A SOFTWARE FOR HARMONIC ANALYSIS OF SWITCHING WAVEFORMS

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Abstract
Electromagnetic interference(EMI) is the major problem in switched mode power supplies(SMPS). The fast switching action which increases the efficiency produces a wide interference spectrum. Since an SMPS can be a source of EMI, this aspect must be considered during the design process. Various switching techniques can be used in parallel resonant bridge inverters both to control the output voltage and minimize its harmonics. In this study, a software is written for analyzing the harmonics of the most common switching techniques.

1.Introduction

Switched mode power supplies have characteristics such as high output power per unit volume and high efficiency, and as a result their use is increasing. If improperly designed, however, they create electromagnetic interference(EMI) that can degrade other systems. A major shortcoming of switching-regular technology is that it is a complex technology that appears to be simpler than it actually is. Because of this apparent simplicity, misjudgements are generally done in the design and application of switched mode power supplies by the engineer. In the Electromagnetic Compatibility(EMC) concept, one of the most important things is to consider the EMI problem in the early design stage [1, 2]. In practice, it is almost impossible to try all the possible techniques since each waveform is required to be produced by a special circuit. The software developed in this study gives us the ability to estimate the harmonic amplitudes by calculating the Fourier coefficients up to high harmonic orders for different switching wave forms. This software allows us to see how the harmonic contents change by differing the switching techniques without setting any circuit.

2. Fourier Series Extension

Any periodic waveform can be represented as composed of the superposition of a D.C. component with a fundamental pure sine-wave component, together with pure sine-waves known as *harmonics* at frequencies which are multiples of the fundamental.
A non-sinusoidal wave can be expressed mathematically as;

$$V(t) = V_0 + V_1.\sin(\omega t + \theta_1) + \ldots + V_n.\sin(n\omega t + \theta_n) \quad (1)$$

where $V(t)$ is the instantaneous value at any time t, V_0 is the value of the D.C. component, V_1 is the maximum value of the fundamental component, V_n is the maximum value of the n[th] harmonic component, and θ_n defines the relative angular reference.

$\omega=2\pi f$, where f is the frequency of the fundamental component .

Using the below equation, the series in equation (1) can alternatively be expressed as in equation (3).

$$V_n \cdot \sin(nx + \theta_n) = A_n \cdot \cos nx + B_n \sin nx \quad (2)$$

$$V(t) = Vo + \sum_{n=1}^{\infty}(A_n \cdot \cos\frac{2n\pi t}{T} + B_n \cdot \sin\frac{2n\pi t}{T}) \quad (3)$$

where T is the period of any periodic function .

Equation (3) is known as the **"Fourier Series"** and the **"Fourier Coefficients"** of $V(t)$ are given by following equations:

$$V_0 = \frac{1}{T}\int_{-T/2}^{T/2} V(t).dt \quad (4)$$

V_0 equals zero if there is no D.C. component of the voltage wave-form.

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} V(t)\cdot\cos\frac{2n\pi}{T}\cdot dt \quad ;\ n=1,2,\ldots \quad (5)$$

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} V(t)\cdot\sin\frac{2n\pi}{T}\cdot dt \quad ;\ n=1,2,\ldots \quad (6)$$

The Fourier Series is used to convert the signal from time domain to the frequency domain in order to analyze the waveform. The frequency content of a waveform can be used to identify the sources of noise, and appropriate remedies can be applied to the system to minimize the EMI problem[3, 4].

The most commonly used switching techniques[5,6] analyzed in this study are :

1. Square wave
2. Quasi-square wave
3. Notched waveform

2.1 Square wave

Square wave is the most commonly used switching waveform in switched mode power conversion systems, especially due to its simplicity. The general waveform of a square wave of which the on and off periods are equal is shown in Figure 1.

