ANALOG MODELS FOR ANALYSIS AND DESIGN OF SOME SWITCH CAPACITOR CIRCUITS

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Abstract
Unitary and systematical analog models for analysis and design of some switched capacitor (SC) circuits are presented. These models permit a direct connection to s domain and a simple determination of the SC circuit function. They also permit the reconfiguration of a RC-OA circuit, so that it should be implemented in the SC mode. The proposed analog models adapted for the SC circuits avoid the confusions introduced by usual analog models used in literature in the same purpose. We analyzed and designed some circuits taken from literature as example and proved the simplicity and efficiency of this method.

1. Introduction

A series of SCC could be designed starting from the continuous RC-AO models [1]. In this case the resistors or some functional blocks (integrators, amplifiers) are replaced by their SC simulated equivalents. Using classical analog models, as it is presently described in literature, does not fully reflect the particularities due to SC and sometimes it can lead to confusion. Besides that, these analogical models do not include quantitative aspects [1]. In this paper we made some specifications regarding the equivalence functionality of the SC circuits with continuous circuits and proposed some specific SC operating mode analog models.

For the resistors (chapter 2) we introduced the term "inverting" or "non-inverting", which avoid the confusion with the negative resistor introduced as (-R) of some non-inverting resistors [1]. We have emphasized four types of resistor connections with usual SC and the dependence of their transfer function on the sampling moment.

The integrators (chapter 3) are seen as fundamental functional blocks, unitary designed, and the integration operator (λ) introduced in this paper, makes connection directly with the plan s. It also includes information regarding the way of the SC implementation of this fundamental block (tab. 1) and the accuracy of the SC circuit behavior correspondent with analog circuit. [3],[4]. The FDT $H(\lambda)$ description of the SC circuit under a unitary form allows the direct design of this one starting from the desired performances showed in the frequency characteristic or FDT H(s) parameters.

We analyzed (chapter 4) some linear and non-linear circuits stressing the fact that the introduced models allow a direct inspection of SCC and these ones are more thorough and more suggestive that the classic functional models. Besides that, the method of analysis, allows also quantitative parameter or FDT in z determinations.

In the fifth chapter we have shown an example of redesigning with SC an analog continuous biquad filter. Its description through SC analog models permitted the simple redesigning and resizing the circuit, obtaining two known in the specialized literature circuits [1].

The qualitative and quantitative correspondence of the SCC with their analog equivalent is valid only for certain frequency ranges, depending of the approximations introduced by s → z transformation reflected in the integrator cofigurations [4],[3] (tab. 1).

2. Functional models of some resistors with SC

Although there are a lot of kinds of SC resistor configurations, in purpose of reducing the parasitic effects, the resistors indicated in their general form [2] in the fig 1.a. are preferred The pin **in** is connected to the input signal. The other three pins have a zero potential. The transfer of the electrical charge q_1 is realized at the pin **out**. Supposing that we have a two-phases system, the electrical charge q_{out} is taken from the input by phase a $(a=\{1,2\})$. If $c=a$, $d=b$, this one is transmitted to the output on the same phase as it had been taken. In this case there won't be a delay between the current given by the input signal source and the output current. The resistor will be a "non-inverting" one. If the input signal taken on the phase a appears at output on phase b $(c=b,\ d=a)$, then there will be a change of current direction. In this case the resistor is acting like an "inverting" for the input "a". We introduced the term "inverting" and "non-inverting" to avoid the confusion with the negative resistor effect, due of some notations $\pm R$ used in literature to indicate this effect [1].

As a result we introduced the symbol of the inverter in the functional model of this kind of resistors. It actually indicates the role of the switch to place at the pins of the simulated resistor the input voltage with the reverse polarity (fig1.d,e).

