Analysis of Chaotic Dynamics of Chua's Circuit with lncosh Nonlinearity

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Abstract

Chua's circuit, which demonstrates one of the most complicated nonlinear dynamical behaviors, i.e. chaos, contains a three-segment Piecewise Affine (PWA) resistor as the unique nonlinear element. In this study, the non-smooth nonlinearity of Chua's circuit represented by absolute value is approximated with employing the (smooth) lncosh nonlinearity. In contrast to the other smooth approximation, the $\frac{1}{\lambda}lncosh(\lambda x)$ approximation has the property of yielding the absolute value nonlinearity $|x|$ as the limit case when λ parameter goes to infinity. The bifurcation maps and attractors of introduced Chua's circuit obtained for different λ parameters are presented in the paper in a comparative way. Computer simulations show that lncosh approximation preserves the chaotic behavior and hence provides the possibility of analyzing the behavior of the Chua's circuit by the methods requiring smoothness.

1. Introduction

Chua's circuit is a simple electronic circuit as shown in Fig.1 that exhibits complex nonlinear dynamics such as chaos. The circuit has a nonlinear (piecewise affine) resistor which results a piecewise linear system in the state space form of

$$\begin{aligned} \dot{V}_{c1} &= C_1^{-1}[R^{-1}(V_{c2}-V_{c1})-\sigma(V_{c1})] \\ \dot{V}_{c2} &= C_2^{-1}[R^{-1}(V_{c2}-V_{c1})+I_L] \\ \dot{I}_L &= -L^{-1}V_{c2}-r_0 I_L \end{aligned} \quad (1)$$

where $\sigma(V_{c1}) = G_b V_{c1} + \frac{1}{2}(G_a - G_b)(|V_{c1}+E| - |V_{c1}-E|)$ is a PWA function describing electrical response of the nonlinear resistor. Since the PWA function which represents the characteristic of the nonlinear resistor is non-smooth and non-differentiable, it has been substituted by various nonlinear smooth functions, e.g. cubic polynomial [1,2], cubic-like functions [3,4,5], sigmoid and signum functions [6], etc. Bifurcation analysis and different attractors of standard Chua's circuit and Chua's circuit with these nonlinearities has been presented extensively in literature [7].

In this study, a nonlinear function is proposed to substitute this PWA function providing smoothness and also differentiability. By adjusting a parameter, the proposed smooth nonlinearity has the property to approximate with a sufficiently small error to the PWA function in standard Chua's circuit. In this regard, for different values of the control parameter, the bifurcation maps and different attractors of Chua's circuit with the smooth nonlinearity are presented.

The organization of the sections of this study is as follows. Section 2 describes the smooth nonlinear function and the emergence of the new mathematical model for Chua's circuit exploiting this nonlinear function. In Section 3, the bifurcation maps and attractors of Chua's circuit with the smooth nonlinearity are presented in order to show the behavior of this circuit. Finally, the main conclusions of the paper are summarized in Section 4.

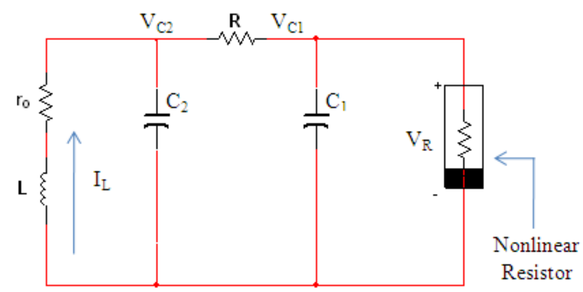

Fig. 1. Chua's circuit

2. Chua's circuit with a smooth nonlinearity

The system in (1) can be written in the dimensionless form [7] as

$$\begin{aligned} \dot{x}_1 &= \alpha[x_2 - x_1 - \sigma(x_1)] \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 - \gamma x_3 \end{aligned} \quad (2)$$

where the PWA function is

$$\sigma(x_1) = m_1 x_1 + \frac{1}{2}(m_0 - m_1)[|x_1+1| - |x_1-1|] \quad (3)$$

The non-smoothness of function $\sigma(.)$ is caused by absolute value functions which can be approximated by $|x_1 - 1| \approx \frac{1}{\lambda}lncosh[\lambda(x_1-1)]$ and $|x_1+1| \approx \frac{1}{\lambda}lncosh[\lambda(x_1+1)]$ with adjusting λ. The approximation becomes an exact representation when $\lambda \to \infty$. For sufficiently large λ it is obvious

