Estimation of Online Transmission Line Parameters and Fault Location by Using Different Differential Equation Algorithms

Dogan Yildiz[1], Serap Karagol[1], and Okan Ozgonenel[1]

[1] Electrical and Electronics Engineering Department, Ondokuz Mayis University, Samsun, Turkey
dogan.yildiz@omu.edu.tr, serap.karagol@omu.edu.tr, okanoz@omu.edu.tr

Abstract

Differential Equation Algorithm (DEA), which is used as an alternative method to Fourier and similar algorithms, is among the online parameter estimation algorithms used in transmission line protection. Pi or T equivalent circuit parameters (R, L and C) of the transmission line can be calculated efficiently by using the fundamental harmonic components of voltage and current. DEA is a general model-based parameter estimation method and it does not depend on the form of the signal (voltage/current). Due to this feature DEA provides an efficient parameter estimation. In addition to traditional trapezoidal DEA algorithm, Simpson 1/3, Simpson 3/8, Simpson 7, Medium Point, Forward Euler, Backward Euler, Runge-Kutta and Boolean algorithms were first proposed in this study and the parameter estimations were performed on an exemplary transmission line. Thusly, it is proven that alternative methods can be used effectively on relays as an algorithm to find the location of a fault in the line.

1. Introduction

Distance protection is implemented in meshed distribution systems to maintain selectivity. Several algorithms have been developed to implement distance relaying, such as symmetrical component traveling wave's algorithm, Fourier, adaptive scheme, artificial neural and differential equation algorithm (DEA). When conventional algorithms for distance relays are applied in parallel line systems, and these are adversely affected by the mutual coupling between lines. The effect of the mutual inductance may cause relays to see the fault as if it had taken place nearer or farer than the protection zone and will produce under-reach or over-reach. Besides, the sensitivity of the signals which are obtained from current/voltage transformers increases the sensitivity of the protection relay [1].

The research on power transmission lines requires transmission line parameters, including series resistance, series reactance and shunt susceptance as vital inputs to various power system analyses and applications, such as the power flow analysis and the protective relaying application. Accurate estimate of line parameters may also be employed for transmission line thermal condition monitoring. So it will be desirable if line parameters can be accurately estimated using on-line voltage and current measurements.

Due to the use of the global positioning services and phase measurement units in recent years, voltage and current samples can be sampled at high resolution and synchronization performance is improved. Thus, the line parameters can be calculated with high accuracy [2].

In the protection of transmission lines; when it comes to speed, reliability and security; especially in short lines, DEA provides a fast and accurate solution. The use of the DEA technique with rapid circuit breakers means the failure to be cleaned as soon as possible and the improvement of system stability. The DEA technique can be rearranged for the faults which is likely to be on the lines phase-to-ground, three-phase and two-phase faults. For the DEA, let us assume the following assumptions:

1. The line is short in the sense that it does not need to be modelled by surge impedance and wave propagation;
2. The voltage and current transformer are ideal in the frequency range of the algorithm, 50-300 Hz;
3. Load current is neglected;
4. The fault resistance is small;
5. The line is perfectly transposed and
6. Shunt capacitance is neglected. [3].

The DEA technique can be applied the voltage and the current signals that include the DC component and harmonics with success. However, parameter estimation using the DEA technique, depends on the sampling frequency, the angle of the voltage at the time of the fault and the frequency dependence of parameters. This is why the DEA is more successful at short lines than long lines. Shunt capacitance influences the DEA's performance negatively at long lines [4].

The frequency response of the capacitive voltage transformer and unmodeled dynamics, such as shunt capacitance, put an upper limit on algorithm speed and cause erroneous parameter estimation. The system matrix is negative or zero in some cases and that is one of the negative aspects of the DEA. The theoretically fastest algorithm is the travelling wave algorithm for transmission lines but the algorithm's implementation is rather complicated, its computation cost is very much and it puts high demands on the dynamics of current and voltage sensors. [5].

DEA technique is generally used with low pass filter (LPF). But the use of LPF decreases the algorithm's speed. In spite of this, the use of LPF in DEA is recommended for sensitive parameter estimation [6].

In this study, instead of the DEA technique based on conventional trapezoidal integration method; Forward Euler, Backward Euler, Mid-point, Runge-Kutta, Simpson 1/3, Simpson 3/8, Boole, Simpson 7 algorithms are proposed and the performances of the algorithms are discussed.