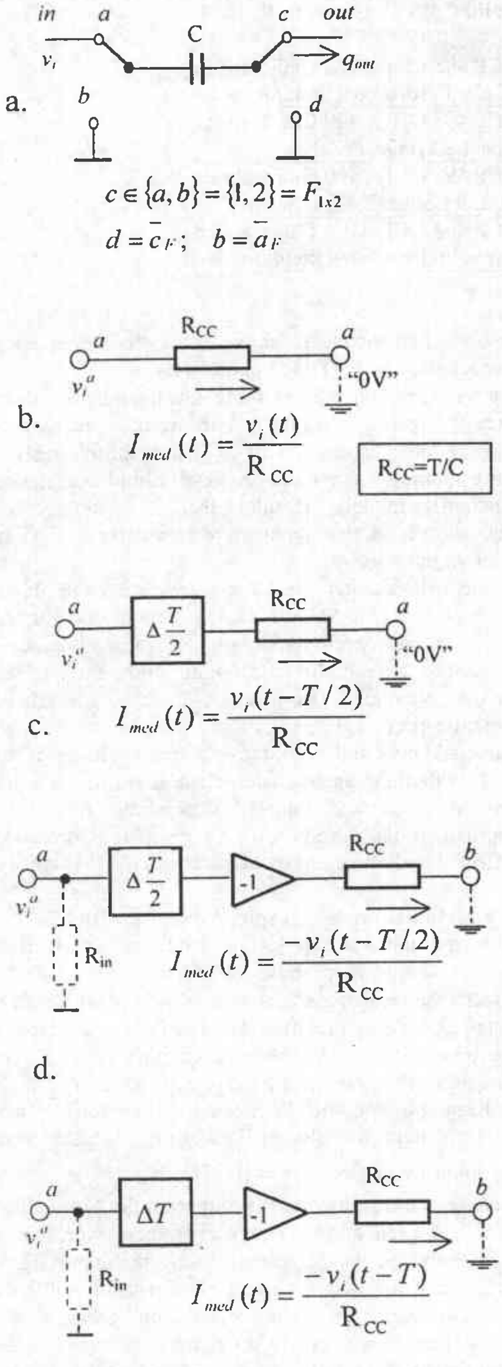


fig.1

Depending on the signal sample-and-hold mode: on phase a (v_i^a) or on phase b (v_i^b) delays between the taken and furnished signal may appear. There are shown in the models from fig.1. through delaying elements with T/2 or T (T means the sampling period). A rigorous model should also emphasize the input resistance of the circuit which is not ∞ as the symbols of the equivalent delaying elements may suggest (fig 1. d, e – dotted line).

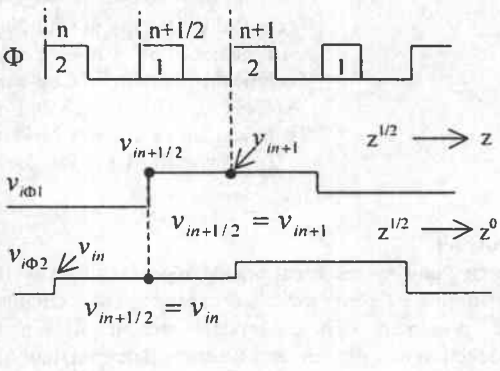


fig.2

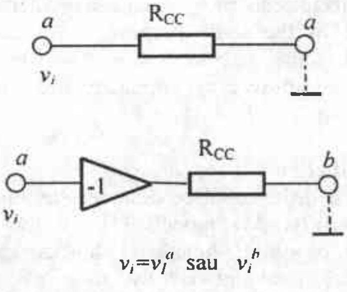


fig.3

The analysis of the sampling moment influence of the input signal shows that (fig.2), if the signal is considered constant for each period T. The delay T/2 in the transfer function could be equivalent with 0 or T, reducing the number of the possible cases. For a simple and better use of the equivalent symbols we draw the resistor without the delaying element and the input resistor (fig.3). The delaying effects appear in the equivalent fundamental macro model expressions of the circuits containing these resistors. We considered as example the integrator as the fundamental block.

3. The unitary modeling of some integrators with SC

The integrators are fundamental functional blocks in a series of linear circuits with SC (RC-AO). There are a lot of kinds of SC integrators which simulate, more or less precisely continuous integrators. The first order integrators the most used, due to their simple configuration [1], [4]. The integrator configurations are chosen regarding the expected precision and they depend of the variable transformation from s plane to z plane [4].

