AN EFFICIENT a-b-c REFERENCE FRAME-BASED ALGORITHM IN AN ACTIVE POWER FILTER FOR REACTIVE POWER COMPENSATION UNDER UNBALANCED CONDITIONS

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ABSTRACT

Shunt active power filters are used to eliminate current harmonics and to improve the power factor in unbalanced systems near non-linear loads. In this paper an instantaneous vector expression for filter current in terms of active and reactive powers using generalized theory of instantaneous reactive power in a-b-c reference frame-based has been derived. It is shown that the algorithm works under balanced and unbalanced source voltage in magnitude and phase angle. This algorithm has been simulated by Matlab, and the results validate the theory for the balanced and unbalanced conditions.

I. INTRODUCTION

In the past, most of consumed power was due to linear loads. In the recent years, the broader application of nonlinear loads such as power electronic equipment or arc furnaces, has led to transmit high levels of power under unbalanced and non-sinusoidal conditions.

Shunt active power filters (APF) are recognized as a better solution to the problems of current harmonic pollution. This operates like a current source, and injects the compensation currents into the AC lines to cancel harmonics and compensate fundamental reactive power consumed by non-linear loads [1]. In an active power filter, a controller determines the harmonics that are to be eliminated. The output of this controller is the references of three-phase current controlled inverter [2]. Figure 1 shows the general scheme for shunt active power filters. Some algorithms and theories for generating reference have been proposed [3-6].

In [2] four different methods have been compared. These methods are the pq theory, the modified pq theory, the synchronous reference frame theory and the modified synchronous reference frame theory. So we don't discuss these methods in this paper. In [5], the general definitions of instantaneous active and reactive power have been presented. In this formulation, the active and reactive

powers are expressed as the dot and the cross product of voltage and current vectors, respectively.

In this paper we will extend the last theory and more general vector equations for compensation currents have been proposed. The proposed method has been modified for systems that have unbalance in voltage magnitudes and phase angles. Finally, this algorithm has been verified by simulation.

II. INSTANTANEOUS ACTIVE AND REACTIVE POWER THEORY

This discussion has been clearly defined in [5]. Consider a three-phase four-wire system as shown in Fig.1. The instantaneous vectors, v and i are defined as follows;

$$v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad and \quad i = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$
(1)

(2)

The subscripts 'a', 'b' and 'c' denote the respective phases.

The instantaneous active and reactive powers are defined as the dot and cross product of vectors v and irespectively, *i.e.*

and

$$q = v \times i = \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

 $p = v \cdot i = v_a i_a + v_b i_b + v_c i_c$

that:

$$q_{a} = \begin{vmatrix} v_{b} & v_{c} \\ i_{b} & i_{c} \end{vmatrix}, \quad q_{b} = \begin{vmatrix} v_{c} & v_{a} \\ i_{c} & i_{a} \end{vmatrix}, \quad q_{c} = \begin{vmatrix} v_{a} & v_{b} \\ i_{a} & i_{b} \end{vmatrix}$$
(3)

Then we can decompose the current into instantaneous active current, i_p and instantaneous reactive current vector, i_q as,



Figure 1. Schematic diagram of the three-phase active power filter compensation system

$$i_{p} = \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \frac{p}{v.v} v$$

$$i_{q} = \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \frac{q \times v}{v.v}$$
(4)
$$(5)$$

The total current vector is the sum of active and reactive current vectors *i.e.*

$$\dot{i} = \dot{i}_p + \dot{i}_q \tag{6}$$

III. GENERAL EXPRESSIONS FOR FILTER AND SOURCE CURRENTS

Using the equations (4), (5) and (6), we can obtain the instantaneous vector of source and active filter current in terms of active and reactive powers of source and active filter, respectively, as follows.