2. Conventional Differential Equation Algorithms

The DEA technique is commonly used in transmission lines because it provides a simple and quick solution for finding fault location in transmission lines. This technique is first derived for the transmission line's serial parameters at various sampling frequencies.

$$v(t) = Ri(t) + L\frac{di(t)}{dt} \tag{1}$$

In Eq. (1), R and L are the serial distributed parameters of transmission line, *v(t)* is the starting line voltage and *i(t)* is the current which flows along the serial branch. Considering the existence of the fault point resistance, Eq. (1) must be rearranged. The *R* and *L* parameters of the line can be readily calculated by the equality (1) written in different time intervals. Conventionally, Eq. (1) is rearranged by using trapezoidal integration rule for different intervals (3 samples and 6 samples used respectively) and used as fault location finding algorithm. Minimization of errors due to the approximate solution of integral and differential equations can also be included into the process. If the effect of the parallel capacity asked to be taken into account in the modelling line, the amount of error in the estimation of line parameters will increase. Using this technique, the line's serial *R* and *L* parameters can be estimated sensitively especially in the short lines which are not effected from parallel capacity [7]. While there are two unknown parameters in Eq. (1), there are two known magnitudes (*v(t)* and *i(t)*) are known. Therefore, if Eq. (1) is rewritten in two different time intervals, Eq. (2) and Eq. (3) are obtained.

$$\int_{t0}^{t1} v(t)dt = R\int_{t0}^{t1} i(t)dt + L[i(t_1) - i(t_0)] \tag{2}$$

$$\int_{t1}^{t2} v(t)dt = R\int_{t1}^{t2} i(t)dt + L[i(t_2) - i(t_1)] \tag{3}$$

The integral terms in Eq. (2) and Eq. (3) are reorganized according to the trapezoidal rule, and Eq. (4) is obtained.

$$\int_{t0}^{t1} v(t)dt = \frac{\Delta t}{2}[v(t_1) + v(t_0)] = \frac{\Delta t}{2}[v_1 + v_2] \tag{4}$$

Δt is the sampling step interval in Eq. (4). If Eq. (2) and Eq. (3) are reorganized in *k, k+1, k+2* discrete moments, *R* and *L* parameters of the line can be calculated as shown in Eq. (5).

$$\begin{bmatrix} \frac{\Delta t}{2}(i_{k+1}+i_k) & (i_{k+1}-i_k) \\ \frac{\Delta t}{2}(i_{k+2}+i_{k+1}) & (i_{k+2}-i_{k+1}) \end{bmatrix} \begin{bmatrix} R \\ L \end{bmatrix} = \begin{bmatrix} \frac{\Delta t}{2}(v_{k+1}+v_k) \\ \frac{\Delta t}{2}(v_{k+2}+v_{k+1}) \end{bmatrix} \tag{5}$$

Eq. (5) also known as Short Window Algorithm and needs 3 voltage and 3 current samples for the purpose of computing the parameters of the line. The revised format of Eq. (5) which uses 6 voltage and 6 current samples is given in Eq. (6). Eq. (6) is also known as Long Window Algorithm. Compared with the Short Window Algorithm, although the Long Window Algorithm makes more sensitive parameter estimation, it requires longer calculation time.

$$\begin{bmatrix} \frac{\Delta t}{2}(i_{k+1}+i_k) & (i_{k+1}-i_k) \\ \frac{\Delta t}{2}(i_{k+2}+i_{k+1}) & (i_{k+2}-i_{k+1}) \\ \frac{\Delta t}{2}(i_{k+3}+i_{k+2}) & (i_{k+3}-i_{k+2}) \\ \frac{\Delta t}{2}(i_{k+4}+i_{k+3}) & (i_{k+4}-i_{k+3}) \\ \frac{\Delta t}{2}(i_{k+5}+i_{k+4}) & (i_{k+5}-i_{k+4}) \end{bmatrix} \begin{bmatrix} R \\ L \end{bmatrix} = \begin{bmatrix} \frac{\Delta t}{2}(v_{k+1}+v_k) \\ \frac{\Delta t}{2}(v_{k+2}+v_{k+1}) \\ \frac{\Delta t}{2}(v_{k+3}+v_{k+2}) \\ \frac{\Delta t}{2}(v_{k+4}+v_{k+3}) \\ \frac{\Delta t}{2}(v_{k+5}+v_{k+4}) \end{bmatrix} \tag{6}$$

Especially Eq. (6) is concerned, singular value decomposition method must be used to solve the line parameters system matrices because there are few known parameters despite the unknown parameters. Eq. (5) and Eq. (6) are still used as a conventional DEA techniques and a numerical distance protection relay algorithm in transmission lines.

