NUMERICAL SIMULATION OF A TWO-DIMENSIONAL CIRCULAR REFLECTOR ANTENNA SYSTEM BY THE METHOD OF MOMENTS

Akın Aybars Hamşioğlu
email:akin.hamsioglu@eee.deu.edu.tr

Taner Oğuzer
email: taner.oguzer@eee.deu.edu.tr

Dokuz Eylül University, Faculty of Engineering, Department of Electrical & Electronics Engineering, Tınaztepe Campus, Buca 35160
İzmir-Turkey

Key words: Method of moments, scattering, complex-source point

ABSTRACT

This study briefly presents the formulation and numerical results of the scattering problem from a two dimensional circular reflector antenna system. The feeder antenna of the system is located at the approximate geometrical focus and modelled by a complex source point method. The method of moments (MoM) has been used for this solution. It has been seen that the results are very close to the exact solutions obtained before in literature. Although in literature it has been applied to small size geometries, we have tried to solve even larger geometries with the same method by combining MoM by complex source point method.

I. INTRODUCTION

The problem of scattering from a perfectly conductive two-dimensional curved structure is one of the traditional problems of the classical electrodynamics. In literature, one of the earliest studies by MoM is performed in [1] and solved moderate size geometries by using different approaches. In [2], an approximate solution is also presented for E-polarized wave about the scattering problem from a concentrially loaded slit cylinder. Later in [3] and [4], a very stable MoM solution even for a narrow, slit cases is formulated. Although this is performed for both polarizations and only for small and medium size geometries, it is also valid for cavity resonances with a high Q factor.

In addition to that, the same problem of the curved and pec strip scattering is also studied by method of regularization and especially semi-inversion by using Riemann-Hillbert Technique is used. The details exist in [5,6]. The circularly curved 2D surface illuminated by a directive feed antenna can also simulate the 2D reflector antenna system. In [7], an accurate simulation of this kind of 2D antenna system is performed. Although this solution is needed some special functions like legendre polynomials and special defined functions, it finally gives very accurate reference data.

On the other hand, an approximate solution is needed for this kind of 2D reflector antenna systems. A popular alternative for this is method of moments [9]. In this method, the integral equation is derived from boundary conditions and then it is discretized by using basis functions and tested by the same type of functions to reduce error in residue in the light of Galerkin's procedure. Finally, it is reduced to an algebraic matrix equation and solved numerically. This gives the approximate surface current density. So we used the method of moments like previously presented in literature but complex source point method is also combined by MoM. This provides us to obtain more realistic datas quickly. Furthermore, this kind of combination is not presented in literature before and we have a chance to check for these approximate results with the exact reference data given in [7].

II.FORMULATION

Our system consists of two parts, which are the metallic part and the slot part. The metallic part is in a shape of a circular arc of 2θ which is far from the center in the amount of "a". The directional point source has been placed in the approximate focus point. In this study, the feed antenna's radiation patterns also modelled by a Complex source point method which is explained in [8] in detail. If a line source located at the real position $\vec{r}_0$ can be converted to an imaginary position by adding a complex vector $i\vec{b}$ so a complex line source at $\vec{r}_s = \vec{r}_0 + i\vec{b}$ then radiates the following directive beam field.

$$\vec{E}_z(\vec{H}_z) = C.H_n^{(2)}\left(k_0\left|\vec{r} - \vec{r}_s\right|\right) \tag{1}$$

$$= C.\sum_{n=-\infty}^{\infty} J_n(k_0\vec{r}_s)H_n^{(2)}(k_0\vec{r})e^{in\varphi}e^{-in\theta_s}, r > \left|r_s\right| \tag{2}$$

where $r_s = \sqrt{r_0{}^2 + 2ir_0 b\cos\beta - b^2}$ (3)

and $\theta_s = \cos^{-1}(\frac{r_0 + ib\cos\beta}{r_s}), \mathrm{Re}(r_s) > 0$ (4)

The b, β, and θ_s parameters represent the complex source beamwidth and direction.

This complex source can be imagined as an aperture antenna having 2b aperture width. So imaginary region cannot be cross the pec reflector surface [7].