$$i_{s} = i_{sp} + i_{sq} = \frac{p_{s}v_{s}}{v_{s}v_{s}} + \frac{q_{s} \times v_{s}}{v_{s}v_{s}}$$
 (7)

$$\dot{i}_{f}^{*} = \dot{i}_{fp}^{*} + \dot{i}_{fq}^{*} = \frac{p_{f}v_{s}}{v_{s}v_{s}} + \frac{q_{f} \times v_{s}}{v_{s}v_{s}}$$
(8)

whereas p_s, q_s, p_f and q_f denote source active power, source reactive power, filter active power and filter reactive power, respectively, that are defined by Equations (2) and (3). In Fig.1, it is shown that

$$i_{jj}^{*} = i_{lj} - i_{sj}$$
 , $j = a, b, c$ (9)

By applying the definition of Eq. (3) and expanding the cross product in Eq. (7), the following time domain expressions for filter currents have been derived.

$$i_{fa}^{*} = i_{la} - \frac{1}{\sum_{j=a,b,c} v_{sj}^{2}} (p_{s} v_{sa} + q_{sb} v_{sc} - q_{sc} v_{sb})$$
(10)

$$i_{jb}^{*} = i_{lb} - \frac{1}{\sum_{j=a,b,c} v_{sj}^{2}} (p_{s} v_{sb} + q_{sc} v_{ac} - q_{sa} v_{sc})$$
(11)

$$i_{fc}^{*} = i_{lc} - \frac{1}{\sum_{j=a,b,c} v_{sj}^{2}} (p_{s} v_{sc} + q_{sb} v_{sb} - q_{sb} v_{sa})$$
(12)

IV. COMPENSATION UNDER UNBALANCED VOLTAGE

By simulation, we can show that the application of filter currents (Eqs. (10)- (12)) to an unbalance system result in distorted source currents so we must modify the algorithm to obtain correct compensation as given below. Consider unbalance in magnitudes and in phase angles of main voltage, *i.e.*

$$v_{sa} = V_{ma} \sin(\omega t) \tag{13a}$$

$$v_{sb} = V_{mb}\sin(\omega t - \frac{2\pi}{3} + a_b)$$
(13b)

$$v_{sc} = V_{mc}\sin(\omega t + \frac{2\pi}{3} + \alpha_c)$$
(13c)

In the above equations, V_{ma} , V_{mb} and V_{mc} are not equal to one another. The phase angles α_b and α_c are showing unbalance in phase angles.

We consider a balanced set of main voltage which yields equal average real power with unbalanced source voltages. This balance set will satisfy Eqs (10)-(12). Consider the balanced set defined as follows;

$$v_{sa1} = V_m \sin(\omega t) \tag{14a}$$

$$v_{sb1} = V_m \sin(\omega t - \frac{2\pi}{3})$$
(14b)

$$v_{sc1} = V_m \sin(\omega t - \frac{2\pi}{3})$$
(14c)

Two sets of main voltage *i.e.* Eqs (13) and (14) must have equal average real power, so from this requirement and zero phase angle between v_{sa} and i_{sa} , we obtain the magnitude V_m as:

The above equation leads to.

$$\frac{3}{2}V_m I_{sm} = \frac{1}{2}(V_{ma}I_{sm} + V_{mb}I_{sm}\cos\alpha_b + V_{mc}I_{sm}\cos\alpha_c)$$
$$V_m = \frac{1}{3}(V_{ma} + V_{mb}\cos\alpha_b + V_{mc}\cos\alpha_c)$$
(15)

Based of above considerations, the modified algorithm for active filter reference currents in unbalanced conditions is as follows,

$$i_{fa}^{*} = i_{la} - \frac{1}{\sum_{j=a,b,c} v_{sj1}^{2}} (p_{s} v_{sa1} + q_{sb} v_{sc1} - q_{sc} v_{sb1})$$
(16)

$$i_{fb}^{*} = i_{lb} - \frac{1}{\sum_{j=a,b,c} v_{sj1}^{2}} (p_{s}v_{sb1} + q_{sc}v_{sa1} - q_{sa}v_{sc1})$$
(17)

$$i_{fc}^{*} = i_{lc} - \frac{1}{\sum_{j=a,b,c} v_{sj1}^{2}} (p_{s} v_{sc1} + q_{sa} v_{sb1} - q_{sb} v_{sa1})$$
(18)

In the case of unbalance in magnitude only i.e. $\alpha_b = \alpha_c = 0$, V_m is the average of the unequal magnitudes V_{ma} , V_{mb} and V_{mc} .