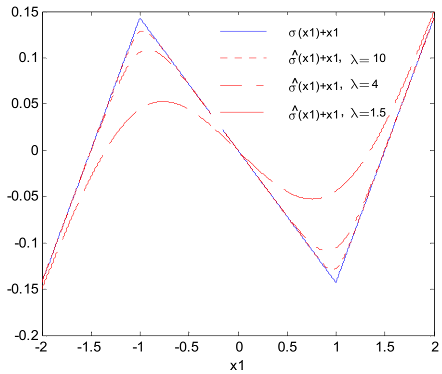

Fig. 2. PWA function and its approximations with different λ values

that the function $\sigma(.)$ can be approximated with sufficiently small error as shown in Fig. 2 and the mathematical model of Chua's circuit with smooth approximation becomes

$$\begin{aligned}\dot{x}_1 &= \alpha[x_2 - x_1 - \hat{\sigma}(x_1)] \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 - \gamma x_3\end{aligned} \quad (4)$$

where the smooth approximated nonlinearity is

$$\hat{\sigma}(x_1) = m_1 x_1 + \frac{1}{2}(m_0 - m_1)\left[\frac{1}{\lambda} lncosh[\lambda(x_1 + 1)] - \frac{1}{\lambda} lncosh[\lambda(x_1 - 1)]\right]. \quad (5)$$

3. Bifurcation maps and attractors of Chua's circuit with the smooth nonlinearity

The Chua's circuit shown in Fig. 1, using the proposed smooth nonlinearity is simulated with the parameters fixed to $\alpha = 9$, $\beta = 100/7$, $\gamma = 0.016$, $m_0 = -8/7$ and $m_1 = -5/7$ with regard to previous works [8].

The effect of the change of parameter λ on the behavior of the circuit with proposed smooth nonlinearity can be observed with the bifurcation maps in Fig. 3. The bifurcation maps are generated from dimensionless equation and show the period doublings as the forking which yields period-2 and period-4 etc.

Fig. 4 shows a simulated period–1, period-2, period-4, period-8 limit cycles as well as single scroll and double scroll attractors with the bifurcation parameters λ =1.5, λ =1.66, λ =1.91, λ =1.95, λ =2.88, λ =4, respectively achieved from the Chua's circuit with smooth nonlinearity. In order to eliminate transient response, the simulation results between 50s and 200s are presented in Fig. 4.

