REAL FREQUENCY TECHNIQUE WITHOUT OPTIMIZATION

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ABSTRACT

In this study, single matching problem is solved without optimization. With respect to the given load data and the desired transducer power gain (TPG) level, back-end impedance values of the matching network which will be designed are obtained. Then this impedance is modelled and after synthesizing the impedance or reflectance function of the model, matching network is obtained. An algorithm and an example are given to illustrate the proposed method.

I. INTRODUCTION

The analytic matching theory needs an explicit expression or, equivalently, a circuit realization for the load and generator impedances. In practice however, one usually meets with experimental real frequency data for the terminating impedances to be matched. In such cases, to be able to use analytic theory, these data should be approximated with a proper equivalent circuit or an analytic realizable function. There is also another problem: Starting with an analytic form of gain function which is the best choice for the given terminating impedances. The terminating impedances are then processed to establish the theoretic gain-bandwidth restrictions which should be satisfied by the assumed form of the transfer function. These restrictions take the form of a set of integral expressions or a set of equations which should be satisfied simultaneously. For problems involving simple terminations which include one or two reactive elements, the difficulties are almost unmanageable. Indeed, the most complicated case treated by the analytic theory in the literature is that of the simple LCR-load (two reactances and one resistor) which is matched over a low-pass band. Even in this case, the solution is not simple. If the number of reactive elements in the load network is greater than two, even when the procedure is applicable in principle, it may lead to unnecessarily complicated and suboptimal equalizer networks [1,2,3].

On the other hand, matching networks can be designed by employing commercially available computer aided microwave circuit design packages. These programs use purely numerical methods, and they are actually devised for the analysis and optimization of given networks based on the circuit elements. That is, the topology of the network and a good estimate of the element values should be supplied to the programs. Here, the major difficulty is the determination of optimum topology which is usually unclear. Moreover, the performance function is in general highly nonlinear in terms of the unknown element values to be optimized. Therefore, it is essential to start with element values which are close enough to the final solution to ensure the convergence to a global optimum. For problems involving narrow band designs with small number of elements, the choice of the circuit topology and the initialization of the elements may not be very critical. In such cases, by trial and error, reasonable solutions can be obtained. However, for complicated broadband problems, if the optimum topology is unknown, the use of these programs is difficult and would yield suboptimal results. Therefore, such computer aided design packages are essential for final trimming of the element values.

In 1977, Carlin has proposed a new method named Real Frequency Technique (RFT) which removes the major difficulties from which the analytic theory suffers [2]. Later on Carlin, Yarman, Fettweis and Pandel have developed some alternative real frequency algorithms [4,5,6]. These are line-segment technique, direct computational technique, parametric representation of Brune functions and scattering approach. The methods use the experimental real frequency data for the generator and the load, and it is not necessary to assume neither the analytic form of the transfer function nor the equalizer topology. It has been also shown for various single matching problems that in compared to a design obtained with analytic theory, the real frequency technique results in better performance with simpler structures [1,2,7]. Because of these advantages, the real frequency technique has become the most feasible approaches to solve broadband matching problems. But, by the invention of efficient and accurate data modeling tools, matching network desing by analytic methods is an unanswered problem for the researchers.

II. MATCHING via MODELLING

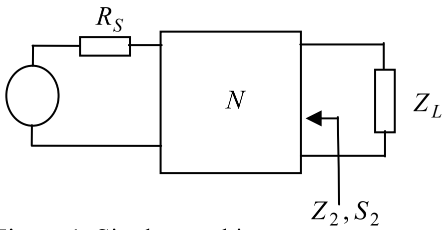

Figure 1. Single matching arrangement

Let us consider the single matching arrangement seen in Figure 1. To be able to synthesize the matching network (N), the function (Z_2 or S_2) that completely describes the network must be obtained. The methods mentioned above can be used to obtain the function Z_2. But all the methods have non-linear optimization part. So it is very important to start the optimization with suitable initial values to make the optimization-time shorter or to make the optimization process convergent.

Here by means of modelling, how suitable initial component values and network topology for final optimization can be obtained is discussed.

