

# EVALUATION OF THE PERFORMANCE OF VARIOUS FUZZY PID CONTROLLER STRUCTURES ON BENCHMARK SYSTEMS

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## ABSTRACT

**There are various fuzzy PID controller structures found in literature. In this paper, performance and robustness analysis of these controllers are investigated. The PID coefficients and/or scaling factors are determined using genetic search algorithms. In conclusion, it has been tried to find out which controller structures are more effective and successful with regard to a certain performance index and uncertainties in system parameters on the various benchmark control systems.**

## I. INTRODUCTION

PID controller is well established in classical control systems and it is often used as a benchmark against the other types controllers [1]. Since the PID controllers are linear they are not usually suitable for strongly nonlinear systems. Fuzzy PID controllers are often mentioned as an alternative to classical PID controllers in such cases [2].

Fuzzy PID controllers in literature can be classified into three major categories as direct action type, fuzzy gain scheduling type, and hybrid type fuzzy PID controllers [3]. The direct action type can also be classified into three categories according to number of inputs as single input, double input, and triple input direct action fuzzy PID controllers. The classification of fuzzy PID controllers can be seen in Figure 1.

In this study, direct action type and hybrid type fuzzy PID controllers are considered [4]. The hybrid PID controller is constructed by the combination of a two input direct action fuzzy PID controller and a conventional PID controller.

The main purpose of this study is to find out which controller structures are more effective and successful with regard to a certain performance index and uncertainties in system parameters on the various benchmark control systems. A meaningful and

measurable performance index (PI) that considers many time-domain control criteria could be defined as follows:

$$PI = \frac{k}{C_1 m_p + C_2 t_p + C_3 t_s + C_4 e_{ss}} \quad (1)$$

where,  $m_p$  is the maximum percent overshoot,  $t_p$  is the peak time,  $t_s$  is the settling time, and  $e_{ss}$  is the steady state error. The coefficients,  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$  should be considered as scaling factors of the above mentioned performance criteria rather than weighting factors, since their main purpose is to bring all quantities to the same comparable level [5]. In this case, the parameters of the performance index are chosen as,  $k=1000$ ,  $C_1=100$ ,  $C_2=6$ ,  $C_3=3$ ,  $C_4=100$ . The PID coefficients and/or scaling factors are determined using genetic search algorithms. Integral square error (ISE) is also noted for the systems with optimum parameters found according to performance index given in Eq. (1).

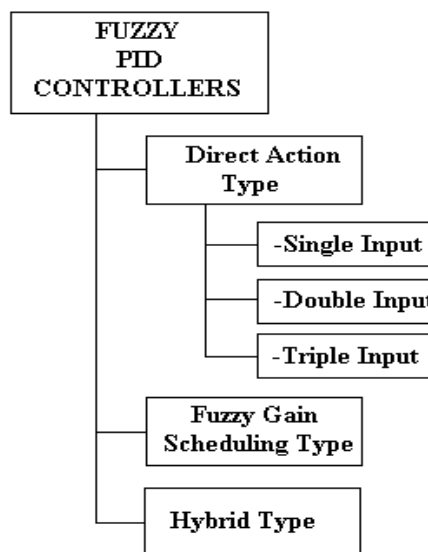


Figure 1. Classification of fuzzy PID controllers

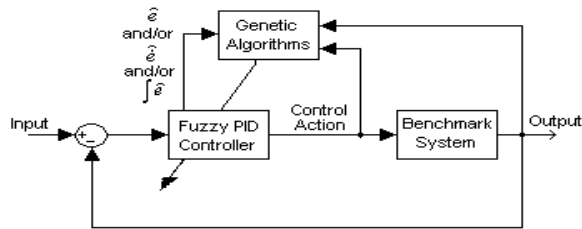


Figure 2. Block diagram of the process

The coefficients of classical PID and scaling factors of fuzzy PID are chosen in interval [0 5] and the control action is kept in interval [-10 10]. Block diagram of the whole process can be seen in Figure 2.