3. Proposed Differential Equation Algorithms

To see proposed algorithms' and conventional DEA techniques' parameter estimation performances, the line shown in Fig. 1 and its electrical equivalent are used.

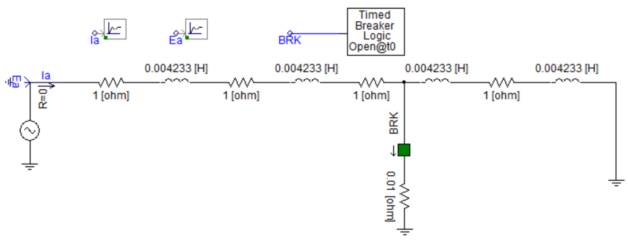

Fig. 1. Single-phase line model

3.1. Forward Euler Method

Forward Euler method is generally expressed as shown in Eq. (7) [8] and this method uses 2 voltage and 3 current samples for the parameter estimation. Eq. (8) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_1} f(x)dx \approx \Delta t \cdot f(x_0) \tag{7}$$

$$\begin{bmatrix} \Delta t \cdot (i_k) & i_{k+1} - i_k \\ \Delta t \cdot (i_{k+1}) & i_{k+2} - i_{k+1} \end{bmatrix} \begin{bmatrix} R \\ L \end{bmatrix} = \begin{bmatrix} \Delta t \cdot (V_k) \\ \Delta t \cdot (V_{k+1}) \end{bmatrix} \tag{8}$$

3.2. Backward Euler Method

Backward Euler method is generally expressed as shown in Eq. (9) [8] and this method uses 2 voltage and 3 current samples for the parameter estimation. Eq. (10) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_1} f(x)dx \approx \Delta t \cdot f(x_1) \tag{9}$$

$$\begin{bmatrix} \Delta t\cdot(i_{k+1}) & i_{k+2}-i_{k+1} \\ \Delta t\cdot(i_{k+2}) & i_{k+3}-i_{k+2} \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=\begin{bmatrix} \Delta t\cdot(V_{k+1}) \\ \Delta t\cdot(V_{k+2}) \end{bmatrix} \tag{10}$$

3.3. Mid-Point Method

Mid-point method is generally expressed as shown in Eq. (11) [8] and this method uses 3 voltage and 3 current samples for the parameter estimation. Eq. (12) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_1} f(x)dx \approx \Delta t\cdot f\left(\frac{x_0+x_1}{2}\right) \tag{11}$$

$$i_{mid}(k)=\frac{i_k+i_{k+1}}{2} \text{ and } V_{mid}(k)=\frac{V_k+V_{k+1}}{2}$$

R and L are parameters to be calculated as follows.

$$\begin{bmatrix} \Delta t(i_{mid}(k)) & i_{k+1}-i_k \\ \Delta t(i_{mid}(k+1)) & i_{k+2}-i_{k+1} \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=\begin{bmatrix} \Delta t(V_{mid}(k)) \\ \Delta t(V_{mid}(k+1)) \end{bmatrix} \tag{12}$$

3.4. Runge-Kutta Method

Runge-Kutta Method is generally expressed as shown in Eq. (13) [10] and this method uses 3 voltage and 3 current samples for the parameter estimation. Eq. (14) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_1} f(x)dx \approx \frac{\Delta t}{6}\cdot(f_0+4f_{mid}(0)+f_1) \tag{13}$$

$$f_{mid}(k)=\frac{f_k+f_{k+1}}{2},\quad i_{mid}(k)=\frac{i_k+i_{k+1}}{2},\quad V_{mid}(k)=\frac{V_k+V_{k+1}}{2}$$

$$\begin{bmatrix} \frac{\Delta t}{6}(i(k+1)+4i_{mid}(k)+i_k) & i_{k+1}-i_k \\ \frac{\Delta t}{6}(i(k+2)+4i_{mid}(k+1)+i_{k+1}) & i_{k+2}-i_{k+1} \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=\begin{bmatrix} \frac{\Delta t}{6}(V(k+1)+4V_{mid}(k)+V_k) \\ \frac{\Delta t}{6}(V(k+2)+4V_{mid}(k+1)+V_k) \end{bmatrix} \tag{14}$$