Transducer power gain for the arrangement seen in Figure 1 can be written as

$$TPG = \frac{4R_2R_L}{(R_2+R_L)^2+(X_2+X_L)^2} \tag{1}$$

where $R_2 = \mathrm{Re}\{Z_2\}$, $X_2 = \mathrm{Im}\{Z_2\}$, $R_L = \mathrm{Re}\{Z_L\}$ and $X_L = \mathrm{Im}\{Z_L\}$.

Let us divide the network N into two parts, a minimum reactive and a Foster part as seen in Figure 2.

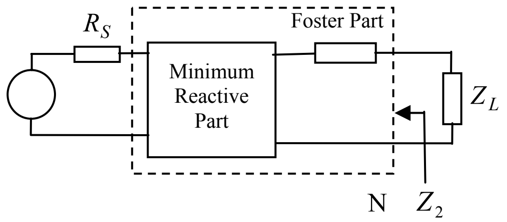

Figure 2. Minimum reactive and Foster parts

Z_2 impedance can be written as

$$\begin{aligned} Z_2 &= R_2 + jX_2 = Z_{MR} + Z_F \\ &= (R_{MR} + jX_{MR}) + jX_F \\ &= R_{MR} + j(X_{MR} + X_F) \end{aligned} \tag{2}$$

where Z_{MR} is the impedance of the minimum reactive part and Z_F is the impedance of the Foster part.

So we can conclude that

$$R_2 = R_{MR} \qquad X_2 = X_{MR} + X_F \tag{3}$$

If (3) is substituted in (1),

$$TPG = \frac{4R_{MR}R_L}{(R_{MR}+R_L)^2+(X_{MR}+X_F+X_L)^2} \tag{4}$$

If it is assumed that the matching is perfect,

$$R_2 = R_{MR} = R_L \text{ and } X_2 = X_{MR} + X_F = -X_L \tag{5}$$

Now let's define a new condition, only imaginary part is perfectly matched and call this situation as semi-perfect match condition. Under this condition transducer power gain will be

$$TPG = \frac{4R_{MR}R_L}{(R_{MR}+R_L)^2} \tag{6}$$

By using this equation, an expression for the ratio of $R_2 = R_{MR}$ to R_L can be found as

$$\alpha = \frac{R_{MR}}{R_L} = \frac{R_2}{R_L} = \frac{2 - TPG \pm 2\sqrt{1-TPG}}{TPG} \tag{7}$$

Note that α is a function of transducer power gain. So if a suitable gain form can be selected, resistive part data of the minimum reactive network can be obtained from (7) as

$$R_2 = R_{MR} = \alpha\, R_L \tag{8}$$

Then by means of Hilbert Transformation, imaginary part data of the minimum reactive network can be found

$$X_{MR} = H\{R_{MR}\} \tag{9}$$

After determining minimum reactive impedance data, Foster data of the matching network can be calculated from (5) as

$$X_F = -(X_L + X_{MR}) \tag{10}$$

So the question here is how to select a suitable gain form. Will it have a flat form, Butterworth form or Chebyshev form? Also another question is how the sign in (7) will be selected, should the plus sign or minus sign be used?

If TPG curves obtained from the matching networks designed by means of the methods mentioned in the previous section are examined, it can be seen that these curves have small fluctuations in the pass-band. So it will be reasonable to select the gain form in (7) as a Chebyshev form. The degree of the selected Chebyshev polynomial will be equal to the degree of the minimum reactive part of the matching network. Also it has been observed that firstly the sign in (7) must be plus sign and then must be changed after each roots of the selected Chebyshev polynomial.

As discussed above, TPG values that will be used in (7) will be obtained from a Chebyshev polynomial which is given in equation (11),

$$TPG = \frac{1}{1+\varepsilon^2 T_n(w)^2} \tag{11}$$

where ε is the ripple factor and T_n is the n^{th} order Chebyshev polynomial.

As pointed out above, Chebyshev polynomial will fluctuate around a flat gain level. This flat gain level can be determined for some simple loads. For example if load is a parallel combination of a capacitance and a resistor, this gain level can be calculated as [8,9,10]

$$TPG_{flat} = 1 - e^{-\frac{2\pi}{RCw_C}} \tag{12}$$

where R is the resistance and C is the capacitance in the load combination and w_c is the maximum frequency in the pass-band. If the load is much more complex than this simple parallel RC load, it is very difficult to obtain this flat gain level. So a suitable gain level must be supplied by the designer. After selecting/calculating TPG_{flat} level, square of ripple factor ε is found as

$$\varepsilon^2 = \frac{1-TPG_{flat}}{TPG_{flat}} \tag{13}$$

The degree of the Chebyshev polynomial must be determined by the designer. This degree will give the number of elements in the minimum reactive part of the matching network.