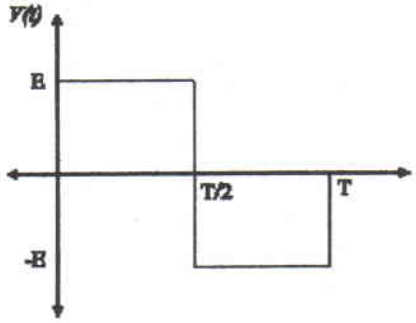

Figure 1 Square wave

$V(t)$ function can be formulated as in equation (7):

$$V(t)=\begin{cases} E \rightarrow 0\langle t\langle T/2 \\ -E \rightarrow T/2\langle t\langle T \end{cases} \quad (7)$$

V_0=0 since there is no D.C. component

A_n and B_n coefficients can be calculated from equations (5) and (6);

$$A_n=\frac{2E}{T}\left[\int_0^{T/2}\cos\frac{2n\pi}{T}\cdot dt-\int_{T/2}^{T}\cos\frac{2n\pi}{T}\cdot dt\right]=0 \quad (8)$$

$$B_n=\frac{2E}{T}\left[\int_0^{T/2}\sin\frac{2n\pi}{T}\cdot dt-\int_{T/2}^{T}\sin\frac{2n\pi}{T}\cdot dt\right]$$
$$=\frac{2E}{n\pi}(1-\cos n\pi) \quad (9)$$

It is clear that B_n values change depending on the ratio of the on and off periods over the period(T) of the waveform.

2.2 Quasi-square wave

Voltage control can be obtained by inserting zero periods into the square wave , giving a shape known as a "Quasi-square wave ".

Duration of the zero periods plays and important role on the amplitudes of the harmonic components . But it is difficult to calculate these values by hand for the different shapes of quasi square wave. General shape of the quasi-square wave is shown in Figure 2.

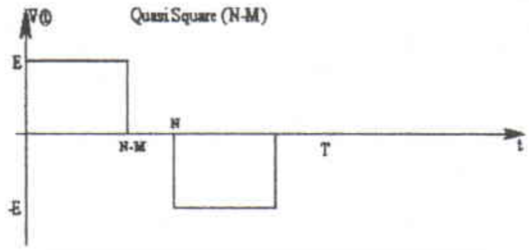

Figure 2 General shape of quasi-square wave

Quasi-square wave shapes and characteristics change depending on the duration of zero periods in a half cycle. A notation can be developed to indicate the ratio of this zero periods over the half period. In this study a quasi-square wave is represented by the name Quasi-square(N-M)
where N is the number of divisions in a half period and M is the number of zero-value divisions in a half period.

For example; Quasi-square(4-1) is a kind of wave such that the zero-value division number is "1" where the number of divisions in a half period is "4". The voltage function of Quasi-square(4-1) waveform can be written as shown in equation(10).

$$V(t)=\begin{cases} E & 0\langle t\langle 3T/8 \\ 0 & 3T/8\langle t\langle T/2, 7T/8\langle t\langle T \\ -E & T/2\langle t\langle 7T/8 \end{cases} \quad (10)$$

From equations (5) and (6), Fourier Coefficients A_n and B_n can be calculated for Quasi-square(4-1) waveform.

$$A_n=\frac{E}{n\pi}\left[\sin\left(\frac{3n\pi}{4}\right)-\sin\left(\frac{7n\pi}{4}\right)\right] \quad (11)$$

$$B_n=\frac{E}{n\pi}\left[1-\cos\left(\frac{3n\pi}{4}\right)-\cos\left(\frac{7n\pi}{4}\right)+\cos(n\pi)\right] \quad (12)$$

2.3 Notched Waveform

Another form of voltage control can be provided by the notched waveform. It can be obtained by notching a square wave in different intervals. Notched

waveforms can be different depending on the number of divisions per half cycle. A notation can be use to represent the type of the notched waveform as
Notched(n)
where n is the number of divisions per half cycle.

Notched(4) waveform is shown in Figure 3 as an example.