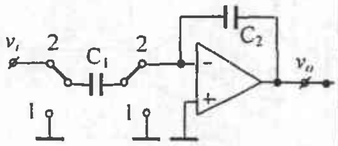
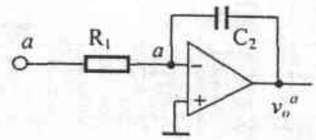
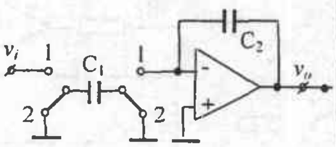
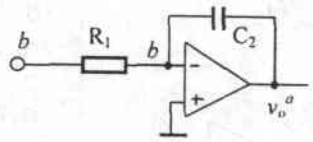
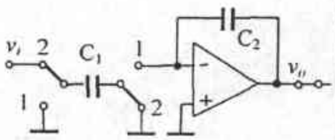
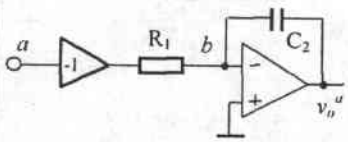
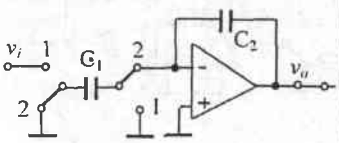
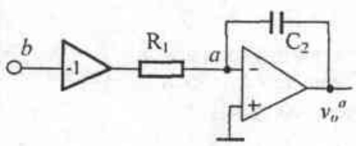
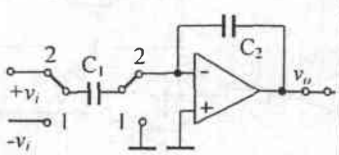
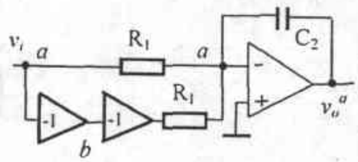
Tip	Integrator	Model	F (dt)
A			$H(z) = -\frac{C_1}{C_2} \cdot \frac{z}{z-1}$ $\lambda_a = \lambda_b = (z-1)/z$
B			$H(z) = -\frac{C_1}{C_2} \cdot \frac{z^{1/2}}{z-1}$ $\lambda_a = z-1$ $\lambda_b = (z-1)/z$
C			$H(z) = \frac{C_1}{C_2} \cdot \frac{1}{z-1}$ $\lambda_a = \lambda_b = (z-1)$
D			$H(z) = \frac{C_1}{C_2} \cdot \frac{z^{1/2}}{z-1}$ $\lambda_a = z-1$ $\lambda_b = (z-1)/z$
E			$H(z) = -\frac{C_1}{C_2} \cdot \frac{z+z^{1/2}}{z-1}$ $\lambda_a = (z-1)/(z+1)$ $\lambda_b = (z-1)/2z$ $s = 2\lambda/T$

Table 1

Analyzing some of the integrators frequently used in realizing SCC (Table 1) it can be noticed that their transfer function has the expression:

$$H(z) = \frac{C_2}{C_1} \cdot \frac{1}{\lambda(z)} = \frac{1}{\tau s}\bigg|_{s=\lambda/T} \qquad (1)$$

where $\lambda(z)/T$ is quite the variable transformation used in approximate determination of TF in z starting from TF in s. The integrating constant τ is given by C_1T/C_2. The generalized transfer function (TF):

$$H(\lambda) = \frac{1}{\tau} \cdot \frac{T}{\lambda} \qquad (2)$$

permits a direct correspondence with plan s, and so, with the continuous integrator based circuits, without being influenced by the implementing way of the integrators.

4. Analysis of some SCC based on continuous models

We exemplify the unitary models utility obtained before, analyzing some circuits [1].

a) Analysis of a comparator (fig. 4,a)

The equivalent model from the fig 4.b. indicates a positive reaction, realized at the AO inverter input, using an inverting resistor. The signal v_i is applied on a non-inverting I/O path and the voltage V, on an inverting path.

