Mathematical Modelling of Human Heart as a Hydroelectromechanical System

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Abstract

Different electrical models of human heart, partial or complete, with linear or nonlinear models have been developed. In the literature, there are some applications of mathematical and physical analog models of total artificial heart (TAH), a baro-receptor model, a state-space model, an electromechanical biventricular model of the heart, and a mathematical model for the artificial generation of electrocardiogram (ECG) signals. Physical models are suitable to simulate real physiological data based on proper experimental set up present. This paper introduces a new mathematical modelling of human heart as a **h**ydro**e**lectro**m**echanical **s**ystem (HEMS). This paper simulates the human heart based on three main functions: hydraulic, electrical and mechanical parameters. Hydro-mechanical model developed then has been transformed into electrical domain and simulation has been carried out according to the mathematical model or formulations obtained using Laplace transform. This electrical model / circuit is then tested by MATLAB based simulations and results found are comparable with the normal ECG waveforms so that these simulated results may be useful in clinical experiments. In this model basic electrical components have been used to simulate the physiological functions of the human heart. The result is a simple electrical circuit consisting of main electrical parameters that are transformed from hydraulic models and medical physiological values. Developed MATLAB based mathematical model will primarely help to understand the proper functioning of an artificial heart and its simulated ECG signals. A comprehensive model for generating a wide variety of such signals has been targeted for future in this paper. This research especially focuses on modelling human heart as a hydro-electro-mechanical system with three case studies.

Keywords: Hydro-electro-mechanical Heart Model, Matlab Simulation, Cardiovascular System, Biomedical, Circulatory System.

1. Introduction

Heart disease is the main cause of death worldwide, especially in the developed and developing countries. Treatments of such diseases have various forms, from the simplest such as following appropriate diets to very complex and dangerous forms such as heart transplantation. Due to improvements in technology, it is possible to find out new opportunities, for example new medical assist devices or predictive medical instruments which help to be aware of the dangerous situations to occur in due time to our health. To realize such goals since the 1960s, various mathematical modeling of human heart have been studied using physical models of the real systems as the reference domain [1-2]. Mathematical models have been widely used in the simulation of cardiovascular systems. The human cardiovascular system is highly complex and involves many control mechanisms. The model of Windkessel is a famous example of such a discrete model. Although computer analysis of numerical models have taken the place of the physical models in many cases, physical electrical models are still indispensable in testing the reliability, functionality and the control of human cardiovascular system. Short term heart rate control is mainly mediated by the central nervous system. Danielson's research group integrated the mathematical models of nervous systems with the Windkessel model, to investigate the control mechanism of the heart in a different way. To replace the functions of real human heart for a limited time, artificial heart pumps have been used. This pump mainly replaces left ventricular failure. Three generations of pumps are present: pulsatile pumps, rotary pumps with contact bearing and rotary pumps with noncontact bearing. The last developed pump as an artificial heart assist device is a magnetically levitated axial flow heart pump. [3]

An electromechanical biventricular model, which couples the electrical property and mechanical property of the heart, has been constructed and the right ventricular wall motion and deformation have been simulated [1]. The excitation propagation has been simulated by electrical heart model, and the resultant active forces have been used to study the ventricular wall motion during systole. The simulation results are found to be compatible with the models theoretical results [1]. It suggests that such electromechanical biventricular model can be used to assess the mechanical function of two ventricles as well. A third-order model of the cardiovascular system designed to illustrate biological system simulation has been described [2]. The model is considered to be extremely simple which allows concentrating on the simulation and computation dimensions but not on the physiological complexity. Heart pumping action is derived from a variable capacitor, and the systemic circulation is modeled as an RC filter. With the rapid development of computer and medical image technology, finite element method (FEM) has become one of the main tools to study cardiovascular mechanics as well [2]. A third-order model of the cardiovascular system uses real geometric shape and fiber structure with a 3-D finite element biventricular model [2].

The surface ECG is the recorded potential difference between two electrodes placed on the surface of the skin at predefined points. The oscillation between systole and diastole states of the heart is reflected in the heart rate (HR). The electrocardiogram (ECG) is a timevarying signal reflecting the ionic current flow which causes the cardiac fibers to contract and subsequently relax. A single normal cycle of the ECG represents the successive atrial depolarization / repolarization and ventricular depolarization / repolarization which occur with every heartbeat. Extracting useful clinical information from the real biomedical systems (or in noisy enviroment) requires reliable ECG signal processing techniques [1]. These include R-peak detection [4], QT-interval detection [5], and the derivation of heart rate and respiration rate from the ECG [3-6]. At present, new biomedical signal processing algorithms are usually evaluated by applying them to ECGs in a large database such as the Physionet database. Baroreceptors, although well established, have been investigated extensively. The goals of such research is to understand the mechanisms of the total artificial heart and obtain a better mathematical model as well as a simple physical analog model of the total artificial heart[5].

