TRANSIENT STABILITY ASSESSMENT OF AN ELECTRIC POWER SYSTEM USING THE BISECTION ALGORITHM COMBINED WITH THE DOT PRODUCT CRITERIA

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ABSTRACT

In the last years the importance of the transient stability assessment has been increasing since the electric power systems are being operated closer to their stability limits. In this paper it is studied and analysed the transient stability of a test power network using the bisection algorithm combined with the dot product criteria. Only the effects of the contingencies that present a high risk level were analysed. It was used the software package TRANsySTEM that the authors developed for transient security assessment of a multimachine power system. These computer programs use the bisection algorithm combined with the dot product criteria in order to increase the simulation efficiency in the evaluation of the critical clearing time. This formulation reduces the computing time, since it only requires solving partially the differential motion equations of the system. The results obtained with this approach were compared with the solutions produced by the full integration scheme as well as by the maximum difference angle criteria.

Keywords: Transient stability; Bisection algorithm; Dot product criteria.

I. INTRODUCTION

In the last years electric power systems grew in size and complexity with a large number of interconnections [1]. Moreover, electric utilities, due to economic, open-market and regulatory constraints, are being forced to operate their power systems in such a way as to make maximum the use of their capacity and operate closer to their transmission limits [2]. This poses a variety of challenging engineering problems at the planning and design stage as well as during the system operation [1], [3].

One of the most important problems in the planning and operation of a multimachine power system is the transient stability assessment. This analysis entails the evaluation of a power system ability to withstand large disturbances and to survive transition to a normal or acceptable operating condition [4]. Conventional transient stability study is normally performed by numerically solving the differential motion equations of the system and analysing the resulting swing curves of its synchronous machines [5]. In order to reduce the computing time that this approach requires, a combined bisection algorithm with the dot product criteria is used in this paper.

Only the effects of the contingencies that present a high risk level were analysed. The risk values were obtained from an off-line evaluation that combines the unstability probabilities with the associated social and economic impact [6]. It was used the software package TRANsySTEM that the authors developed for transient security assessment of a multimachine power system. These computer programs use the bisection algorithm combined with the dot product criteria in order to increase the simulation efficiency in the evaluation of the critical clearing time (CCT). This formulation reduces the computing time, since it only requires solving partially the differential motion equations of the system. The results obtained with this approach were compared with the solutions produced by the full integration scheme as well as by the maximum difference angle criteria. The developed computer programs that use the Runge-Kutta method to solve the motion equations were applied to a multimachine test power network.

This paper is organised as follows. Section II is devoted to the formulation of the problem using the bisection algorithm combined with the dot product criteria. In Section III is presented the New England test power network (10 synchronous machines, 39 busbars). Section IV shows the results obtained using the proposed formulation and the solutions produced by full integration scheme as well as by the maximum difference angle criteria. Finally, in section V, some conclusions that provide a valuable contribution to the understanding of the transient stability assessment of a multimachine power system are pointed out.

II. FORMULATION OF THE PROBLEM

The bisection algorithm combined with the dot product criteria is a very efficient tool to assess transient stability of a multimachine power network.

The motion of a multimachine power system is described by the following ordinary differential equations [7]:

$$\dot{\delta}_i = \omega_i \qquad (1)$$

$$M_i \dot{\omega}_i = P_{mi} - P_{ei} \qquad (2)$$

with

$$P_{ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) \qquad (3)$$

where

δ_i - rotor angle of the ith synchronous machine
ω_i - speed of the ith synchronous machine
M_i - inertia coefficient of the ith synchronous machine
P_{mi} - mechanical power of the ith synchronous machine
P_{ei} - electric power of the ith synchronous machine
E_i - voltage behind the direct axis transient reactance of the ith synchronous machine
n - number of machines in the power system
G_{ij} - real part of the ijth element of the nodal admittance matrix
B_{ij} - imaginary part of the ijth element of the nodal admittance matrix

The dot denotes the first order derivative.