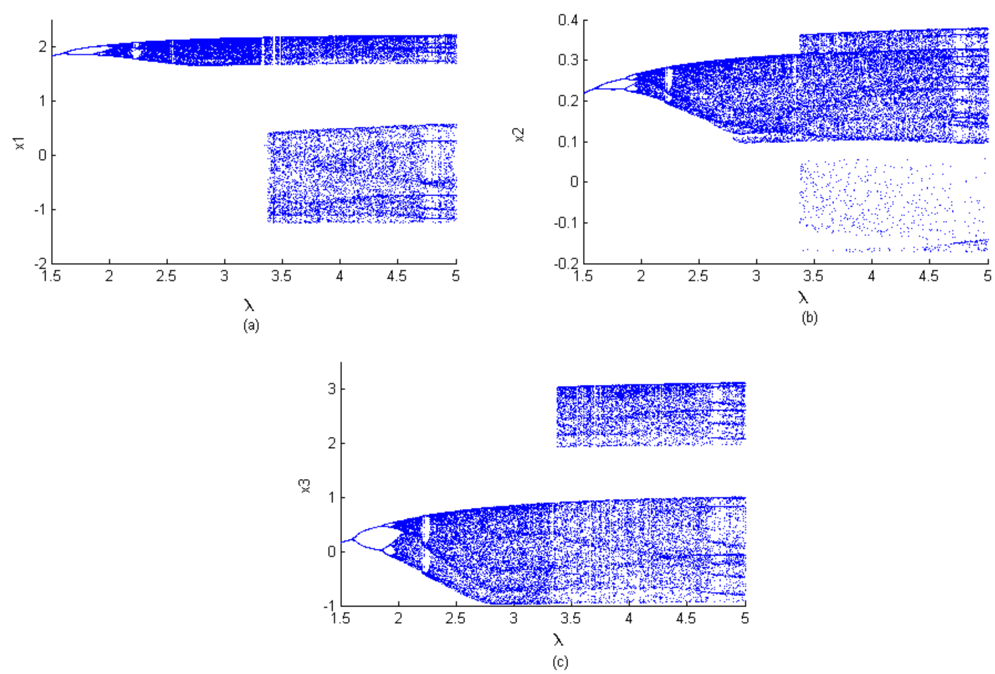

Fig. 3. Bifurcation maps for (a) λ vs. x1 (b) λ vs. x2 (c) λ vs. x3

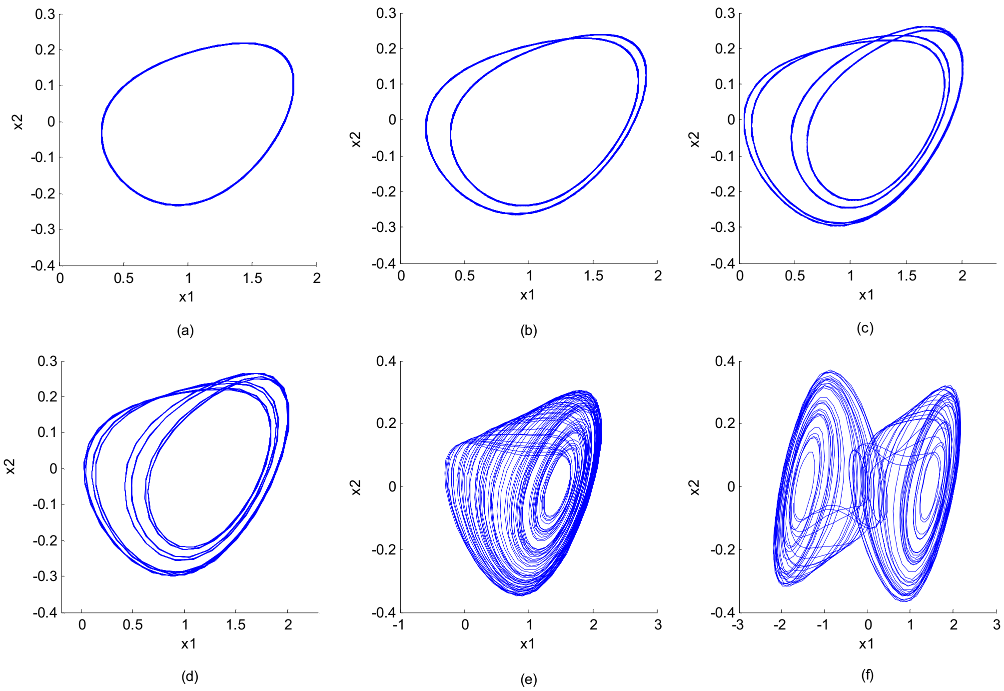

Fig. 4. Bifurcation sequence with respect to parameter (a) λ =1.5, period-1 limit cycle (b) λ =1.66, period-2 limit cycle (c) λ =1.91, period-4 limit cycle (d) λ =1.95, period-8 limit cycle (e) λ =2.88, single scroll Chua's attractor (f) λ =4, double scroll Chua's attractor

4. Conclusions

Chua's circuit contains a three-segment Piecewise Affine (PWA) resistor and demonstrates a wide variety of nonlinear behavior one of which is chaos. In this study, the non-smooth nonlinearity of Chua's circuit represented by absolute value is approximated with employing the (smooth) lncosh nonlinearity. Both the bifurcation maps and attractors of introduced Chua's circuit are presented for different λ parameters. Computer simulations show that lncosh approximation, when an appropriate parameter λ is chosen, preserves the chaotic behavior and provides the possibility of analyzing the behavior of the Chua's circuit by the methods requiring smoothness.

As a future work, the work may be extended by implementing the nonlinear resistor based on lncosh nonlinearity by using opamps and diode elements. Considering the diode V-I characteristic is defined by exponential nonlinearity as $I = I_o(e^{V/V_T} - 1)$, ln nonlinearity can be realized by using opamp-diode circuit where the diode takes place at the feedback loop of the opamp. Cosh nonlinearity consisting of exponentials can also be realized by opamp-diode circuit such that the diode takes place at the feed-forward part of the opamp connecting the input to the (negative) terminal of the opamp.

The work may also be extended by comparing the lncosh nonlinearity to the cubic and other nonlinearities available in the literature defining the nonlinear resistor, in terms of some performances such as robustness and richness of the chaotic behavior.

5. References

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