2. Methods

S. Wenyu and M. S. Chew developed a mathematical and physical analog model of total artificial heart (TAH) in order to simulate human cardiovascular system [4-5].

Y. Wu, P. E. Allaire, G. Tao, and D. Olsen introduce a state–space model which is developed through theoretical analysis and numerical solutions to approximate total artificial heart response of human circulatory system [3]. This system model has one critical time-varying parameter: total artificial heart (TAH) resistance of peripheral blood vessels. An optimal adaptive controller is proposed to control total artificial heart estimated aortic pressure to track a reference signal updated by a nonlinear function of TAH pump head to meet TAH physiological need. A Matlab simulation program is used as test environment for TAH human circulatory systems with a left ventricular assist device and their physiological controllers. Different physiological conditions are evaluated to test TAH designed physiological control system. Simulation and experimental results consistently show that TAH aortic pressure estimation error is small, and that TAH abnormal hemodynamic variables of a congestive heart failure patient are restored back to TAH normal physiological range [3].

The HEMS model developed by this paper is given in Fig. 1 below. The model is based on symmetry property of human cardiovascular and circulatory system and is made of two parallel equivalent circuits. The compartmental electrical model of Yalcinkaya, Kizilkaplan and Erbas with the defined system parameters (hydraulic and electrical parameters and their units) are given as follows in Figures [1-2], and Tables [1-5], below.

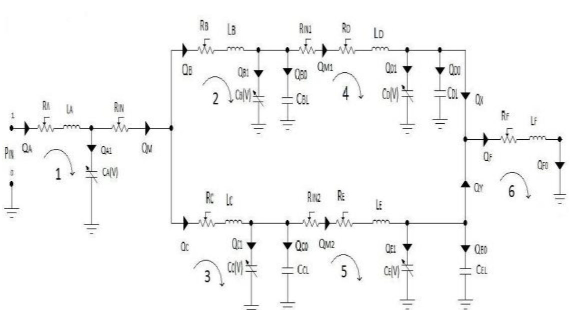


Fig. 1. The compartmental electrical model of **h**ydro**e**lectro**m**echanical **s**ystem (HEMS)

Table 1. Different system parameters and their units

Parameters	Annotations of the model parameters	Units
Q	Flow	ml/s
P	Pressure	mmHg
U	Voltage	Volt
C	Capacity	F
L	Inductance	H
R	Resistance	Ω
$dP_{IN}(t)/dt$	Systemic pressure	mmHg
P_{IN}	Aortic pressure	mmHg
R_A	Fluid variable resistance of aortic valve	Ω
R_{IN}	Fluid variable resistance of pulmonary valve	Ω
R_B	Fluid variable resistance of right-atrium valve	Ω
R_{IN1}	Fluid variable resistance of mitral valve	Ω
R_D	Fluid variable resistance of right-ventricle valve	Ω
R_C	Fluid variable resistance of left-atrium valve	Ω
R_{IN2}	Fluid variable resistance of tricuspit valve	Ω
R_E	Fluid variable resistance of left-ventricle valve	Ω
R_F	Fluid variable resistance of ventricle and atrium interaction	Ω
L_A	Aortic inductance	H
L_B	Right-atrium inductance	H
L_D	Right-ventricle inductance	H
L_C	Left-atrium inductance	H
L_E	Left-ventricle inductance	H
L_F	Ventricle and atrium interaction inductance	H
C_A	Aortic variable capacitance	F
C_B	Right-atrium variable capacitance	F
C_{BL}	Right-atrium systemic capacitance	F
C_D	Right-ventricle variable capacitance	F
C_{DL}	Right-ventricle systemic capacitance	F
C_C	Left-atrium variable capacitance	F
C_{CL}	Left-atrium systemic capacitance	F
C_E	Left-ventricle variable capacitance	F
C_{EL}	Left-ventricle systemic capacitance	F
C_F	Ventricle and atrium interaction variable capacitance	F
C_{FL}	Ventricle and atrium interaction systemic capacitance	F

