Economic Dispatch by the Combination of HNN and the Lagrange Method

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***Abstract*—In this work, we are going to solve the ED (economic dispatch) problem by the use of an HNN (Hopfield neural networks) model combined with the Lagrange method noted LHNN. In contrary to the techniques that use the linear function of the total inputs to neurons, we are going to use it at the non-linear form that allows the parameters of NN to contain all information of the original problem, so the HNN become able to reach exactly the solutions. At the end, simulations will be presented, with promising results, of a multi-objective economic environmental problem.**

Keywords*—*HNN; Lagrange method; ED; optimization

I. Introduction

The problem of the load economic distribution between the production units is important to increase the profits of production for any variation in demand of the electrical energy. Since 1930, engineers were concerned with the problem of good economic distribution of active power between the production units [1]. Their techniques were based on the successive loading of production units according to their capacities compared to other units [1]. After the development of the computing machine the problem had known a great change. Sometimes, by the use of the so-called classical or deterministic methods such as the Lambda iteration method, Newton's method and Lagrange [2-3]. These methods provide accurate results and can be blocked in local solutions or may not converge to the solution when the problem becomes non-convex. On the other hand, we find modern techniques or artificial intelligence techniques such as GA (genetic algorithms), the HNN, SA (simulated annealing), PSO (particle swarm optimization) ... [3-5]. Among the ancient techniques of artificial intelligence, we find the HNN. Generally, the neurons are completely interconnected and constitute a single layer network. This technique has simplified calculations compared to classical methods, especially when the problem becomes non-convex such as the use of multiple fuels in the thermal production unit [6-8]. The results obtained by the HNN are approximate. Some authors have improved the performance of the HNN technique by iterative adjustments of the values of bias and / or shape parameter of the transfer function which had improved the results and accelerated the convergence [8]. By other means, based on HNN in quadratic problems, E. Dudnikov presented an improved model more stable and faster by doubling the number of variables and layers of HNN structure [9]. Most of these works had considered the equality constraints as linear by calculating the active losses separately from neural computation. R. S. Hartati, and M. E. El-Hawary had given the whole calculation to the HNN by the linearization of the calculation formula of active losses with the use of Taylor series expansions; so they introduced the approximate losses coefficient in the HNN parameter [10]. In reality, for application, these approaches had preferred to handle the ED problem without introducing the power losses of the electrical network [7-8],[11-16] in order to avoid the non-linearity of the equality constraint ; if it wasn't the case, the techniques provided a mismatch in the equality constraint; and not attained the real solution [17-19], so, it became weak in fast calculation problem. In [20] for the environment ED, the authors faced great problems in determining the penalty constants introduced at each time for each new objective to treat.

In this work, we will avoid the problem of choice of penalty parameters, the separate calculation of active losses, and the linearization of its formula. Thus, we will be based on the combination of the HNN with the Lagrange method.

The number of output neurons is equal to n control variables plus one (neuron) which represents the Lagrange parameter corresponding to the equality constraint.

To treat the non-linearity of the constraints, we will use the modified HNN, where the states of the neurons obtained at time t are produced by a combination of products and summations of output states of the neurons at the instant t-1.

II. Economic problem

The problem of ED is to seek the optimal solution of the active powers generated by the production units by minimizing the cost of total production, with the satisfaction of the constraints of production capacity and balance of powers. Mathematically, the problem is as follows:

$$C_{tot} = min\left(\sum_{i=1}^{n} C_i\right) \quad (1)$$

Subject to:

$$P_D + P_L = \sum_{i=1}^{n} P_i \quad (2)$$

$$P_i^{min} \le P_i \le P_i^{mix} \quad (3)$$

With:

$$C_i = a_i + b_i P_i + c_i P_i^2 \quad (4)$$

$$P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} P_i P_j \quad (5)$$

C_{tot} : the cost of the total production.

C_i : the cost of production of the unit *i*.

a_i, b_i, c_i : coefficients of the cost of the unit *i*.

n : the number of units.

P_D : the total demand of the active power.

P_L : transmission losses.

P^{imin} and P^{imax} : the production minimal and maximal limits of the unit *i,* respectively.

B_{ij} : the coefficients of the matrix losses.

III. MODEL OF LAGRANGE HOPFIELD

When the economic problem is presented by the non linear equations that must be calculated by the neural networks the known architecture, of single layer with linear function of the total inputs to neurons, become unable to deal with the original problem.

The problem in the form of Lagrange is:

$$\Im = C_{tot} + \lambda\left(P_D + P_L - \sum_{i=1}^{n} P_i\right) \quad (6)$$

λ : the Lagrange multiplier.

