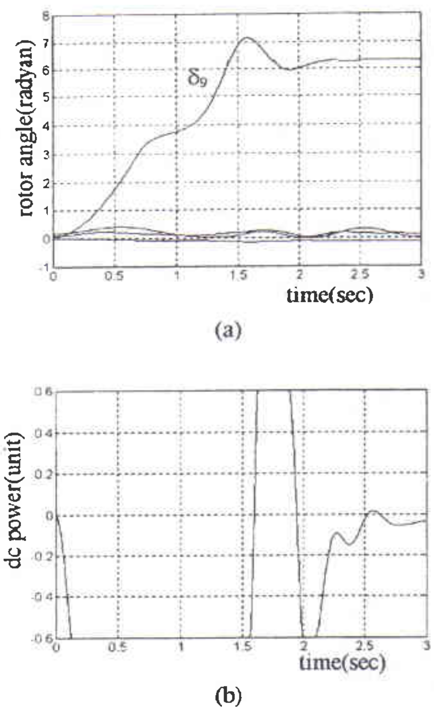

Figure 4a-4b. Rotor angles and dc line power. DC line is connected the buses of number 2-5 and $P_{dc} = \mp 0.6$

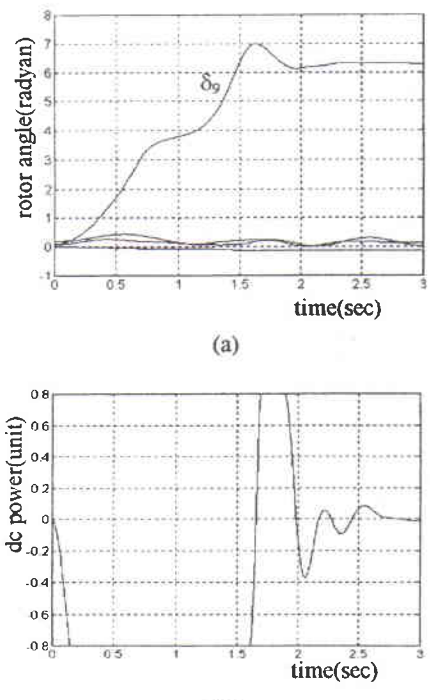

Figure 5a-5b. Rotor angles and dc line power. DC line is connected the buses of number 2-5 and $P_{dc} = \mp 0.8$

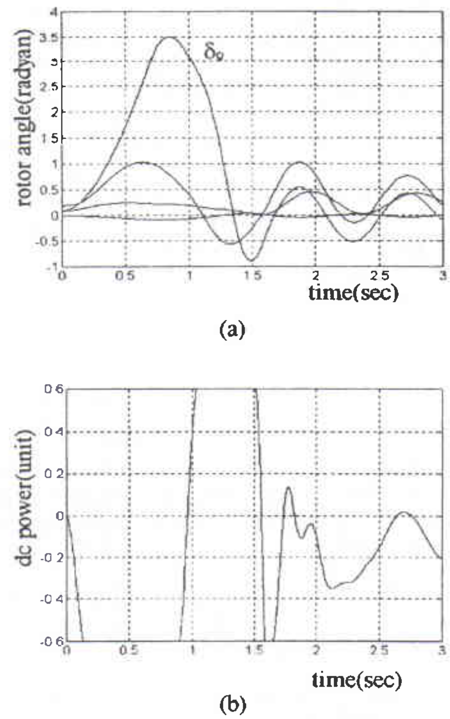

Figure 6a-6b. Rotor angles and dc line power. DC line is connected the buses of number 3-4 and $P_{dc} = \mp 0.6$

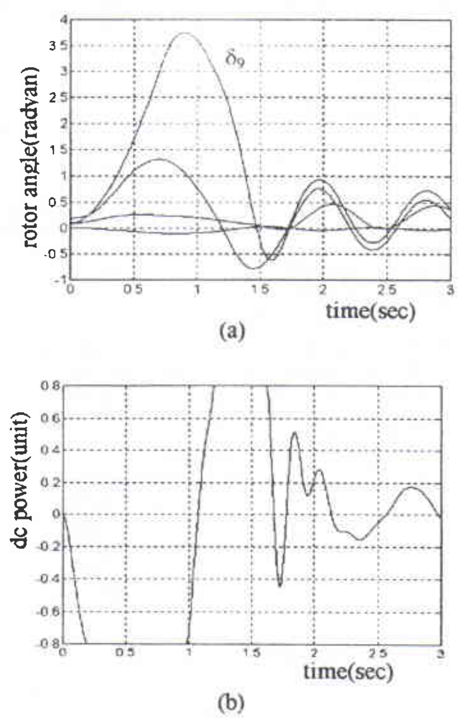

Figure 7a-7b. Rotor angles and dc line power. DC line is connected the buses of number 3-4 and $P_{dc} = \mp 0.8$

As shown from figure 4a, 5a, 6a, and 7a, the machine of number 9 tends to lose the synchronism. If the fault occurs on ac line of number 3 that is parallel to the dc line, then the better results are obtained for CCT. When the power limit transmitted through the dc line increase, CCT increase. However, after the fault is cleared, it is shown obviously that the power transmitted through the dc line returns the prefault value in a short time as approximately three seconds. From figure 4a and 4b, it can be shown that the machine of number 9 set in a new equilibrium point after fault.

X. REFERENCES

[1]. E.W. Kimbark, *Direct Current Transmission*, Wiley-Interscience, 1971.

[2]. G. Anderson, "A Way to Improve AC Power Transfer Limits, HVDC Links Used to Stabilize Electric Power Systems," *ABB Review, Power System Stability*, February 1992, pp. 15-22

[3]. P.M. Anderson, A.A. Fouad, *Power System Control and Stability*, Iowa State University Press, Ames, Iowa, USA, 1977

[4]. W.F. Tinney, W.L. Powell, "The REI Approach to Power Network Equivalents," *1977 PİCA Conference*, Toronto, Canada, 1977

[5]. K. Ayan, *A different approach* for transient *stability of integrated alternating current-direct current systems*, Ph.D. Thesis, İstanbul Technical University, İstanbul, 1998

[6]. J.L. Willems, and J.C. Willems, "The application of Lyapunov Methods to the Computation of Transient Stability Regions of Multimachine Power Systems," *IEEE Trans. PAS*, Vol. 89, No. 5, May/June 1970, pp. 795-801

[7]. F.S. Prabhakara, A.H. El-Abiad, "A Simplified Determination of Transient Stability Regions for Lyapunov Methods," *IEEE Trans. PAS*, Vol. 94, No. 2, 1975, pp. 672-680

[8]. A.H. El-Abiad, K. Nagappan, "Transient Stability Regions of Multimachine Power Systems", IEEE Trans. PAS, Vol. 85, No. 2, 1966, pp.169-178