In order to apply the proposed method the above equations should be modified considering the centre of inertia (COI) coordinates [8]. The centre of inertia is defined as follows:

$$\delta_o = \frac{1}{M_T} \sum_{i=1}^{n} M_i \delta_i \qquad (4)$$

Thus the swing equations (1) and (2) are modified into the following expressions:

$$\dot{\theta}_i = \tilde{\omega}_i \qquad (5)$$

$$M_i \dot{\tilde{\omega}}_i = f_i(\theta_i) \qquad (6)$$

with

$$\theta_i = \delta_i - \delta_o \qquad (7)$$

$$\tilde{\omega}_i = \dot{\delta}_i - \dot{\delta}_o \qquad (8)$$

$$f_i(\theta_i) = P_{mi} - P_{ei} - \frac{M_i}{M_T} P_{COI} \qquad (9)$$

$$P_{COI} = \sum_{i=1}^{n} \left(P_{mi} - P_{ei} \right) \qquad (10)$$

$$M_T = \sum_{i=1}^{n} M_i \qquad (11)$$

The transient stability assessment is based on the concept of dot the products from the potential energy boundary surface method (PEBS) [9]. In this analysis the post-fault trajectory is checked whether it passes the PEBS or not. If the fault is cleared after the CCT, the trajectory is unstable and will cross the PEBS. If the fault is cleared before the CCT, the trajectory is stable and will not cross the PEBS. Solving the equations (5) and (6), two dot products are evaluated in every integration step. These dot products DP_1 and DP_2 are calculated using the following expressions:

$$DP_1 = f^T(\theta) \cdot (\theta - \theta_{cl}) \qquad (12)$$

$$DP_2 = \omega^T \cdot (\theta - \theta_{cl}) \qquad (13)$$

The subscript *cl* denotes the value at the clearing time of its vector.

During the simulation time, a change in sign in the dot products means that the projection of the vector *f* or *ω* on the vector *(θ - θ_{cl})* changes its direction. The DP_1 indicates that the post-fault trajectory crosses the PEBS. The DP_2 indicates that the post-fault trajectory swings back before it crosses the PEBS (system is first-swing stable) [10].

To evaluate the CCT for a given contingency the bisection algorithm is used (figure 1). At the beginning of this numeric approach, the upper (FCTu) and the lower (FCTs) limits are specified for the fault clearing time (FCT). Starting with the upper clearing time the system transient stability is checked. If it is found that the system is unstable, then the upper stability limit of the interval is replaced by the middle point value. Otherwise, the lower value is assumed equal to the middle point value. The procedure is repeated until reaching the required accuracy and the CCT will be the last clearing time of a stable situation (conservative assessment).

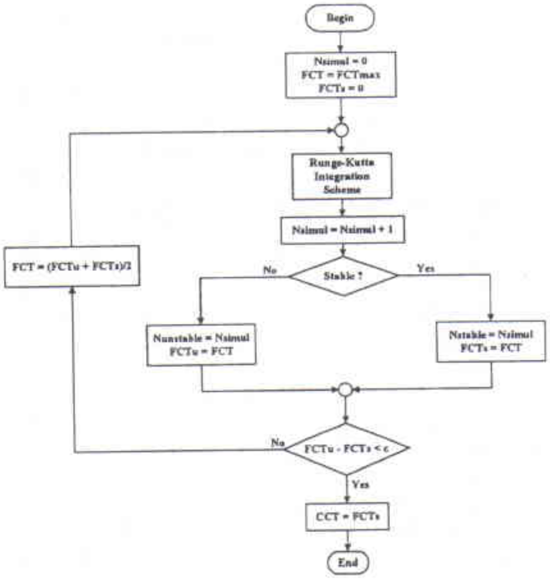

Figure 1. The Bisection Algorithm

The dot products and a tolerance value ε were used to detect (un)stability during the iterative process. If the DP_1 changes sign before DP_2 then the simulation is stopped and the system is assumed as unstable. If the DP_2 changes sign before DP_1 then the simulation is finished and the system is assumed as first-swing stable. The tolerance value is assumed equal to the difference between the fault clearing times of the last unstable and the last stable simulation.