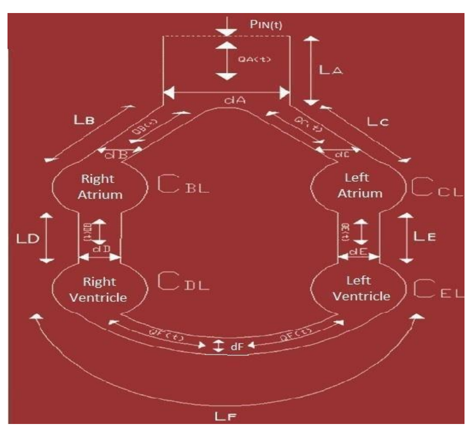


Fig. 2. Compartmental hydroelectromechanical model (HEMS)

Table 2. Flow through parameters of physiological valves and their units

Parameters	Annotations of the model parameters	Units
Q_A	Flow through aortic valve	ml/s
Q_M	Flow through pulmonary valve	ml/s
Q_B	Flow through right-atrium valve	ml/s
Q_{BO}	Flow through right-atrium systemic valve	ml/s
Q_{M1}	Flow through right-ventricle valve	ml/s
Q_{DO}	Flow through right-ventricle systemic valve	ml/s
Q_C	Flow through left-atrium valve	ml/s
Q_{CO}	Flow through left-atrium systemic valve	ml/s
Q_{M2}	Flow through left-ventricle valve	ml/s
Q_{EO}	Flow through left-ventricle systemic valve	ml/s
Q_F	Flow in ventricle and atrium interaction systemic valve	ml/s
Q_{FO}	Flow through ventricle and atrium interaction systemic valve	ml/s

The Equations governing the whole circulatory system could be written down as in the following with the boundary conditions taking into account;

Q_A (t) =Q_{A1} (t) + Q_M (t), Q_M (t) = Q_B (t) + Q_C (t), Q_B (t) = Q_{B1} (t) +Q_{BO} (t) + Q_{M1} (t), Q_{M1} (t) = Q_{D1} (t) + Q_{DO} (t) +Q_X (t), Q_C (t) = Q_{C1} (t) + Q_{CO} (t) + Q_{M2} (t), Q_{M2} (t) = Q_{E1} (t) + Q_{EO} (t) + Q_Y (t), Q_F (t) = Q_X (t) + Q_Y (t), and Q_F (t) = Q_{F0} (t) + Q_{F1} (t)

$$P_1(t) + P_2(t) + P_4(t) + P_6(t) = P_{IN}(t)$$

Writing down the first group of compartmental equations and differentiating both sides of the equations;

$$R_A \cdot Q_A(t) + L_A \cdot \frac{dQ_A(t)}{dt} + \frac{1}{C_A} \cdot \int Q_A(t) \cdot dt + R_{IN} \cdot Q_M(t) + R_B \cdot Q_B(t) + L_B \cdot \frac{dQ_B(t)}{dt} + \frac{1}{C_B} \cdot \int Q_{B1}(t) \cdot dt +$$
$$(R_{IN1} + R_D) \cdot Q_{M1}(t) + L_D \cdot \frac{dQ_{M1}(t)}{dt} + \frac{1}{C_D} \cdot \int Q_{D1}(t) \cdot dt + R_F \cdot Q_F(t) + L_F \cdot \frac{dQ_F(t)}{dt} t = P_{IN}(t)$$

$$R_A \cdot \frac{dQ_A(t)}{dt} + L_A \cdot \frac{d^2Q_A(t)}{dt^2} + \frac{1}{C_A} \cdot Q_{A1}(t) + R_{IN} \cdot \frac{dQ_M(t)}{dt} + R_B \cdot \frac{dQ_B(t)}{dt} + L_B \cdot \frac{d^2Q_B(t)}{dt^2} + \frac{1}{C_B} \cdot Q_{B1}(t) +$$
$$(R_{IN1} + R_D) \cdot \frac{dQ_{M1}(t)}{dt} + L_D \cdot \frac{d^2Q_{M1}(t)}{dt^2} + \frac{1}{C_D} \cdot Q_{D1}(t) + R_F \cdot \frac{dQ_F(t)}{dt} + L_F \cdot \frac{d^2Q_F(t)}{dt^2} = \frac{dP_{IN}(t)}{dt}$$