III. PROPOSED ALGORITHM

- Select/calculate TPG_{flat} level.
- Calculate square of ripple factor, $\varepsilon^2 = \frac{1-TPG_{flat}}{TPG_{flat}}$.
- Select the degree of the Chebyshev polynomial.
- Calculate transducer power gain level, $TPG = \frac{1}{1+\varepsilon^2 T_n(w)^2}$.
- Obtain α values, $\alpha = \frac{2-TPG \pm 2\sqrt{1-TPG}}{TPG}$
- Find the data for resistive part of the minimum reactive part of the matching network, $R_{MR} = \alpha \cdot R_L$.
- By means of Hilbert transformation obtain imaginary part data of the minimum reactive network, $X_{MR} = H\{R_{MR}\}$.
- Calculate Foster data, $X_F = -(X_L + X_{MR})$.
- Model the data obtained for minimum reactive part and Foster part.
- After synthesizing the functions obtained from modelling process, initial component values and the topology of the matching network for final optimization routine are found.

IV. EXAMPLE

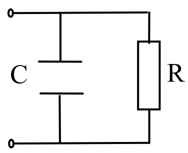

Figure 3. Selected load arrangement

Let us use the load arrangement seen in Figure 3. Real and imaginary values of the load impedance are given in Table 1.

Table 1. Real and imaginary values of the load impedance

w	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\mathrm{Re}\{Z_L\}$	1	0.8621	0.6098	0.4098	0.2809	0.2000	0.1479	0.1131	0.0890	0.0716	0.0588
$\mathrm{Im}\{Z_L\}$	0	-0.3448	-0.4878	-0.4918	-0.4494	-0.4000	-0.3550	-0.3167	-0.2847	-0.2579	-0.2353

TPG_{flat} level for the selected load is

$$\begin{aligned} TPG_{flat} &= 1 - \exp(-2\pi / RCw_c) \\ &= 1 - \exp(-2\pi / 1 \cdot 4 \cdot 1) = 0.7921 \end{aligned}$$

So square of ripple factor can be calculated as

$$\varepsilon^2 = \frac{1 - TPG_{flat}}{TPG_{flat}} = \frac{1 - 0.7921}{0.7921} = 0.2625$$

Let us select 4^{th} degree Chebyshev polynomial, so minimum reactive network will have four components,

$$TPG = \frac{1}{1 + \varepsilon^2 T_4(w)^2} \text{ where } T_4(w) = 8w^4 - 8w^2 + 1 .$$

TPG , α , real and imaginary values of minimum reactive network and Foster data are given in Table 2.

Table 2. Calculated TPG , α , R_{MR} , X_{MR} and X_F values

w	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
TPG	0.7921	0.8690	0.9496	0.9953	0.9930	0.9532	0.8978	0.8457	0.8087	0.7925	0.8003
α	0.3737	0.4685	0.6334	0.8712	1.1830	1.5520	1.9401	2.2937	2.5548	2.6730	2.6161
R_{MR}	0.3737	0.4039	0.3862	0.3570	0.3323	0.3104	0.2870	0.2595	0.2273	0.1915	0.1539
X_{MR}	0	-0.0361	-0.0943	-0.1338	-0.1619	-0.1881	-0.2147	-0.2410	-0.2656	-0.2892	-0.3341
X_F	0	0.3809	0.5821	0.6256	0.6114	0.5881	0.5698	0.5578	0.5503	0.5470	0.5694

Obtained minimum reactive and Foster data can be modelled by means of any modelling method. Here to form model with only lumped components, conjugate gradient method is used and the following model is obtained.

$$h_{MR} = -2.8635p^4 - 2.8266p^3 - 3.6571p^2 - 1.7143p - 0.4469$$

$$g_{MR} = 2.8635p^4 + 3.9942p^3 + 5.0477p^2 + 3.1250p + 1$$

After synthesizing minimum reactive network and modelling Foster data (a simple series inductor, $L = 0.6078\,H$), the following matching network is obtained,