In figure 1 Nsimul, Nunstable and Nstable stand for the number of simulations, the number of last unstable simulations and the number of last stable simulations respectively.

III. APPLICATION EXAMPLE

The bisection algorithm combined with the dot product criteria was applied to study the transient stability of the New England test power network shown in figure 2 (10 synchronous generators, 39 busbars). The electric power system data is presented in [8]. It was simulated a three-phase fault in some of the transmission lines. The short-circuit was cleared by the simultaneous tripping of the two line circuit breakers.

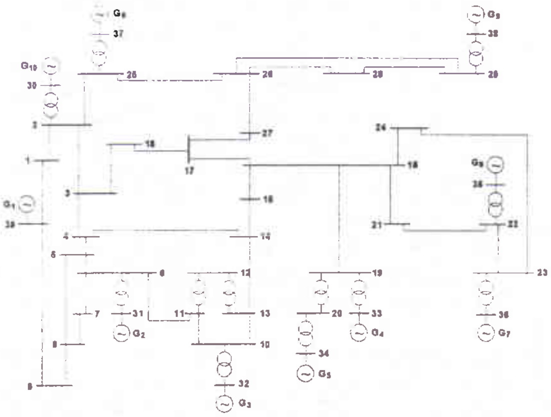

Figure 2. Electric Power System

IV. RESULTS

The results obtained using the bisection algorithm combined with the dot product criteria considering some of the must severe disturbances are presented in Tables 1 and 2.

Table 1 shows the critical clearing times obtained using the dot product approach, the full integration scheme and the maximum difference angle criteria. All the three numeric approaches use the bisection algorithm with a tolerance value ε equal to 5 ms. In order to obtain each one of the CCT values 9 simulations were required.

Table 1. Critical Clearing Times [s]

Faulted Bus	Line Tripped	Dot Prod.	Max. Ang.	Full Integ.
2	2 - 3	0.25	0.26	0.25-0.26
6	6 - 11	0.22	0.24	0.21-0.22
10	10 - 13	0.23	0.24	0.22-0.23
15	15 - 16	0.22	0.23	0.22-0.23
16	16 - 21	0.15	0.17	0.15-0.16
22	22 - 23	0.21	0.22	0.21-0.22
25	2 - 25	0.12	0.13	0.12-0.13
26	26 - 27	0.14	0.14	0.13-0.14
29	26 - 29	0.07	0.08	0.07-0.08

In table 2 are presented the seconds of time domain integration (sTDI) that were required to obtain the critical clearing times in the three different approaches.

Table 2. Seconds of time domain integration

Faulted Bus	Line Tripped	Dot Prod.	Max. Ang.	Full Integ.
2	2 - 3	7.8	11.4	13.5
6	6 - 11	8.5	12.1	13.5
10	10 - 13	7.6	11.9	13.5
15	15 - 16	8.4	12.6	13.5
16	16 - 21	7.8	11.5	13.5
22	22 - 23	7.5	11.8	13.5
25	2 - 25	8.0	10.4	13.5
26	26 - 27	5.5	10.4	13.5
29	26 - 29	6.5	9.6	13.5

Figures 3 and 4 present the swing curves corresponding to one of the analysed situations, when the fault occurs in line 6-11 near busbar 6. They were obtained using the Runge-Kutta method, considering the centre of inertia formulation.