Applying Laplace transform to both sides of the equations with the proper initial conditions, ending with the basic equation of the system:

$$Q_A(s) \cdot \left[s^2 \cdot L_A + s \cdot (R_A + R_{IN})\right] + Q_{A1}(s) \cdot \left[-s \cdot R_{IN} + \frac{1}{C_A}\right] + Q_B(s) \cdot \left[s^2 \cdot (L_B) + s \cdot (R_B)\right] +$$
$$Q_{M1}(s) \cdot \left[s^2 \cdot L_D + s \cdot (R_{IN1} + R_D)\right] + \frac{1}{C_B} \cdot Q_{B1}(s) + \frac{1}{C_D} \cdot Q_{D1}(s) + Q_F(s) \cdot \left[s^2 \cdot L_F + s \cdot R_F\right] = \left[s \cdot P_{IN}(s)\right]$$

Writing down the second group of compartmental equations and differentiating the both sides of the equations;

$$P_1(t) + P_3(t) + P_5(t) + P_6(t) = P_{IN}(t)$$

$$R_A \cdot Q_A(t) + L_A \cdot \frac{dQ_A(t)}{dt} + \frac{1}{C_A} \cdot \int Q_A(t) \cdot dt + R_{IN} \cdot Q_M(t) + R_C \cdot Q_C(t) + L_C \cdot \frac{dQ_C(t)}{dt} + \frac{1}{C_C} \cdot \int Q_{C1}(t) \cdot dt +$$
$$(R_{IN2} + R_E) \cdot Q_{M2}(t) + L_E \cdot \frac{dQ_{M2}(t)}{dt} + \frac{1}{C_E} \cdot \int Q_{E1}(t) \cdot dt + R_F \cdot Q_F(t) + L_F \cdot \frac{dQ_F(t)}{dt} = P_{IN}(t)$$

Taking the differentiation of both sides of the above equation,

$$R_A \cdot \frac{dQ_A(t)}{dt} + L_A \cdot \frac{d^2Q_A(t)}{dt^2} + \frac{1}{C_A} \cdot Q_{A1}(t) + R_{IN} \cdot \frac{dQ_M(t)}{dt} + R_C \cdot \frac{dQ_C(t)}{dt} + L_C \cdot \frac{d^2Q_C(t)}{dt^2} + \frac{1}{C_C} \cdot Q_{C1}(t) +$$
$$(R_{IN2} + R_E) \cdot \frac{dQ_{M2}(t)}{dt} + L_E \cdot \frac{d^2Q_{M2}(t)}{dt^2} + \frac{1}{C_E} \cdot Q_{E1}(t) + R_F \cdot \frac{dQ_F(t)}{dt} + L_F \cdot \frac{d^2Q_F(t)}{dt^2} = \frac{dP_{IN}(t)}{dt}$$

Applying Laplace transform to both sides of the equations with the initial conditions, finally derived equations are as follows:

$$Q_A(s) \cdot \left[s^2 \cdot L_A + s \cdot (R_A + R_{IN})\right] + Q_{A1}(s) \cdot \left[-s \cdot R_{IN} + \frac{1}{C_A}\right] + Q_C(s) \cdot \left[s^2 \cdot (L_C) + s \cdot (R_C)\right] +$$
$$Q_{M2}(s) \cdot \left[s^2 \cdot (L_E) + s \cdot (R_{IN2} + R_E)\right] + \frac{1}{C_C} \cdot Q_{C1}(s) + \frac{1}{C_E} \cdot Q_{E1}(s) + Q_F(s) \cdot \left[s^2 \cdot L_F + s \cdot R_F\right] = \left[s \cdot P_{IN}(s)\right]$$

Table 3. Parameters associated with different pressure and valves

Parameters	Annotations of the model parameters	Units
Q_A , Q_{A1} , Q_{A2} , Q_{A3}	Flow through aortic valve	ml/s
P_{IN}	Aortic Pressure	mmHg
P_{CS}	Circulatory System of Systolic Pressure	mmHg
P_{CD}	Circulatory System of Diastolic Pressure	mmHg
Q_{ACS1},Q_{ACS2}, Q_{ACS3}	Flow Through Systolic of Aortic Value	ml/s
Q_{ACD1},Q_{ACD2}, Q_{ACD3}	Flow Through Diastolic of Aortic Value	ml/s