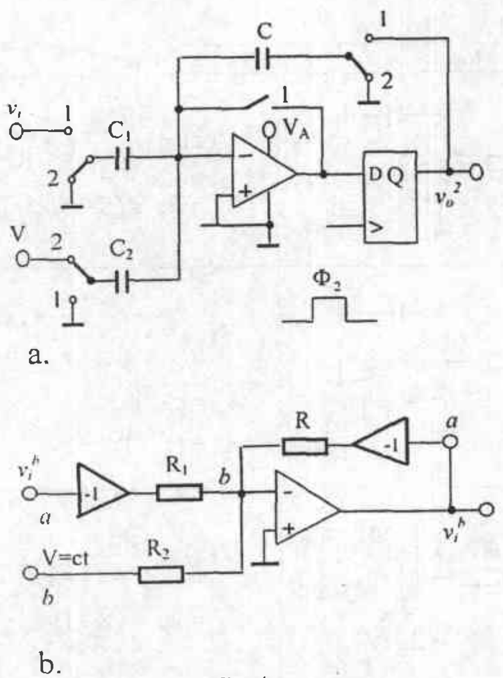


fig.4

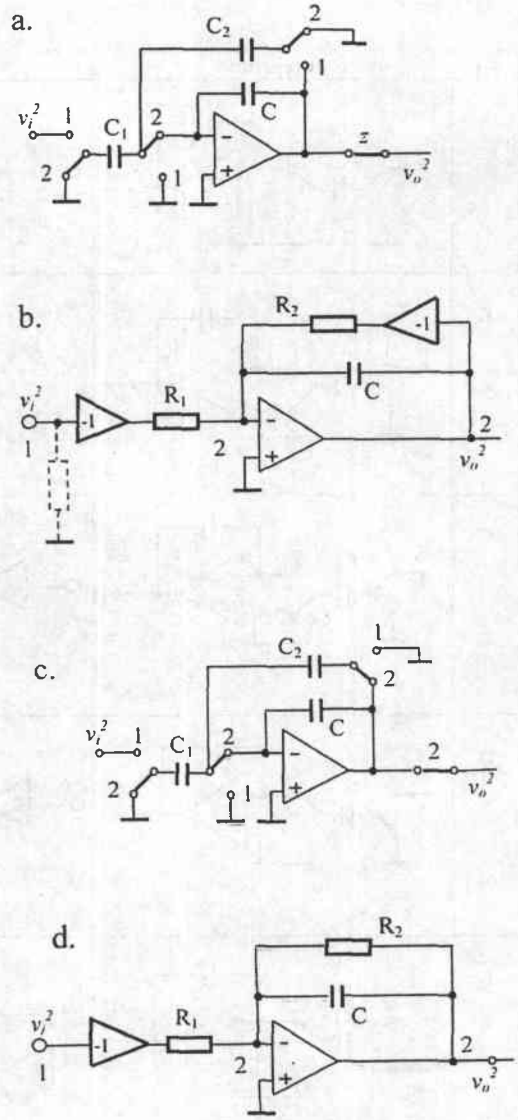


fig.5

Having a positive reaction, the circuit is a comparator. Classical analysis of the circuit from fig. 4.b. leads to the expression of the hold voltages. voltages.

$$V_{PH} = V \cdot \frac{R_1}{R_2} + |V_{OL}| \cdot \frac{R_1}{R}; \qquad (3)$$

$$V_{PL} = V \cdot \frac{R_1}{R_2} - V_{OH} \cdot \frac{R_1}{R}. \qquad (4)$$

Identifying the equivalent continuous model with the scheme's elements with SC, we have

$$V_{PH} = V \cdot \frac{C_2}{C_1}; \quad V_{OL} = 0; \qquad (5)$$

$$V_{PL} = V \cdot \frac{C_2}{C_1} - V_{OH} \cdot \frac{C}{C_1}. \qquad (6)$$

which have the same results as direct analysis at the level of capacitor-switch of SCC.

Starting from models from fig.1, we can see that the signal v_i have to be sampled in phase 2, in order to get a good phase correspondence.

b) The analysis of a loss integrator

It's obvious that the scheme from fig. 5.a having the continuous model b is unstable, while the scheme from fig.c equivalent with d. has a stable function.