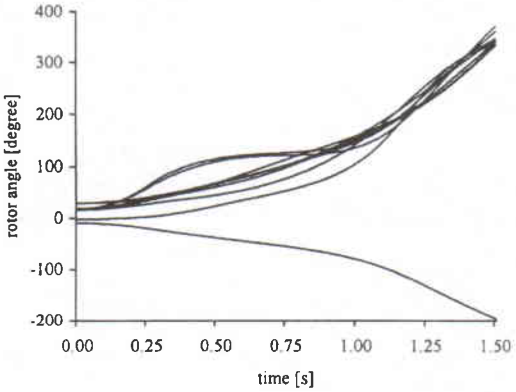

Figure 3. Swing curves when the fault occurs on line 6-11 near bus 6 for a clearing time of 0.25 [s]

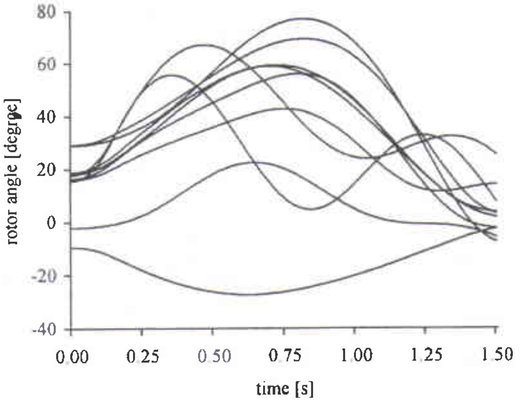

Figure 4. Swing curves when the fault occurs on line 6-11 near bus 6 for a clearing time of 0.15 [s]

Figures 5 and 6 show the swing curves corresponding to another analysed situation, when the fault occurs in line 26 - 29 near busbar 29. They were also obtained using the Runge-Kutta method, considering the centre of inertia formulation.

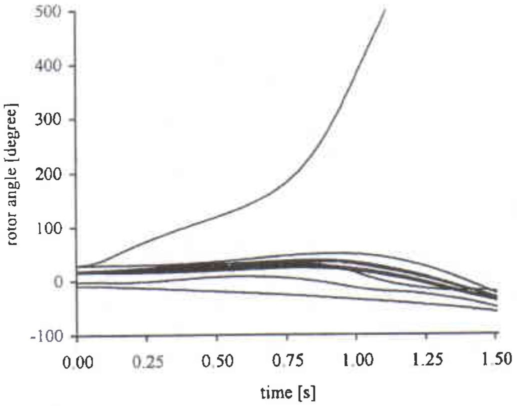

Figure 5. Swing curves when the fault occurs on line 26-29 near bus 29 for a clearing time of 0.05 [s]

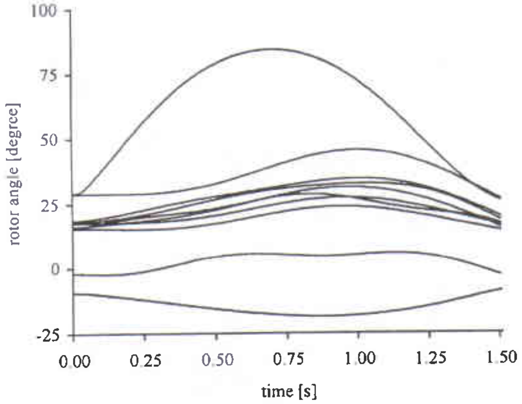

Figure 6. Swing curves when the fault occurs on line 26-29 near bus 29 for a clearing time of 0.10 [s]

V. CONCLUSIONS

From the results presented in section IV the following conclusions can be extracted:

- the solution obtained with the bisection algorithm combined with the dot product criteria used in this study are in accordance with the results calculated with the full integration scheme as well as with the maximum difference angle formulation;
- the proposed algorithm is simple and efficient since it uses the dot product criteria to assess the transient stability of the power network;
- from table 2 it can be shown that the proposed formulation presents the smaller sTDI values and consequently this approach can be used successfully to save computing time;
- the bisection algorithm combined with the dot product criteria may be used as an efficient filtering tool for contingencies screening and ranking in transient stability studies.

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