## II. FUZZY PID CONTROLLERS

### DIRECT ACTION FUZZY PID CONTROLLERS

#### i) Single input fuzzy PID controllers (FPID-SI)

This structure uses error as the only input and has a one-dimensional rule-base. As it is seen in Figure 3 [6], it is simply a nonlinear mapping of error into fuzzy proportional action cascaded to a conventional PID controller [3].

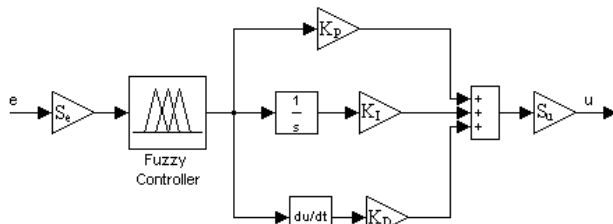


Figure 3. Single input fuzzy PID controller

#### ii) Double input fuzzy PID controllers (FPID-DI)

Double input fuzzy PID controller could be obtained by the combination of fuzzy PD and fuzzy PI [7] controllers. A fuzzy PD controller PI controller that has two-dimensional rule-base is shown in Figure 4(a)-(b), respectively.

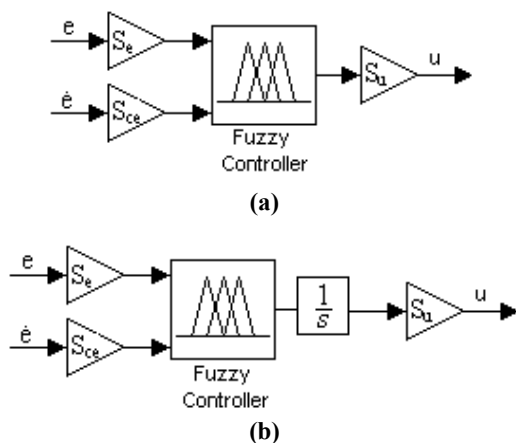


Figure 4. (a) Fuzzy PD Controller (b) Fuzzy PI Controller

Fuzzy PD and PI actions can be combined to form a fuzzy PID [8-12]. If the rule-bases of fuzzy PD and fuzzy PI are defined as the same, these two structures can be combined to form a fuzzy PID controller as shown in Figure 5.

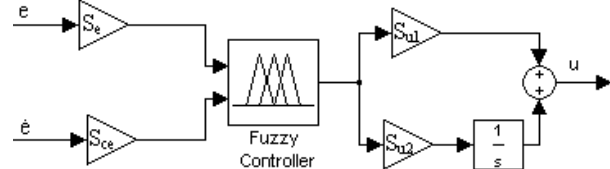


Figure 5. Two input fuzzy PID controller

#### iii) Triple input fuzzy PID controllers (FPID-TI)

Fuzzy PID controllers can also be defined with inputs error, change of error, and sum of error [13]. A fuzzy PID controller with three inputs and has one three-dimensional rule-base is shown in Figure 6.

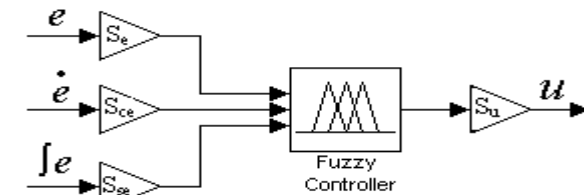


Figure 6. Fuzzy PID controller with one three-input rule-base (FPID-TI(Type1))

Also another fuzzy PID controller with three inputs, but with three one-dimensional rule-bases is shown in Figure 7.

