# MODELING MAGNETIZATION CHARACTERISTICS OF MATERIALS USED IN POLE LEG OF DC MACHINES WITH ARTIFICIAL NEURAL NETWORKS AND ITS COMPARISON WITH CUBIC SPLINE METHOD

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***Abstract:** In this study, magnetization characteristics of materials constituting polar leg in DC machines is modeled with artificial neural networks applied to finite elements method. Cubic spline method is preferred for modeling magnetization curves of electrical materials when solving non-linear field problems with finite elements method. As the method results in large errors at the points where the curve changes fast, modeling is processed after the curve is divided into several parts at those points. This leads to an increase in modeling costs. Such a disadvantage is dismissed by modeling the curves with artificial neural networks. For learning processes in the artificial neural networks, the back propagation training algorithm is used.*

## 1.INTRODUCTION

This present paper deals first with Cubic spline method in numerical modeling of $\upsilon$-$B^2$ curves when solving non-linear problems with the finite elements method and then the $\upsilon$-$B^2$ curve obtained by using the magnetization curve of the chosen DC machine is modeled with cubic spline and artificial neural networks in order to compare the results.

## 2. OBTAINING $\upsilon$ - $B^2$ CURVES FOR ELECTRICAL MATERIALS

In order to apply Newton-Raphson method on non-linear isotropic problems in solving non-linear problems with finite elements method, reluctivity of elements and their slopes with respect to $B^2$ must be calculated [1].
By using the energy expression,

$$W = B \cdot H \qquad (1)$$

curves $\upsilon = f(B^2)$ are obtained from B-H curves by using the following:

$$W = \upsilon \cdot B^2 \qquad (2)$$

$$B^2 = W / \upsilon \qquad (3)$$

## 3.MODELLING $\upsilon$ - $B^2$ CURVES WITH CUBIC SPLINE

The cubic spline method, one of the exponential function approximation methods for B-H curves, is highly preferred in the application of finite elements method to electromagnetic problems. In this paper, the algorithm given below [2] is used to model the magnetization curve of a DC shunt motor (3.5 kW, 440 V, 9.5 A, 2900 rpm)with cubic spline. Used material is rolled out vertically and has a thickness of 5 mm.

In order to construct and evaluate a cubic spline interpolant S(x) for N+1 data points $(x_0, y_0)$, $(x_1, y_1)$ ,....., $(x_N, y_N)$, provision is made for the following choices of endpoint constraints:

( i ) " Clamped cubic spline" ; specify $S'(x_0)$ and $S'(x_N)$.

( ii ) " Natural cubic spline"

( iii ) Extrapolate $S''(x)$ to the endpoints.

( iv ) $S''(x)$ is constant near the endpoints.

( v ) Specify $S''(x)$ at each endpoint.

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H(0):=X(1)-X(0)                 (Difference in abcissa)
D(0):=[Y(1)-Y(0)]/H(0)          (Difference quotient)

FOR    K=1 TO N-1 DO

H(K):=X(K+1)-X(K)               (Difference in abcissa)
D(K):=[Y(K+1)-Y(K)]/H(K)        (Difference quotient)
A(K):=H(K)                      (Subdiagonal elements)
B(K):=2*[H(K-1)+H(K)]           (Diagonal elements)
C(K):=H(K)                      (Superdiagonal elements)

FOR   K=1 TO N-1 DO (Determine the column vector)

V(K) := 6*[D(K) - D(K-1)]

CASES (Modify the matrix and/or column vector)

(i)   Set B(1):= B(1) - H(0)/2
          V(1):=V(1)-3*[D(0)-S'(x0)]     (Input S'(x0))
          B(N-1):=B(N-1) - H(N-1)/2
          V(N-1):=V(N-1)-3*[S'(xN)-D(N-1)]
                                         (Input S'(xN))
(ii)  Set M(0):= 0 and M(N) :=0