The optimal solution of the problem is given by:

$$\nabla \Im = 0 \quad (7)$$

Thus, we obtain:

$$\begin{cases} \dfrac{\partial \Im}{\partial P_i} = b_i + 2c_i P_i + \lambda\left(2\sum_{j=1}^{n} B_{ij} P_j - 1\right) = 0 \\ \dfrac{\partial \Im}{\partial \lambda} = P_D + \sum_{i=1}^{n} \sum_{j}^{n} B_{ij} P_i P_j - \sum_{i=1}^{n} P_i = 0 \end{cases} \quad (8)$$

With *i* = 1, ... *n*.

For the calculation of the solution by HNN, the energy function is given by:

$$E = \frac{1}{2}\left(\sum_{i=1}^{n}\left(\frac{\partial \Im}{\partial V_i}\right)^2 + \left(\frac{\partial \Im}{\partial V_\lambda}\right)^2\right) \quad (9)$$

V_i indicates the powers P_i.

V_λ represents the λ parameter.

This function can make the neural network. The dynamic change of the inputs of neurons is given by:

$$\begin{cases} \dfrac{dU_k}{dt} = -\dfrac{\partial E}{\partial V_k} \\ \dfrac{dU_\lambda}{dt} = -\dfrac{\partial E}{\partial V_\lambda} \end{cases} \quad (10)$$

$$\begin{cases} \dfrac{dU_k}{dt} = -\sum_{i=1}^{n} \dfrac{\partial \Im}{\partial V_k} \dfrac{\partial^2 \Im}{\partial V_k \partial V_i} - \dfrac{\partial \Im}{\partial V_\lambda} \dfrac{\partial^2 \Im}{\partial V_k \partial V_\lambda}, \quad k = 1,\ldots,n \\ \dfrac{dU_\lambda}{dt} = -\sum_{i=1}^{n} \dfrac{\partial \Im}{\partial V_\lambda} \dfrac{\partial^2 \Im}{\partial V_\lambda \partial V_i} - \dfrac{\partial \Im}{\partial V_\lambda} \dfrac{\partial^2 \Im}{\partial V_\lambda^2} \end{cases} \quad (11)$$

U_k and U_λ are, respectively, the total inputs to neurons *k* and λ.

After development and simplification of the system equations (11), we write:

$$\begin{cases} \dfrac{dU_k}{dt} = -\begin{pmatrix} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{s=1}^{n} T_{kijs} V_i V_j V_s + \\ \sum_{i=1}^{n} \sum_{j=1}^{n} T_{kij} V_i V_j + \\ \sum_{i=1}^{n} T_{ki} V_i + I_k \end{pmatrix}; \; k = 1,\ldots,n \\ \dfrac{dU_\lambda}{dt} = -\left(\sum_{i=1}^{n} \sum_{j=1}^{n} T_{\lambda ij} V_i V_j + \sum_{i=1}^{n} T_{\lambda i} V_i + I_\lambda\right) \end{cases} \quad (12)$$

Where:

$$T_{kijs} = 2B_{ij} B_{ks} \quad (13)$$

$$T_{kij} = -\left(2B_{kj} + B_{ij}\right) \quad (14)$$

$$T_{ki} = 2P_D B_{ki} + 1 + 4\lambda\left(c_k + c_i\right)B_{ki} + (2\lambda)^2 \sum_{j=1}^{n} B_{ij} B_{kj} \quad (15)$$

$$I_k = 2\lambda \sum_{i=1}^{n} \left(b_i - \lambda\right) B_{ki} - P_D \quad (16)$$

$$T_{\lambda ij} = 4\lambda \sum_{s=1}^{n} \left(B_{sj} B_{si}\right) + 4c_i B_{ij} \quad (17)$$

$$T_{\lambda i} = 2\sum_{j=1}^{n} B_{ij} b_j - 2c_i - 4\lambda \sum_{j=1}^{n} B_{ij} \quad (18)$$

$$I_{\lambda} = \lambda - \sum_{i=1}^{n} b_i \quad (19)$$

If $i = k$, we add the following terms:

$$T_{ki} = T_{ki} + (2c_k)^2 \quad (20)$$

$$I_k = I_k + 2c_k (b_k - \lambda) \quad (21)$$

For the solution, the dynamic function will be on the discontinuous form:

$$\begin{cases} U_k^{t+1} = U_k^t - \dfrac{dE^t}{dV_k}, \quad k = 1, \dots n \\ U_\lambda^{t+1} = U_\lambda^t - \dfrac{dE^t}{dV_\lambda} \end{cases} \quad (22)$$