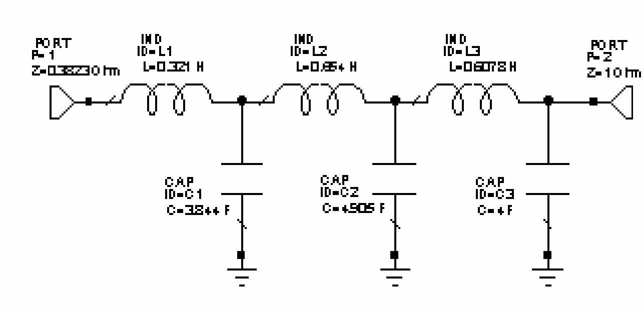

Figure 4. Obtained lumped matching network for parallel RC load

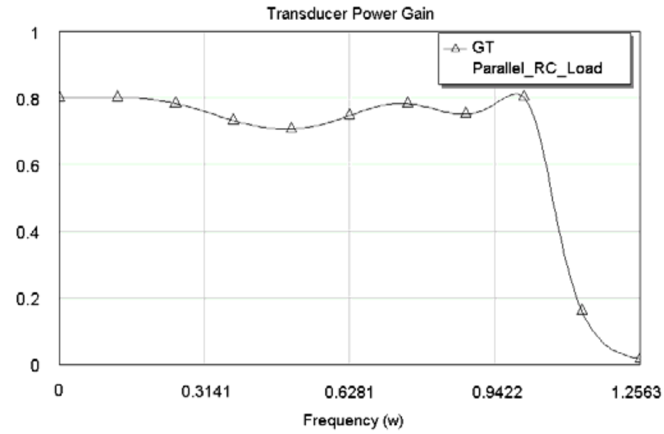

Figure 5. Transducer power gain vs frequency

This network is used as an initial network for final optimization and the following network and transducer power gain curve is obtained.

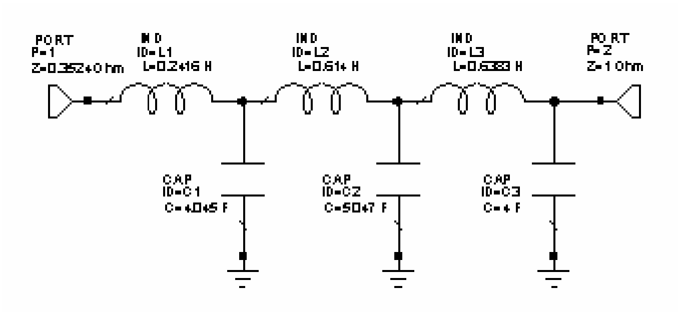

Figure 6. Matching network after optimization

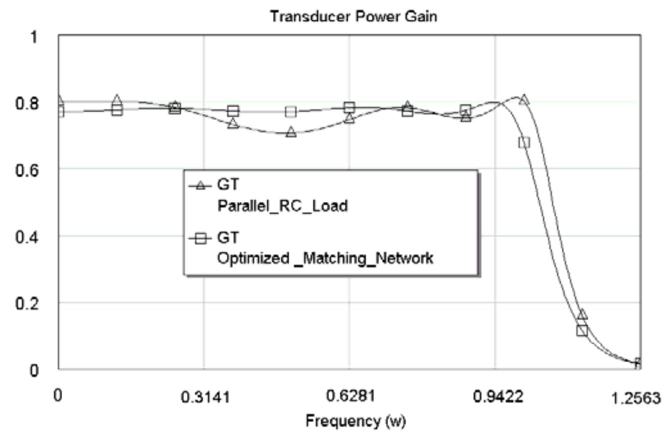

Figure 7. Transducer power gain of optimized matching network

V. CONCLUSION

As mentioned above, matching networks can be designed by employing commercially available computer aided microwave circuit design packages. But these programs use purely numerical methods, and the topology of the network and a good estimate of the element values should be supplied to the programs. Here, the major difficulty is the determination of optimum topology which is usually unclear. Moreover, the performance function is in general highly nonlinear in terms of the unknown element values to be optimized. Therefore, it is essential to start with element values which are close enough to the final solution to ensure the convergence to a global optimum. Here an algorithm to obtain suitable initial element values for the commercially available computer aided microwave circuit design tools is given. After obtaining the topology and initial element values, such computer aided design packages are essential for final trimming of the element values.

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