(iii) Set B(1):= B(1) + H(0) + H(0)*H(0)/H(1)
          C(1):= C(1) - H(0)*H(0)/H(1)
          B(N-1):=B(N-1) + H(N-1)
                       + H(N-1)*H(N-1)/H(N-2)
          A(N-2):= A(N-2) - H(N-1)*H(N-1)/H(N-2)

(iv)  Set  B(1):= B(1) + H(0) and
                      B(N-1):= B(N-1) + H(N-1)
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(v) SetV(1):=V(1)-H(0)* S''(x0)                (Input S''(x0))
        V(N-1):=V(N-1)-H(N-1)* S''             (Input S''(xN))

END

FOR     K=2  TO  N-1  DO

T:= A(K-1)/B(K-1)
B(K):= B(K) - T*C(K-1)
V(K):= V(K) - T*V(K-1)
M(N-1):=V(N-1)/B(N-1)
                        (Previous step is used to find mk)

FOR     K=N-2  DOWNTO  1  DO
M(K):= [V(K) - C(K)*M(K+1)]/B(K)

CASES{Determination of the values of M(0) and M(N)}

(i) Set M(0):=3*[D(0)-S'(x0)]/H(0) - M(1)/2
        M(N):=3*[S'(xN) - D(N-1)]/H(N-1) - M(N-1)/2

(ii) Set M(0) := 0  and  M(N) := 0

(iii) Set M(0) := M(1) - H(0)*[M(2) - M(1)]/H(1)
        M(N) := M(N-1) + H(N-1)*
                        [M(N-1) - M(N-2)]/H(N-2)

(iv) Set M(0) := M(1)  and  M(N) := M(N-1)

(v) Set M(0) := S''(x0) and  M(N) := S''(xN)

END

FOR     K=0  TO  N-1  DO

S(K,0):=Y(K)  (Coefficients for each cubic polynomial
               Sk(x) are computed and stored)
S(K,1):=D(K)-H(K)*[2*M(K)+M(K+1)]/6
S(K,2):= M(K)/2
S(K,3):= [M(K+1) - M(K)]/[6*H(K)]

{Procedure for evaluating the cubic spline above on
[x0,xN] }

INPUTX                  (Input the independent variable)

FOR     J=1  TO  N  DO

IF  X(J-1)≤X≤X(J)  THEN
SetK:=J-1 and J:=N                  (Finding the interval)
IF  X=X(0)  THEN  Set K := 0
W:=X-X(K)                           (Spline evaluation)
Z:= [[S(K,3)*W + S(K,2)]*W + S(K,1)]*W +S(K,0)