With:

$$\begin{cases} V_k = g(U_k) = \dfrac{1}{1 + exp\left(-\dfrac{U_k}{u_0}\right)}, k = 1, \dots n \\ V_\lambda = g_\lambda(U_\lambda) = \dfrac{1}{1 + exp\left(-\dfrac{U_\lambda}{u_0}\right)} \end{cases} \quad (23)$$

u_0 characterizes the shape of the sigmoid function. To provide solutions within limits:

$$V_i = \left(P_i^{max} - P_i^{min}\right) \frac{1}{1 + exp\left(-\dfrac{U_i}{u_0}\right)} + P_i^{min} \quad (24)$$

$$V_\lambda = \left(\lambda_i^{max} - \lambda_i^{min}\right) \frac{1}{1 + exp\left(-\dfrac{U_i}{u_0}\right)} + \lambda_i^{min} \quad (25)$$

IV. Stability of the Function of Energy

To ensure the stability of the network, the energy function must exhibit the behavior of stability. This is in relation with the dynamic changes of the neuron states. We set for each neuron:

$$\begin{cases} \dfrac{dE_k}{dt} = \dfrac{\partial E_k}{\partial V_k} \dfrac{dV_k}{dt}, k = 1, \dots n \\ \dfrac{dE_\lambda}{dt} = \dfrac{\partial E_\lambda}{\partial V_\lambda} \dfrac{dV_\lambda}{dt} \end{cases} \quad (26)$$

After substitution of equations (11) and (23) into equation (26), we obtain:

$$\begin{cases} \dfrac{dE_k}{dt} = -\left(\dfrac{dU_k}{dt}\right)^2 \dfrac{dg(U_k)}{dU_k}, k = 1, \dots n \\ \dfrac{dE_\lambda}{dt} = -\left(\dfrac{dU_\lambda}{dt}\right)^2 \dfrac{dg_\lambda(U_\lambda)}{dU_\lambda} \end{cases} \quad (27)$$

The system equations (27) show that each neuron reduces its energy in function of time. So, the HNN has the behavior of stability.

V. Applications

To evaluate our method, we have applied it to the environmental ED problem, (28), defined by the quadratic equations of the cost and polluter emissions of sulfur dioxide (SO_2) and nitrogen oxide (NO_X) (29)-(30).

For the simulation, we will follow the same applications of [21].

The application system data and the objective functions parameters are shown in tables 1-3, the system demand is 850 [MW] in all simulations and the active power transmission losses will be calculated by the relation (31).

$$Minimize[C_{tot}, E_{SO2}, E_{NOX}] \quad (28)$$

$$E_{SO_2} = \sum_{i=1}^{n} \left(a_{Si} + b_{Si} P_i + c_{Si} P_i^2\right) \quad (29)$$

$$E_{NO_x} = \sum_{i=1}^{n} \left(a_{Ni} + b_{Ni} P_i + c_{Ni} P_i^2\right) \quad (30)$$

$$P_L = \left(3P_1^2 + 9P_2^2 + 12P_3^2\right) 10^{-5} \quad (31)$$

VI. Simulation Results

Several simulations were carried out by the LHNN method.

The obtained results will be compared with those of the conventional HNN method, TS (Tabu search) method of [22], NSGA-II (Non-dominated Sorting Genetic Algorithm - II) technical of [21], and with those of the 2008 MATLAB's fmincon command which combines three calculation algorithms (trust-region-reflective, active-set and interior-point).

In case of cost minimization, the lowest cost compared to the other methods is obtained in table 4. By cons, in tables 4-5, the best minimums of SO_2 and NO_X emissions are not obtained. In fact, in table 4, the results were accurate and identical to those of fmincon command while those of the others methods were approximates.

In case of multiple objectives, we show the results of 3 minimizations: the cost with SO_2 emission in table 7, the cost with NO_X emission in table 8 and the cost with both of SO_2 and NO_X emissions in table 9.

Through these 3 tables, the best solutions and the maximum benefit compared to other methods were obtained.

From these simulations, we notice that our method has greatly reduced the cost objective unlike the emissions Gas, because they don't have high values compared to the cost which is the preponderate purpose of the minimization.

TABLE I. DATA SYSTEM WITH THREE UNITS

P_i	a_i [\$/h]	b_i[\$/MWh]	c_i [\$/MWh²]	Pi^{min}[MW]	Pi^{max}[MW]
1	0.00156	7.92	561	150	600
2	0.00194	7.85	310	100	400
3	0.00482	7.97	78	50	200

TABLE II. EMISSION SO_2 COEFFICIENTS

P_i	a_{Si}[ton/h]	b_{Si}[ton/MWh]	c_{Si}[ton/MWh²]
1	$1.6103\ 10^{-6}$	0.00816466	0.5783298
2	$2.1999\ 10^{-6}$	0.00891174	0.3515338
3	$5.4658\ 10^{-6}$	0.00903782	0.0884504