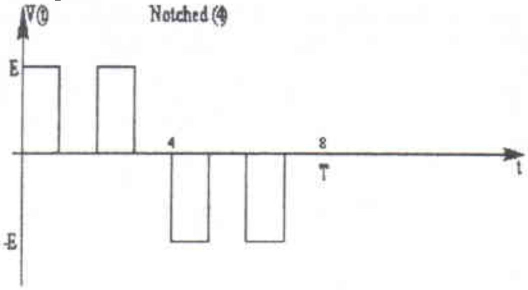

Figure 3 Notched(4) waveform

The function of V(t) in Figure 3 can be written in the following form:

$$V(t)=\begin{cases} E & 0<t<T/8,\ T/4<t<3T/8 \\ 0 & T/8<t<T/4,\ 3T/8<t<T/2,\ 5T/8<t<4,\ 7T/8<t<T \\ -E & T/2<t<5T/8,\ 3T/4<t<7T/8 \end{cases} \tag{13}$$

Fourier Coefficients of the notched waveform change depending on the number of notches in each half cycle. A_n and B_n are calculated for Notched(4) waveform from equations (5) and (6) as follows:

$$A_n=\frac{E}{n\pi}\left[\sin\left(\frac{n\pi}{4}\right)+\sin\left(\frac{3n\pi}{4}\right)-\sin\left(\frac{n\pi}{2}\right)-\sin\left(\frac{5n\pi}{4}\right)-\sin\left(\frac{7n\pi}{4}\right)+\sin\left(\frac{3n\pi}{2}\right)\right] \tag{14}$$

$$B_n=\frac{E}{n\pi}\left[1-\cos\left(\frac{n\pi}{4}\right)-\cos\left(\frac{3n\pi}{4}\right)+\cos\left(\frac{5n\pi}{4}\right)-\cos(n\pi)+\cos\left(\frac{7n\pi}{4}\right)\right] \tag{15}$$

3. Harmonic Analysis Software

A special software is written in "C' programming language particularly for this study to calculate the amplitudes of harmonic components of the above mentioned switching techniques. It can easily be seen from the examples solved for Quasi(4-1) and Notched(4), which are not really complicated, that the mathematical calculations are long and take much time. It is really difficult to calculate harmonics up to 100 or 150 orders. Besides the waste of time, there is a high probability to make mistakes during these long calculations. The software gives the advantage of calculating the harmonics for large harmonic orders without any mistake.