Table 4. System parameters and simulation results

The Real Cardiovascular Performance Under Resting Conditions			
Parameters	**Values**	**Units**	**Results**
P_{IN}	0	mmHg	105
P_{CS}	0	mmHg	120
P_{CD}	0	mmHg	70
Initial values for the Simulation			
Q_{A1}	0	ml/s	24,5
Q_{A2}	0	ml/s	22,41
Q_{A3}	0	ml/s	15,75
Q_{ACS1}	0	ml/s	28
Q_{ACS2}	0	ml/s	25,61
Q_{ACS3}	0	ml/s	18
Q_{ACD1}	0	ml/s	16,3
Q_{ACD2}	0	ml/s	14,94
Q_{ACD3}	0	ml/s	10,5

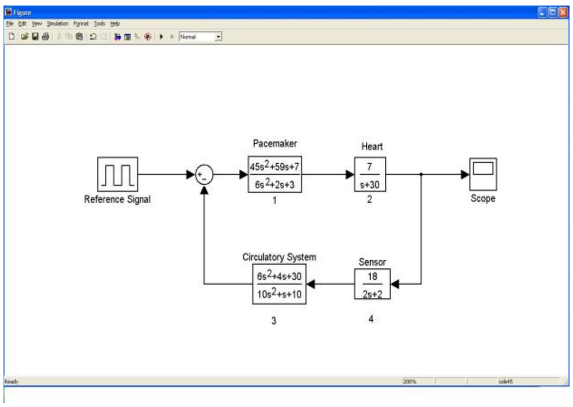
Table 5. Block Parameters: Reference Signal or Pulse Generator Parameters

Pulse Type	**Time-based**
Amplitude	1 mV
Period	4 sec
Pulse Width(% of period)	1 sec
Phase delay	0-1 unit

3. Results

Based on the mathematical formulation obtained, by using the MATLAB simulation facility, six case studies have been done. The system is made of four main elements with their mathematical models defined separately, the main elements are pacemaker, heart, sensor and circulatory system; simulation-1, simulation-2, and simulation-3 are given below producing data according to the main four elements in the feedback control loop.

1) $$\frac{Q_A(s)}{P_{IN}(s)} = \frac{s}{s^2 \cdot L_A + s \cdot (R_A + R_{IN})} = \frac{7}{s+30}$$

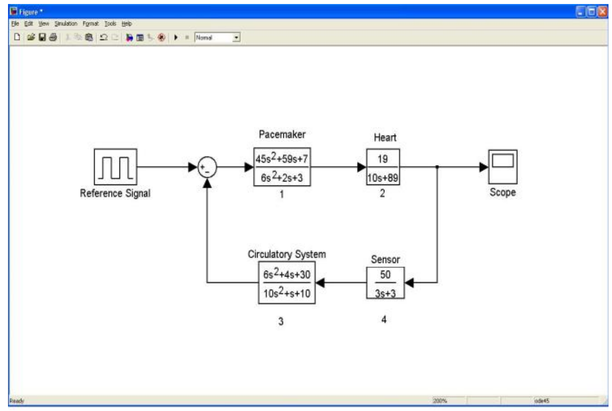


U ≅ [-0.7(mV)]-[1.2(mV)];

U(mV)-t(sec)

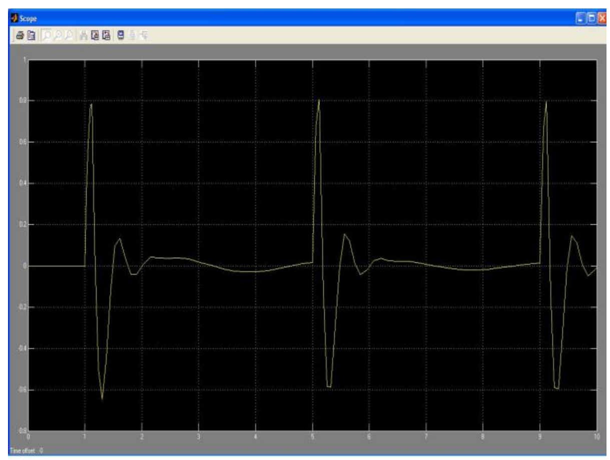
2) $$\frac{Q_A(s)}{P_{IN}(s)} = \frac{s}{s^2 \cdot L_A + s \cdot (R_A + R_{IN})} = \frac{19}{10s+89}$$

U [-0.7(mV)]-[0.75(mV)];