There are some requirements for MoM, the most basic one is the problem must satisfy the Helmholtz' Equation, and it must satisfy the Sommerfeld radiation conditions while it also have to satisfy the specified boundary conditions of the problem itself.

$$\vec{E}^{in} = \int_M \vec{J}^{sc}(\vec{r}')G(\vec{r},\vec{r}')dr' \quad (5)$$

$$\frac{d\vec{H}_z^{in}}{dn} = -\frac{d}{dn}\int_M J_\phi(\vec{r}')\frac{dG}{dn'}dr' \quad (6)$$

where

$$\vec{H}_z^{in} = H_0^{(2)}\left(\left|\vec{r} - \vec{r}_s\right|\right) = \sum_n J_n(k_0\vec{r}_s)H_n^{(2)}(k_0\vec{r})e^{-in\varphi_s}$$

These two equations are called as the electrical field integral equation and the magnetic field integral equation denoted as EFIE and HFIE, respectively.

Finally we end up with the algebraic matrix equation for both polarization in the AX=B form as follows:

For E Polarization:

$$[A_{mn}] = -\frac{ja\pi}{2}\sum_{p=-\infty}^{\infty} J_p(k_0a)H_p^{(2)}(k_0a)w_m^{p*}2\pi \quad (7)$$

and

$$[B_m] = \sum_{p=-\infty}^{\infty} w_m^{p*}J_p(k_0\vec{r}_s)H_p^{(2)}(k_0a) \quad (8)$$

Similarly, for H Polarization:

$$[A_{mn}] = -\frac{jka\pi}{2}\sum_{p=-\infty}^{\infty} w_m^p J_p'(k_0a)H_p^{(2)}{}'(k_0a)f_n^{p*} \quad (9)$$

and

$$[B_m] = \sum_{p=-\infty}^{\infty} w_m^{p*}J_p(k_0\vec{r}_s)H_p^{(2)}{}'(k_0a)$$

A remarkable difference of this formulation from the previous one, is that, we used triangle function as the basis and testing function, instead of using pulse function, since H-solution cannot converge as rapidly as E-solution. Although the results are not as accurate as it is in E-polarization, choosing triangle function enables us to obtain more accurate and convergent results for H-polarization.

III. NUMERICAL RESULTS

It has been seen that by applying not suitable functions, singularities observed in the values of the induced current on the system of interest. Therefore, testing functions has been choosen very carefully so that the inner products of these functions at the both left and right hand side of the main equation, not to be close to zero in order to prevent singularities in the value of current induced at the edges of the reflector. After several experiments, acceptable results achieved choosing pulse function for the E solution, and triangle function for the H solution of the system.

In Figure 1, the geometry of the reflector antenna system is shown.

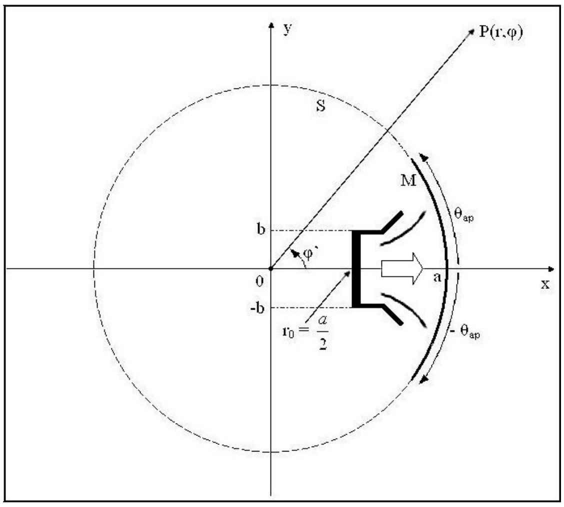

Figure 1. System Geometry

As it can be seen from the following two figures, there is so small differences between the exact and obtained results for the radiation patterns for both E and H Polarization, however, between 60 and 90 degrees, results differ from each other caused by the effect of the edge conditions illustrated by Figure 2, and Figure 3.

The difference between the exact solution and numerical solution becomes more apparent as it can be seen from the following figure which is for H-Polarization solution as it converges slower than the case of E-Polarization.