TABLE III. EMISSION NO_X COEFFICIENTS

P_i	a_{Ni}[ton/h]	b_{Ni}[ton/MWh]	c_{Ni}[ton/MWh²]
1	$1.4721848\ 10^{-7}$	$-9.4868099\ 10^{-5}$	0.04373254
2	$3.0207577\ 10^{-7}$	$-9.7252878\ 10^{-5}$	0.055821713
3	$1.9338531\ 10^{-6}$	$-3.5373734\ 10^{-4}$	0.02773152

TABLE IV. TABLE 4. COST OPTIMIZATION

	LHNN	HNN	TS [22]	NSAG-II [21]	fmincon
P_1	435.198	401.0567	435.69	436.366	435.198
P_2	299.970	340.7315	298.828	298.187	299.970
P_3	130.660	124.7812	131.28	131.228	130.660
λ	9.528	-	-	-	-
P_L	15.829	17.1426	15.798	15.781	15.829
Cost	**8344.593**	**8346.138**	**8344.598**	**8344.606**	**8344.593**
ESO_2	9.0220	9.056	9.02146	9.02083	9.0220
ENO_X	0.0987	0.1008	0.09863	0.0987	0.0987

TABLE V. SO_2 OPTIMIZATION

	LHNN	HNN	TS [22]	NSAG-II [21]	fmincon
P_1	430.762	433.80	549.247	541.308	271.410
P_2	289.319	288.114	234.582	223.249	400.000
P_3	145.561	143.678	81.893	99.919	200.000
λ	0.0105	-	-	-	-
P_L	15.643	15.593	15.722	14.476	21.410
Cost	8346.271	8345.994	8403.485	8387.518	8450.829
ESO_2	**9.0280**	**9.0247**	**8.974**	**8.967**	**9.296**
ENO_X	0.10037	0.1000	0.0974	0.09637	0.1284

TABLE VI. NO_X OPTIMIZATION

	LHNN	HNN	TS [22]	NSAG-II [21]	fmincon
P_1	430.759	432.918	502.914	505.810	378.703
P_2	289.320	288.534	254.294	252.951	287.857
P_3	145.563	144.156	108.592	106.023	200.000
λ	$1.1\ 10^{-4}$	-	-	-	-
P_L	15.643	15.609	15.8	14.784	16.560
Cost	8346.271	8346.054	8371.143	8363.627	8379.573
ESO_2	9.028	9.0257	8.9861	8.9747	9.115
ENO_X	**0.1004**	**0.1001**	**0.0958**	**0.09593**	**0.1161**

TABLE VII. COST AND SO_2 OPTIMIZATION

	LHNN	HNN	NSAG-II [21]	fmincon
P_1	435.324	401.214	485.886	379.323
P_2	299.883	340.615	263.670	287.218
P_3	130.620	124.734	115.381	200.000
λ	9.539	-	-	-
P_L	15.826	17.137	14.937	16.541
Cost	**8344.593**	**8346.089**	**8354.419**	**8379.488**
ESO_2	**9.0218**	**9.0564**	**8.98383**	**9.1143**
ENO_X	0.0987	0.1008	0.0962	0.1161

TABLE VIII. COST AND NO_X OPTIMIZATION

	LHNN	HNN	NSAG-II [21]	fmincon
P_1	435.208	401.071	470.957	271.410
P_2	299.967	340.721	280.663	400.000
P_3	130.654	124.775	113.675	200.000
λ	9.528	-	-	-
P_L	15.8288	17.142	15.294	21.410
Cost	**8344.593**	**8346.135**	**8349.722**	**8450.828**
ESO_2	9.0219	9.0566	8.9932	9.2958
ENO_X	**0.0987**	**0.1008**	**0.09654**	**0.1284**

TABLE IX. COST, NO_X AND SO_2 OPTIMIZATION

	LHNN	HNN	NSAG-II [21]	fmincon
P_1	435.334	401.229	496.328	275.384
P_2	299.880	340.606	260.426	395.790
P_3	130.613	124.728	108.144	200.000
λ	9.539	-	-	-
P_L	15.826	17.137	14.898	21.174
Cost	**8344.592**	**8346.086**	**8358.896**	**8446.145**
ESO_2	**9.0218**	**9.0564**	**8.97870**	**9.2868**
ENO_X	**0.0987**	**0.1008**	**0.09599**	**0.1278**

VII. CONCLUSION

In this study, we have combined the determinist method of Lagrange with the HNN technical, where, by a simple algorithm of Euler, we are able to optimize the non-linear problem, without introducing the penalty parameters or calculating matrices and their inverses.

Also, we have used the non-linear function of the total inputs to neurons in the HNN, because it allows to obtain the just weight parameters and to reach the exact solutions. Unlike the methods based on the linear function. the applied technique has provided accurate results and it converges rapidly that can be applied for the resolution of the ED problem in real time.

The applied method provides the best optimums especially in case of objectives with large values. So, we propose to study the modification of our original problem by another equivalent with a parameterized objective function.

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