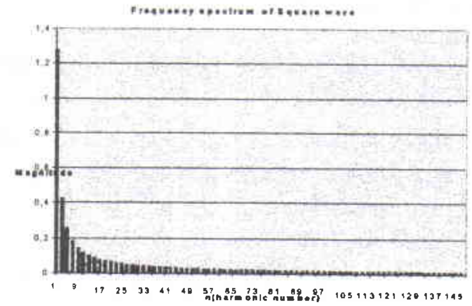

Figure 4 Frequency spectrum of square wave

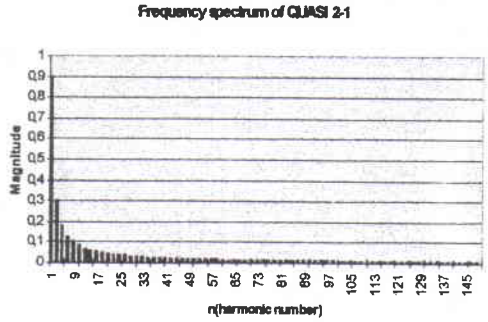

Figure 5-(a) Frequency spectrum of Quasi 2-1

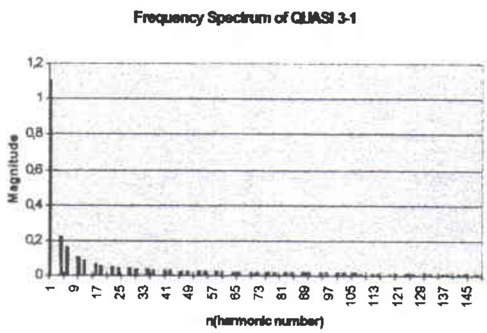

Figure 5-(b) Frequency spectrum of Quasi 3-1

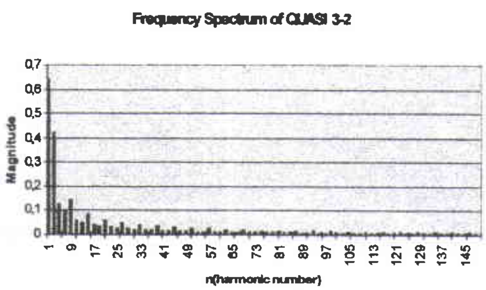

Figure 5-(c) Frequency spectrum of Quasi 3-2

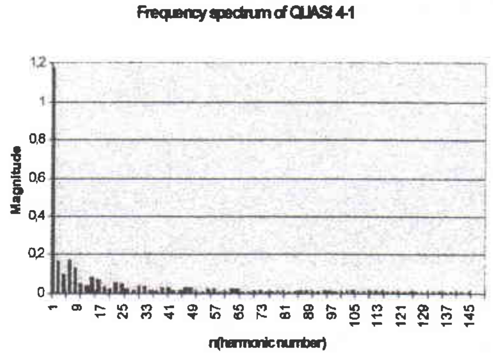

Figure 5-(d) Frequency spectrum of Quasi 4-1

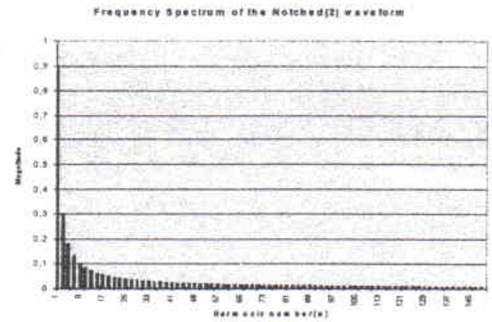

Figure 6-(a) Frequency spectrum of Notched(2)

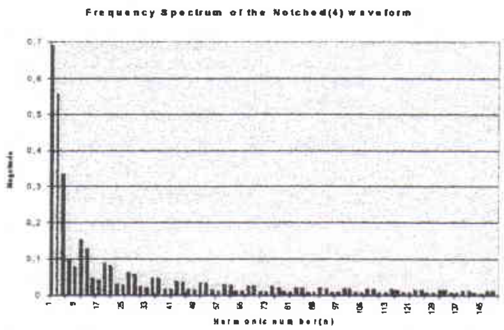

Figure 6-(b) Frequency spectrum of Notched(4)

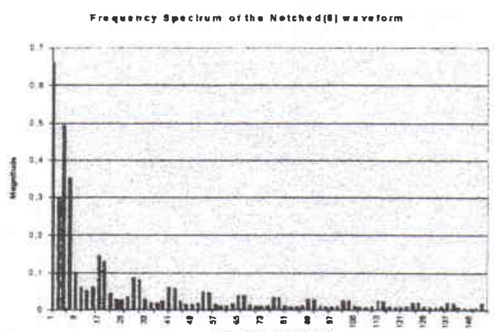

Figure 6-(c) Frequency spectrum of Notched(6)

4. Conclusion

The harmonic analysis software written specially in this study gives us the ability to predict the magnitudes of the harmonic components up to high orders without any mistake and time consumption. According to the frequency spectrum of the harmonic contents, the necessary precautions can be taken or switching technique can be changed. The best solution can be chosen depending on the simulation results.

This software also allows to see how the harmonic contents change by differing the switching techniques without setting any circuit. In practice, it is almost impossible to try all the possible techniques since each waveform is required to be produced by a special circuit. The harmonic analysis software written for this study gives the designer an opportunity to predict the possible problems resulting from the switching action and to take the necessary precautions.

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