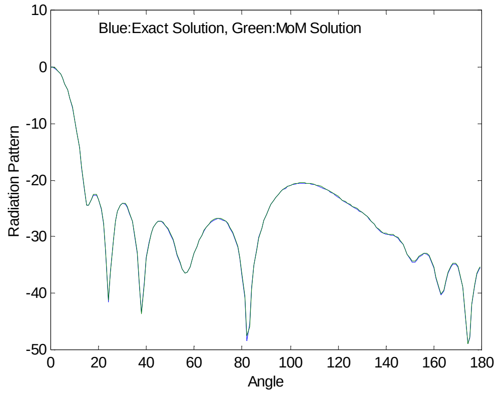

Figure 2. Radiation Pattern versus Angle for E-Polarization

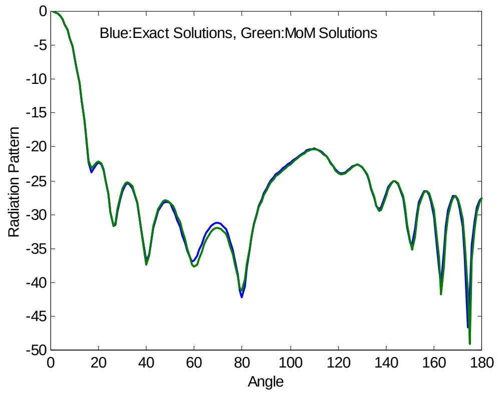

Figure 3. Radiation Pattern versus Angle for H-Polarization

One of the most important issue in MoM, which is the convergence, can also be seen in the results of the induced current compared with the exact solution for both E and H Polarization cases. These results are shown below with the following two figures.

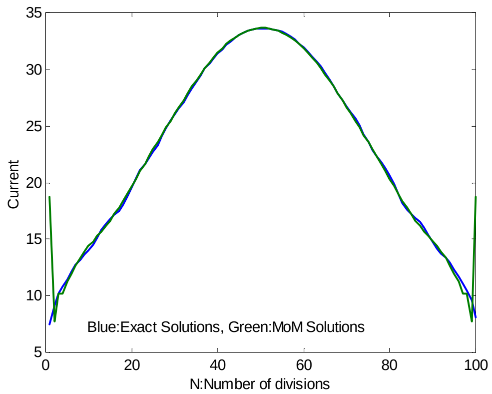

Figure 4. Current Solution for E Polarization

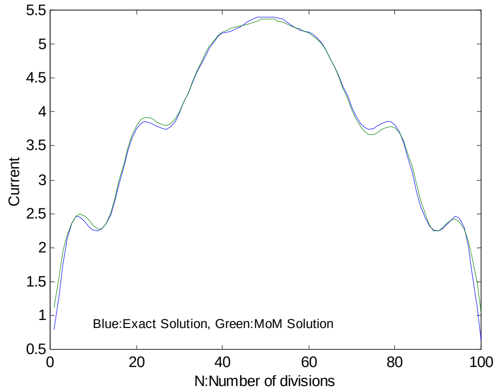

Figure 5. Current Solution for H Polarization

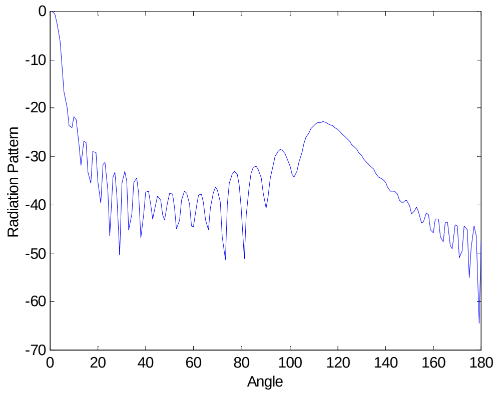

Figure 6. Radiation Pattern versus Angle for E-Polarization for N(number of divisions) = 150

We have solved the same problem for larger size, such as 150 divisions, and we saw that the amount of variation at the radiation pattern after 140 degrees, become higher which is caused by the effect of edge conditions. This is shown in Figure 6.

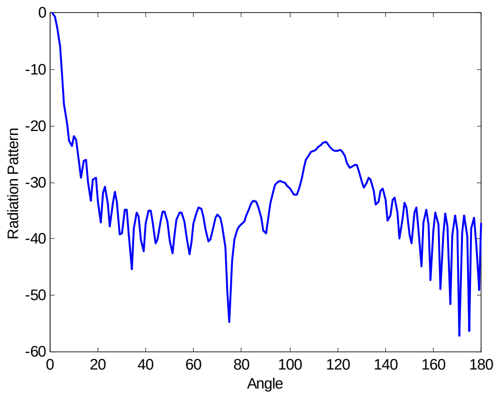

Figure 7. Radiation Pattern versus Angle for H-Polarization for N(number of divisions) = 150

This variation in H Solution exists much more than it does in E Solution which is again caused by the slower convergence of H Polarization case. In the following figure, this can be compared to the previous result.