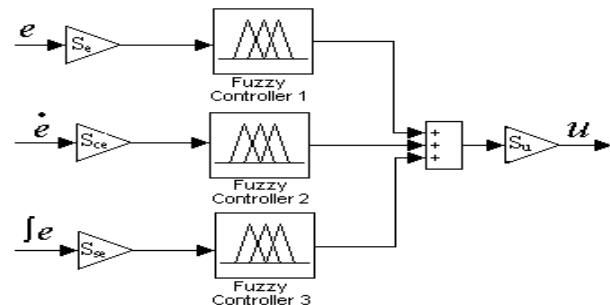


Figure 7. Fuzzy PID controller with three one-input rule-bases (FPID-TI (Type2))

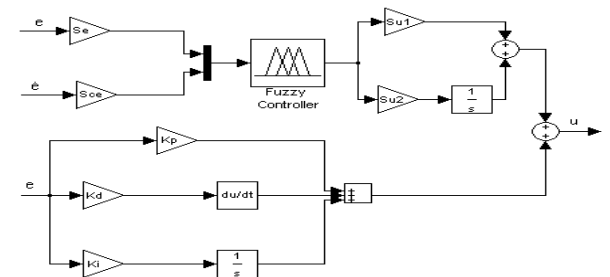


Figure 8. Hybrid PID controller

### HYBRID PID CONTROLLER

The hybrid PID controller is constructed by the combination of a two-input direct action fuzzy PID controller and a conventional PID controller. This structure is shown in Figure 8.

### III. BENCHMARK SYSTEMS

Various benchmark systems for analysis of classical PID control are presented in [14]. We have considered some of these benchmark systems for performance and robustness analysis of various fuzzy PID controller structures given in the previous section. The benchmark systems considered here are as follows:

i) *Multiple equal poles:*

$$G(s) = \frac{1}{(s+1)^n} \quad n=2, 4, 8 \quad (2)$$

These systems are very common. For  $n=1$  and 2 anything can be achieved by PI or PID control respectively. For large values of  $n$  the system behaves like systems with long dead times.

ii) *Time delay and lag:*

$$G(s) = \frac{1}{1+sT} e^{-s} \quad T=0.5 \quad (3)$$

This is the classical system that has been used in many investigations of PID control. The system reduces to a pure time delay for  $T=0$  and represents lag dominated systems for large  $T$ . Many of the early tuning rules are based on this model. A drawback with the model is that it has slow roll-off at high frequencies.

iii) *Time delay and double lag:*

$$G(s) = \frac{1}{(1+Ts)^2} e^{-s} \quad T=0.5 \quad (4)$$

This system is similar (3) but it has more high frequency roll-off. The system reduces to a pure time delay for  $T=0$ .

iv) *Oscillatory system:*

$$G(s) = \frac{\omega_0^2}{(s+1)(s^2 + 2\xi\omega_0s + \omega_0^2)} \quad \begin{matrix} \xi = 0.1 \\ \omega_0 = 2 \end{matrix} \quad (5)$$

Systems of this type with small damping  $\xi$  are not good candidates for PID control. The system is easy to control if  $\omega_0$  is large. The performance can often be improved drastically by more general controller structures.

v) *Unstable pole:*

$$G(s) = \frac{1}{s^2 - 1} \quad (7)$$

This is a simple model of an inverted pendulum. An unstable batch reactor is an example from industry. Notice that particular care must be taken with saturating actuators in this case.

### IV. PERFORMANCE AND ROBUSTNESS ANALYSIS

Rule-bases of the investigated fuzzy controllers are generated in MATLAB Fuzzy Logic Toolbox. Input and output membership functions for each type of controllers

are defined uniformly distributed in interval  $[-1 \ 1]$ . Five triangle fuzzy sets are defined for each input of every rule-base. In the outputs, 5 triangle fuzzy sets are defined for one-dimensional rule-base, 9 triangle fuzzy sets rule bases are defined for two-dimensional rule-base, and 13 triangle fuzzy sets are defined for three-dimensional rule-base. The number of rules is chosen to be 5, 25 and 125 in one-, two- and three-dimensional rule-bases, respectively. Analyses are performed on the simulation of models in MATLAB/Simulink and the performance indices are calculated by taking samples from the control variables at each 0.1 seconds during 50 seconds of each process.