PRINT "The value of the spline S(x) is" Z        (Output)
```

B-H curve which represents magnetic flux density varies as a function of magnetic field intensity for this DC shunt motor is given in Figure 1.

When equation (2) is used in equation (1) in order to obtain the reluctivity in terms of H and B,

$$B.H = \upsilon.B^2 \quad (4)$$

is obtained. After the necessary simplifications, magnetic reluctivity is calculated as:

$$\upsilon = H / B \quad (5)$$

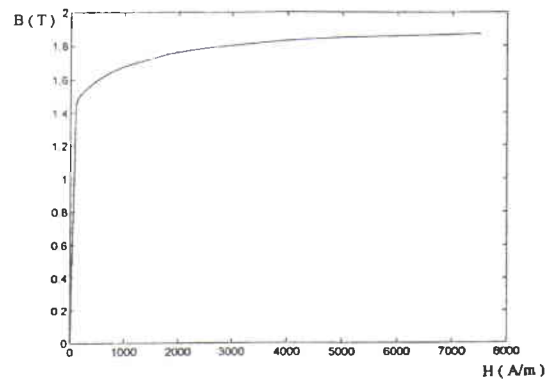


Figure 1. Magnetic flux variation as a function of magnetic field intensity.

Reluctivity values, as obtained by applying B-H values read from the magnetization curve in Figure 1 in the equation (5), are given in Table 1. The variation of reluctivity as a function of the square of magnetic flux density is given in Figure 2.

Table 1. Reluctivity values calculated as related to B and H.

| H (A/m) | B (T) | υ (A/mT) |
|---|---|---|
| 0 | 0 | 0 |
| 100 | 1.45 | 69 |
| 150 | 1.5 | 100 |
| 300 | 1.55 | 193.6 |
| 500 | 1.6 | 312.5 |
| 600 | 1.62 | 370.4 |
| 750 | 1.65 | 454.6 |
| 1000 | 1.68 | 595.2 |
| 1200 | 1.7 | 705.9 |
| 1500 | 1.725 | 869.6 |
| 1800 | 1.75 | 1028.6 |
| 2300 | 1.775 | 1295.8 |
| 2900 | 1.8 | 1611.1 |
| 3800 | 1.825 | 2082.2 |
| 5000 | 1.85 | 2702.7 |
| 7500 | 1.87 | 4010.7 |
| 8700 | 1.88 | 4627.7 |
| 11500 | 1.9 | 6052.6 |
| 13200 | 1.91 | 6911 |

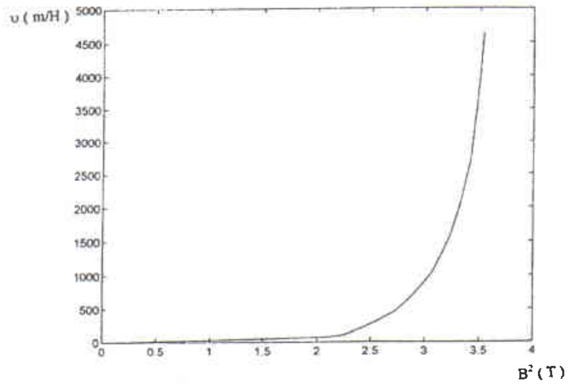


Figure 2. υ-$B^2$ characteristics obtained by using the magnetization curve in Fig. 1.

Coefficients $S_{k,0}$, $S_{k,1}$, $S_{k,2}$ and $S_{k,3}$ , obtained by using the program written with the previously given algorithm, are used for modeling υ-$B^2$ curve with cubic spline and given in Table 2.

Table 2. Coefficients calculated according to the algorithm used in modeling υ-$B^2$ curve with cubic spline.

| $B^2_k$ | $S_{k,0}$ | $S_{k,1}$ | $S_{k,2}$ | $S_{k,3}$ |
|---|---|---|---|---|
| 0.00 | 0.00 | 2.92 | 0.00 | 6.78 |
| 2.10 | 68.97 | 92.68 | 42.74 | 4789.84 |
| 2.25 | 100.00 | 428.82 | 2198.17 | -5994.67 |
| 2.40 | 193.55 | 683.63 | -499.43 | 5457.57 |
| 2.56 | 312.50 | 942.96 | 2120.20 | -29352.3 |
| 2.62 | 370.37 | 880.38 | 3163.22 | 27774.72 |
| 2.72 | 454.55 | 1080.97 | 5169.19 | -19099.1 |
| 2.82 | 595.24 | 1541.84 | -560.55 | 15012.95 |
| 2.89 | 705.88 | 1697.28 | 2781.17 | -15002.1 |
| 2.98 | 869.57 | 1811.47 | -1512.41 | 46410.01 |
| 3.06 | 1028.5 | 2460.56 | 9626.00 | -44198.3 |
| 3.15 | 1295.7 | 3119.22 | -2307.55 | 73116.26 |
| 3.24 | 1611.1 | 4480.58 | 17433.84 | -100667 |
| 3.33 | 2082.1 | 5172.46 | 9746.32 | 320897.0 |
| 3.42 | 2702.7 | 11215.9 | 76895.88 | -158998 |
| 3.50 | 4010.7 | 20466.4 | 38736.28 | -1181385 |
| 3.53 | 4627.6 | 19600.9 | -67588.2 | 565354.3 |
| 3.61 | 6052.6 | 19641.6 | 68096.65 | -567472 |
| 3.65 | 6910.9 | 0.00 | 0.00 | 0.00 |