Numerical results of the tangential total electric field components also ensures the boundary condition that the total tangential E field tends to be zero on the metallic part of the system as it can be seen clearly from the results as our system's boundary conditions are proven by the obtained values on the surface of the metallic part, namely the total tangential electric fields for both E and H polarizations vanish as the sum of the incident and scattered Electric fields tends to be zero on the specified part of the antenna system. Figures 5.8 and 5.9 are the numerical solutions that we have obtained by using the induced current that we have found by MoM.

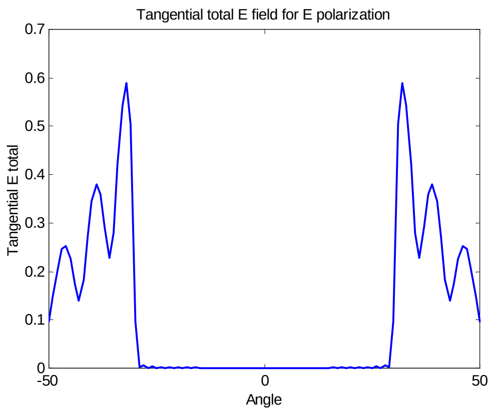

Figure 8. Tangential Total E field for E Polarization

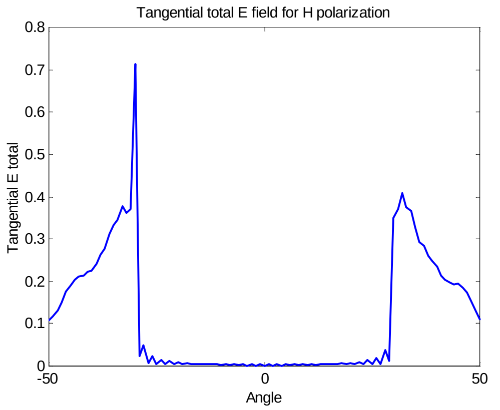

Figure 9. Tangential Total E field for H Polarization

IV.CONCLUSION

As it has been predicted before, current and radiation pattern values are converged rapidly in E field solution compared to the H field solution. As we used the method of moments as the numerical solution, easier matrix equation is solved instead of solving multi-level integral equations, as a result, numerically sufficient solutions are obtained by the use of Complex Source Dual Series approach.

REFERENCES

1. Jeffrey A. Beren, Diffraction of an H-Polarized Electromagnetic Wave by a Circular Cylinder with an Infinite Axial Slot, IEEE Transactions on Antennas and Propagation Magazine, vol.AP-31, no.3, May 1983
2. Alireza H.Mohammadian, Approximate solution for the scattering of an E-Polarized wave from concentrically loaded slit cylinder, Department of Electrical and Computer Engineering, University of Michigan-Dearborn, Dearborn, Michigan, 48128
3. J.R.Mautz and R.F.Harrington, Electromagnetic Penetration into a conducting circular cylinder through a narrow slot, TM case, Journal of Electromagnetic Waves and Applications, vol.2, no: 3/4, pp.269-293, 1988
4. J.R.Mautz and R.F.Harrington, Electromagnetic Penetration into a conducting circular cylinder through a narrow slot, TE case, Journal of Electromagnetic Waves and Applications, vol.3, no: 4, pp.307-336, 1989
5. M.Hashimoto, M. Idemen and O.A. Tretyakov, Alexander I.Nosich, Green's function-dual series approach in wave scattering by combined resonant scatterers in Analytical and Numerical Methods in Electromagnetic Wave Theory, Science House, 1963, pp 419-469, Tokyo
6. Alexander I. Nosich, The Method of Analytical Regularization in Wave Scattering and Eigenvalue Problems: Foundation and Review of solutions, IEEE Transactions on Antennas and Propagation Magazine, vol.41, no.3, June 1999
7. Taner Oguzer, Ayhan Altıntaş, and Alexander I. Nosich, Accurate Simulation of Reflector Antennas by the Complex Source-Dual Series Approach, IEEE Transactions on Antennas and Propagation Magazine, vol.43, no.8, pp.793-801, August 1995
8. Leopold B.Felsen, Complex Source Point Solutions of the field equations and their relation to the propagation and scattering of Gaussian Beams, Symp. Matemat. Istituto Nazionale di Alta Matematica,Academic,London, XXVIII: 40
9. R.F. Harrington, Field Computation by Moment Methods, Macmillan, New York, 1968