The performance index values of performance and robustness analysis determined using PID coefficients and/or scaling factors found using genetic algorithms for each controller structure on the benchmark systems are given in Tables 1-7.

In Table 1, best performance is achieved by FPID-SI both in PI and ISE. FPID-SI is also the most robust controller structure. In general, while all structures have satisfactory performance it is seen that FPID-TI (Type1), FPID-TI (Type2), and classic PID are not so robust.

In this case, classical PID achieved the best performance in Table 2. However, hybrid PID has the smallest ISE value. Moreover, hybrid PID is the most robust controller structure for this system.

In Table 3, It is obvious that FPID-TI(Type1) achieved the best performance and FPID-SI is the most robust controller structure in every situation for this case.

As it can be seen from the Table 4, FPID-TI(Type1) has the best performance, however, the smallest ISE is achieved by FPID-TI(Type2). FPID-TI(Type1) is also the most robust controller structure.

It can be deduced from Table 5 that FPID-SI achieves the best performance and it is closely followed by FPID-DI while the smallest ISE is achieved by FPID-SI and FPID-TI(Type1). FPID-DI(Type2) is the most robust structure. Classical and hybrid PID controllers exhibit poor performance and robustness for this system.

In this case, FPID-TI(Type1) and FPID-SI are the best structures in performance in Table 6. However, FPID-SI, hybrid PID, and classic PID have the smallest ISE value. It is possible to say that all controllers except FPID-DI and FPI are robust for this system.

As it can be seen from the Table 7, FPID-DI and hybrid PID achieve the best performance. However, FPID-SI has the smallest ISE value, also FPID-DI is the most robust structure for this system.

Table 1. (a) Performance analysis (b) Pole and gain robustness analysis on  $1/(s+1)^2$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	9.9759	8.5970	8.3681	9.0496	9.2492	8.3725	9.4287
	ISE	0.34	0.7998	0.9145	0.6991	0.4956	1.183	0.503

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$1/(s+1)^2$	9.9759	8.5970	8.3681	9.0496	9.2492	8.3725	9.4287
	$1/(s+1.1)^2$	10.049	7.1364	5.3620	4.3025	9.3413	8.4787	5.2675
	$1/(s+0.9)^2$	9.7998	7.9507	8.2444	8.9546	9.2000	7.7996	9.1196
	$1.25/(s+1)^2$	10.023	8.3352	8.7158	9.2222	9.4149	7.2410	9.3245
	$0.75/(s+1)^2$	9.9173	8.2561	5.1172	4.2261	9.0188	8.0788	6.0549

Table 2 (a) Performance analysis (b) Pole and gain robustness analysis on  $1/(s+1)^4$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	7.2736	7.1193	6.9967	6.5093	6.9672	5.7729	7.6137
	ISE	2.28	2.445	2.428	2.774	1.84	4.23	1.955

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$1/(s+1)^4$	7.2736	7.1193	6.9967	6.5093	6.9672	5.7729	7.6137
	$1/(s+1.1)^4$	3.7734	3.9375	2.7690	4.3042	4.1577	3.7436	4.1498
	$1/(s+0.9)^4$	5.6587	5.6241	5.6392	5.2411	5.0777	3.2469	5.8218
	$1.25/(s+1)^4$	6.5985	6.4419	6.6814	5.3741	6.5760	4.8768	6.5265
	$0.75/(s+1)^4$	4.3733	4.7620	3.3125	4.5100	6.9911	4.0285	3.8149

Table 3. (a) Performance analysis (b) Pole and gain robustness analysis on  $1/(s+1)^8$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	3.5748	4.3150	4.9296	4.0091	3.8673	3.3851	3.8796
	ISE	8.677	7.365	6.428	8.379	6.161	10.39	6.482