Under balanced conditions, i.e. $V_{ma} = V_{mb} = V_{mc}$ and $\alpha_b = \alpha_c = 0$, the Eqs. (16)-(18) converge to that given in Eqs. (10)- (12). It is to be noted that if we consider v_{sa1}, v_{sb1} and v_{sc1} as balanced supply voltages, then the main source voltage set should supply average load power and zero mean oscillating active and reactive powers.

V. SIMULATION OF CASE STUDY

The Simulink Matlab software is used for simulations. The simulated system parameters are as follows:

$$v_{sa1} = 220\sqrt{2} \sin(100\pi t)$$

$$v_{sb1} = 1.2 \times 220\sqrt{2} \sin(100\pi t - \frac{2\pi}{3} + \frac{\pi}{6})$$

$$v_{sc1} = 0.8 \times 220\sqrt{2} \sin(100\pi t + \frac{2\pi}{3} + \frac{\pi}{18})$$

The load consists of (i) unbalanced three – phase RL load: $R_a = 1\Omega$, $L_a = 100mH$, $R_b = 3\Omega$,



Figure 2. Unbalanced source voltages

 $L_b = 50mH$, $R_c = 5\Omega$, $L_C = 100mH$; (ii) three – Diode phase full bridge converter with $R = 10\Omega, L = 1mH$, load. The results of this simulation are showed in Figs. (2)- (7). The active filter is applied at the end of second cycle (40 msec), so before this time the source currents will be equal to load currents and after 40 msec., the source currents have been compensated. Fig (2) shows the unbalanced source voltages. And Fig (3) shows the source currents. It has been seen that after applying active power filter, the source currents are sinusoidal.

Active power of source and load are plotted in Fig. (4). The compensation currents are shown in Fig. (5). Reactive power of source and load are plotted in Fig. (6). It has seen that the source supply the average and zero mean oscillating active and reactive powers. Harmonic spectra and THD of load and source currents are presented in Fig (7). It has been seen that amplitude of first harmonic decreased and THD of source has been modified.



Figure 3. Compensated source currents



Figure 4. Instantaneous active power of source and load



Figure 5. Compensation currents



Figure 6. Instantaneous reactive power of load and source



Figure7. Harmonic spectra with THD of load and source current respectively

VI. CONCLUSION

In this paper, based on generalized active and reactive powers theory, in a-b-c reference frame-based a compensation algorithm for unbalanced conditions is proposed. This method has been verified by MATLAB simulation. The proposed method can be applied to balanced and unbalanced source voltages in magnitude and phase angle.

VII. REFERENCES

- M. Machmoum, N. Bruyant, "Control Methods for Three-phase Active Power Filters under Non-ideal Mains Voltages", in proceedings Power System Technology, Powercon 2000, International Conference on, Vol. 3, 2000, pp. 1613-1618.
- [2] G.D. Marques, "A Comparison of Active Power Filter Control Methods in Unbalanced and Non-Sinusoidal Conditions", IECON'98, August 31-September 4, pp. 444-449.

- [3] V. Soares, P. Verdelho, G.Marques, "A Control Method for Active Power Filters under Unbalanced Non-Sinusoidal Conditions", in Proc. PEVD'96, 23-25 Sep. 1996, pp.120-124.
- [4] Shyh-Jier Huang, Jing-Chang wu, Hurng-Liang Jou, "A Study of Three-Phase Active Power Filters under Non-ideal Main Voltage", Electric Power Systems Research 49 (1999), pp. 129-137.
- [5] F.Z. Peng, J.S.Lai, "Generalizard Instantaneous Reactive Power Theory for Three – Phase Power Systems", IEEE Transaction on Instrumentation and Measurement Vol. 45, no.1, 1996.
- [6] Cheng-Che Chen, Yuan Yih Hsu, "A Novel Approach to Design of a Shunt Active Filter for an Unbalanced Three-Phase Four – Wire System Under Non-Sinusoidal Conditions", IEEE Transactions on Power Delivery, Vol. 15, no. 4, October 2000, pp. 1258-1264.