Reluctivity values obtained for intermediate values as the result of modeling with cubic spline are given in Table 3.

Table 3. υ-$B^2$ values obtained as the result of modeling with cubic spline.

| $B^2$ | υ |
|---|---|
| 0.50 | 2.31 |
| 0.75 | 5.05 |
| 1.00 | 9.71 |
| 1.50 | 27.28 |
| 2.00 | 60.12 |
| 2.10 | 68.97 |
| 2.15 | 74.31 |
| 2.20 | 83.46 |
| 2.30 | 126.19 |
| 2.45 | 227.17 |
| 2.50 | 262.38 |
| 2.60 | 351.73 |
| 2.66 | 394.68 |
| 2.75 | 491.12 |
| 2.85 | 641.42 |
| 2.94 | 795.71 |
| 3.00 | 905.57 |
| 3.10 | 1139.57 |
| 3.20 | 1455.10 |
| 3.30 | 1920.96 |
| 3.47 | 3435.86 |
| 3.57 | 5339.74 |
| 3.62 | 6255.29 |
| 3.65 | 6910.93 |

In modeling with cubic spline, initial values of the curve deviate excessively as shown in Figure 3.

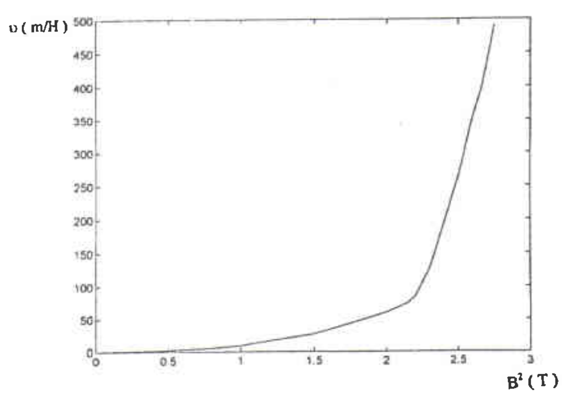


Fig. 3. υ-$B^2$ curve obtained in the result of modeling with cubic spline.

## 4. MODELLING υ - $B^2$ CURVES WITH ARTIFICIAL NEURAL NETWORKS

The cubic spline method generates large errors at points where the curve varies drastically; therefore modeling operation must be carried out after dividing the curve into several parts at those points. This requirement increases modeling costs. This flaw is corrected by the application of artificial neural networks for modeling the curves. In artificial neural networks method, back-propagation of error algorithm is used for learning operation. [3]

As it is known, back propagation algorithm is used in order to minimize the squared error function between the desired output and actual output of a multi-layer feedforward perceptron. In this present paper, the algorithm is applied as it is shown below in order to model the curve with artificial neural networks.

STEP 1. Designate the initial weight values as small random values.

STEP 2. Enter input and desired outputs. Present a continuous input vector $x_0, x_1, \ldots, x_{N-1}$ and determine outputs $d_0, d_1, \ldots, d_{M-1}$. If the net is used as a classifier, all outputs except the one corresponding to the class from which the input originates are set to 0. The desired output is 1. Input may be a new one at each trial or samples from a training set can be presented over and over until the weights become balanced.

STEP 3. Calculate actual outputs.

STEP 4. Adjust weights. Use an algorithm that works backwards starting from the output nodes and repeats itself by returning to the first hidden layer. Adjust weights with the following formula:

$$w_{ij}(t+1) = w_{ij}(t) + \eta\delta_j x_j'$$

In this equation, $w_{ij}(t)$ is a weight from a hidden node i or is a weight from an input at time t from node j. $x_j'$ is either an output from node j or an input, $\eta$ is a gain term and $\delta_j$, is the error term for node j.

STEP 5. Go back to Step 2 and repeat [4].