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$1/(s+1)^8$	3.5748	4.3150	4.9296	4.0091	3.8673	3.3851	3.8796
	$1/(s+1.1)^8$	1.8619	1.9840	-	-	2.8129	1.9259	3.5042
	$1/(s+0.9)^8$	2.4706	-	2.4561	2.3917	-	-	-
	$1.25/(s+1)^8$	3.3580	3.4977	3.7287	2.9946	3.0061	2.7612	3.1881
	$0.75/(s+1)^8$	2.4295	2.8699	-	1.9648	3.3245	2.7305	3.7919

Table 4. (a) Performance analysis (b) Time delay robustness analysis on  $e^{-s}/(0.5s+1)$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	8.3435	8.1636	8.5149	7.7488	6.3154	6.8711	8.2072
	ISE	1.502	1.518	1.317	1.309	1.676	2.183	1.341

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$e^{-s}/(0.5s+1)$	8.3435	8.1636	8.5149	7.7488	6.3154	6.8711	8.2072
	$e^{-1.5s}/(0.5s+1)$	5.8292	5.6474	5.9301	5.6387	4.8005	5.5457	5.8513

Table 5. (a) Performance a (b) Time delay robustness analysis on  $e^{-s}/(0.5s+1)^2$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	8.2300	8.1604	7.5943	7.7417	5.1431	7.0041	5.9814
	ISE	1.664	1.8	1.699	1.771	1.942	2.612	1.819

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$e^{-s}/(0.5s+1)^2$	8.2300	8.1604	7.5943	7.7417	5.1431	7.0041	5.9814
	$e^{-1.5s}/(0.5s+1)^2$	5.6606	4.9715	4.7734	6.4006	3.4783	5.0798	4.1249

Table 6. (a) Performance (b) Pole robustness analysis on  $4/(s+1)(s^2+0.4s+4)$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	6.9909	5.3663	7.0574	6.8707	6.6522	5.2280	6.5583
	ISE	1.04	2.265	1.218	1.489	1.068	2.669	1.042

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$4/(s+1)(s^2+0.4s+4)$	6.9909	5.3663	7.0574	6.8707	6.6522	5.2280	6.5583
	$4/(s+1)(s^2+0.2s+4)$	6.6760	-	5.5786	5.7607	6.1929	-	5.8365
	$4/(s+1)(s^2+0.4s+2)$	5.9707	-	6.5485	6.1320	5.8220	-	6.0997
	$4/(s+1)(s^2+0.2s+2)$	5.3301	-	4.6584	5.9951	5.6779	-	5.3283

Table 7. (a) Performance analysis (b) Pole robustness analysis on  $1/(s^2-1)$

(a)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	PI	7.7897	8.8827	-	5.4373	8.8482	-	6.0855
	ISE	0.3637	0.8282	-	1.366	0.682	-	0.8037

(b)		FPID-SI	FPID-DI	FPID-TI (Type1)	FPID-TI (Type2)	Hybrid PID	FPI	Classical PID
	$1/(s^2-1)$	7.7897	8.8827	-	5.4373	8.8482	-	6.0855
	$1/(s^2-0.5)$	8.3034	6.3230	-	6.0108	5.4618	-	6.9010
$1/(s^2-1.5)$	6.8356	6.2847	-	3.4535	5.7245	-	4.6435	

## V. CONCLUSION

After the analysis performed on various benchmark systems, no supreme controller structure for every system and condition is determined. Thus, it is better to choose a controller according to the type of the system to be controlled and the types of uncertainties in system parameters. Besides, it is also observed that as uncertainty and nonlinearity increases some fuzzy type PID controllers can achieve fine performance while the performance of the systems with the classic PID controller degrades and fuzzy PID controllers are more robust while classical PID controllers are not. However, in general, it is could be deduced that FPID-SI can be a 'good' choice for controller structure with regard to its performance and robustness issues in most of the benchmark systems.

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