The scheme from figure 5,d is described by the following relations:

$$v_0 = \frac{1}{R_1 \cdot C \cdot s} \cdot v_i - \frac{1}{R_2 \cdot C \cdot s} \cdot v_0 \qquad (7)$$

and its discrete equivalent, using the integrators from table 1, is:

$$v_0 = \frac{C_1}{C \cdot \lambda_1} \cdot v_i - \frac{C_2}{C \cdot \lambda_2} \cdot v_0; \qquad (8)$$

$$\lambda_1 = z - 1; \quad \lambda_2 = \frac{z-1}{z}. \qquad (9)$$

As a result we have:

$$H(z) = \frac{C_1}{C + C_2} \cdot \frac{1}{z - C/(C + C_2)} \quad (10)$$

5. The design of SCC based on continuous models
In figure 6 the configuration Tow Thomas of a biquad filter is presented.
The equivalent model is shown in figure 7.
Using the following notations: $R_3C_1 = 1/k_3$; $R_1C_1 = 1/k_1$; $R_0C_1 = 1/k_0$; $R_2C_2 = 1/k_2$ and considering the integrators reported to every input as being the same type , it could be deduced the equations which define the circuit: $\lambda = (z-1)/z$.

$$H(\lambda)_{LP} = \frac{v_{02}}{v_i} = \frac{k_0 k_2 N_i}{\lambda^2 + k_3\lambda + k_1 k_2} \quad (11)$$

$$H(\lambda)_{BP} = \frac{v_{01}}{v_i} = -\frac{k_0 \lambda v_i}{\lambda^2 + k_3\lambda + k_1 k_2} \quad (12)$$

if $\lambda = Ts$, identifying the parameters it results

$$\omega_0 = \sqrt{k_1 k_2}/T \ ; \ Q = \omega_0 / Tk.$$

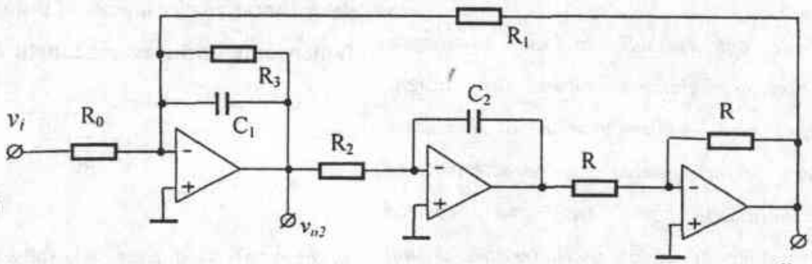


fig.6

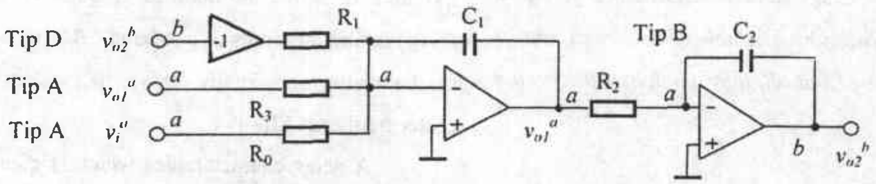


fig.7

The ciruit can be implemented like in figure 8.

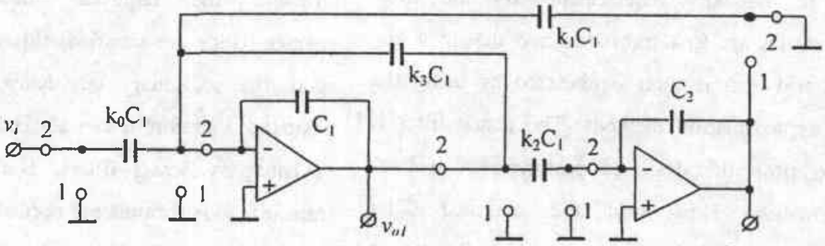


fig.8

Conclusions
The method presented in this paper can be used as a first step in the re-designing continuous time circuits, which already exist as Resistor Capacitor – OpAmp circuits. The proposed continuous models of some SC circuits can be used in a unitary way for designing or analyzing different variants.

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