The net is trained first by choosing small random weights and internal thresholds and then by introducing all training data over and over again. Weights are adjusted after each trial until the correct approach is obtained by using additional data and the cost function has an acceptable value. In back propagation, methods such as permitting extra hidden layers, lowering the gain term used in order to adjust weights and repeating many training trials with different random weight sets are proposed in order to increase performance and decrease formation of local minimums. A difficulty arising from back propagation error is the fact that in most cases, number of training data presentations necessary for approximation is high.

As there is only one input and one output for artificial neural networks in modeling the curve $\upsilon$ -$B^2$, number of nodes in input and output layers is identical and equal to 1. Four nodes are chosen in hidden layer. Network structure for this architecture is presented in Figure 4.

Results of modeling with artificial neural networks has been calculated for different iteration values and learning coefficients. As there is not any linear relationship between iteration and learning, appropriate number of iterations has been investigated and 10,000 iterations have been found to be sufficient.

Error variation as related to iteration as a result of modeling with artificial neural networks is presented in Figure 5. Absolute error is approximately 2% around 10,000 iterations.

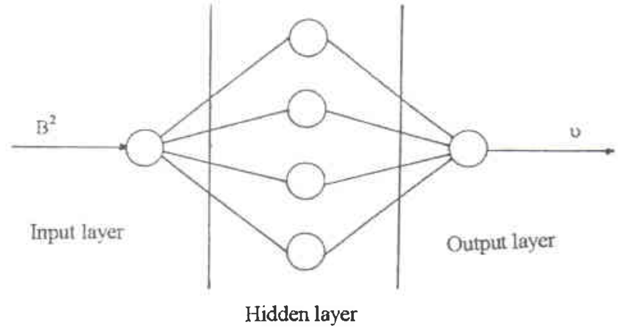


Figure 4. Network architecture used in modeling.

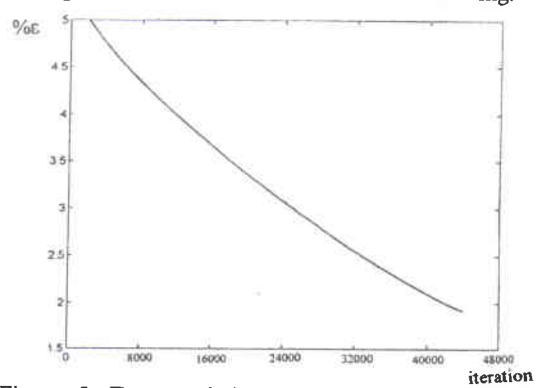


Figure 5. Error variation as related to iteration in modeling with artificial neural networks

Curve $\upsilon$-$B^2$ as a result of this modeling is presented in Figure 6.

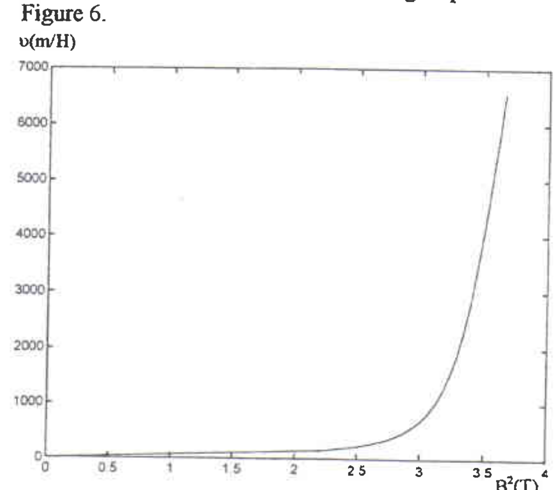


Figure 6. Curve $\upsilon$-$B^2$ obtained as a result of modeling with artificial neural networks

## 5 RESULTS

Curve $\upsilon$-$B^2$ obtained by modeling with artificial neural networks method, when compared to the curve obtained by modeling with cubic spline method, shows that the former approach is more successful. Application of artificial neural networks is not susceptible to extreme deviations in the initial values as it is the case in modeling with cubic spline and both modeling costs and duration decrease considerably.

A single hidden layer is used in modeling with artificial neural networks. The effects of an increase in the number of hidden layers on the modeling results are not investigated. As other parameters (momentum coefficient, learning coefficient and number of iterations) vary with the number of hidden layers, learning is affected largely but no linear relationship exists between an increase in the values of those parameters and improvement in learning.

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