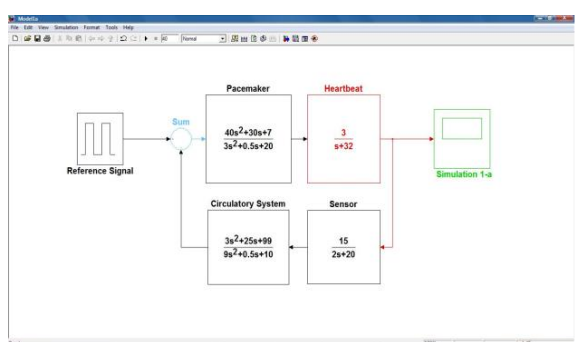
U(mV)-t(sec)

U ≅ [-0.7(mV)]-[0.75(mV)];

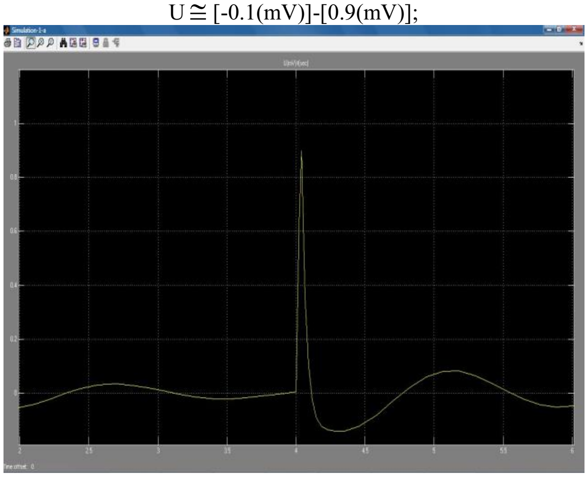


U(mV)-t(sec)

3) $$\frac{Q_A(s)}{P_{IN}(s)} = \frac{s}{s^2 \cdot L_A + s \cdot (R_A + R_{IN})} = \frac{3}{s+32}$$



U ≅ [-0.1(mV)]-[0.9(mV)];



U(mV)-t(sec)

Table 7. The comparison table of three case studies / simulations (1 Unit=0.04sec)

Parameters	Simulation-1	Simulation-2	Simulation-3
P-wave	0.07 sec	No P-Wave	Immeasurable
QT-interval	0.375 sec	0.39 sec	Immeasurable
PR-interval	0.20 sec	No PR-Interval	Immeasurable
QRS-complex	0.139 sec	0.15 sec	Immeasurable
T-wave	0.175 sec	0.20 sec	Immeasurable

4. Discussion and Conclusions

Various theoretical mathematical models and their simulations using different software environments have been developed. Heart wall motion, deformations caused by cardiac electrical activation, displacements and minimum principal strains are some, and have been used to describe the simulation results of the system developed in the literature. Simple cardiovascular models are designed to demonstrate biological system simulations. The physiological knowledge required is deliberately minimized for such system. The computer results are close enough to the physiological data so that verification of the model is possible. These methods are classified in two major classes, linear and non-linear. However the linear methods, are simple and easy to adjust, the nonlinear methods are closer to the real operating, because the origin of this signal is completely non-linear.

This research is based on the mathematical modelling of human heart as a **h**ydro**e**lectro**m**echanical **s**ystem (HEMS). A mathematical model and its component based physical electrical model have been developed, and the MATLAB simulation results agree well with the data obtained from clinical studies. To be able to handle dynamic changes in the cardiovascular system, a multi-variable adaptive model of the human heart and the vascular system will be needed to be developed. The approach employed is to determine the appropriate physical analogs, write the system equations, and formulate the computer simulation. This allowed us consider on the methods of transforming physiological data into useful model parameters, and to establish the analogies between electrical, mechanical, and hydrolic systems. Three Matlab based models, case studies, have been developed and tested with good feedbacks. The system is made of a reference signal, a pacemaker, an artifial heart, a group of sensors and a circulatory system. The results are given in the tables and figures above. As seen from Table-7 above, the three case studies were not able to produce P-wave at all; this needs more attention for future case studies. Secondly the case studies or simulation results produce quite sharp or sharp RS complex which also needs further investigation in term of simulation parameters or feedback control loop used. Real RS complex makes no sharp curve, that point needs attention as well. Although reasonable T waves are obtained from the results of simulation-1 and simulation-2, acceptable T wave is obtained from the results of simulation-3. The four main building blocks of the feedback control loops used in simulations need improvements as the ECG is not only a result of proper functioning of heart but also needs proper and parallel functions of circulatory and cardiovascular subsystems of human whole biological system.

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