3.5. Simpson 1/3 Method

Simpson 1/3 method is generally expressed as shown in Eq. (15) [9]. In here, 1/3 represents the division Δt into three. This method uses 5 voltage and 5 current samples for the parameter estimation. Eq. (16) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_2} f(x)dx \approx \frac{\Delta t}{3}(f_0+4f_1+f_2) \tag{15}$$

$$\begin{bmatrix} \frac{\Delta t}{3}(i_k+4i_{k+1}+i_{k+2}) & (i_{k+2}-i_k) \\ \frac{\Delta t}{3}(i_{k+2}+4i_{k+3}+i_{k+4}) & (i_{k+4}-i_{k+2}) \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=\begin{bmatrix} \frac{\Delta t}{3}(V_k+4V_{k+1}+V_{k+2}) \\ \frac{\Delta t}{3}(V_{k+2}+4V_{k+3}+V_{k+4}) \end{bmatrix} \tag{16}$$

3.6. Simpson 3/8 Method

Simpson 3/8 method is generally expressed as shown in Eq. (17) [9].

$$\int_{x_0}^{x_3} f(x)dx \approx \frac{3\Delta t}{8}(f_0+3f_1+3f_2+f_3) \tag{17}$$

Unlike the previous method, 7 voltage samples and 7 current samples are needed in this method. Eq. (18) illustrates the format of the equation adapted to Fig. 1.

$$\begin{bmatrix} \frac{3\Delta t}{8}(i_k+3i_{k+1}+3i_{k+2}+i_{k+3}) & i_{k+3}-i_k \\ \frac{3\Delta t}{8}(i_{k+3}+3i_{k+4}+3i_{k+5}+i_{k+6}) & i_{k+6}-i_{k+3} \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=\begin{bmatrix} \frac{3\Delta t}{8}(V_k+3V_{k+1}+3V_{k+2}+V_{k+3}) \\ \frac{3\Delta t}{8}(V_{k+3}+3V_{k+4}+3V_{k+5}+V_{k+6}) \end{bmatrix} \tag{18}$$

3.7. Boole Method

Boole method is generally expressed as shown in Eq. (19) [9] and this method uses 9 voltage and 9 current samples for the parameter estimation. Eq. (20) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_4} f(x)dx \approx \frac{2\Delta t}{45}(7f_0+32f_1+12f_2+32f_3+7f_4) \tag{19}$$

$$\begin{bmatrix} \frac{2\Delta t}{45}(7i_k+32i_{k+1}+12i_{k+2}+32i_{k+3}+7i_{k+4}) & i_{k+4}-i_k \\ \frac{2\Delta t}{45}(7i_{k+4}+32i_{k+5}+12i_{k+6}+32i_{k+7}+7i_{k+8}) & i_{k+8}-i_{k+4} \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=\begin{bmatrix} \frac{2\Delta t}{45}(7V_k+32V_{k+1}+12V_{k+2}+32V_{k+3}+7V_{k+4}) \\ \frac{2\Delta t}{45}(7V_{k+4}+32V_{k+5}+12V_{k+6}+32V_{k+7}+7V_{k+8}) \end{bmatrix} \tag{20}$$

3.8. Simpson 7 Method

Simpson 7 method is generally expressed as shown in Eq. (21) [9] and this method uses 13 voltage and 13 current samples for the parameter estimation. Eq. (22) illustrates the format of the equation adapted to Fig. 1.

$$\int_{x_0}^{x_6} f(x)dx \approx \frac{\Delta t}{3}(f_0+4f_1+2f_2+4f_3+2f_4+4f_5+f_6) \tag{21}$$

$$\begin{bmatrix} \frac{\Delta t}{3}(i_k+4i_{k+1}+2i_{k+2}+4i_{k+3}+2i_{k+4}+4i_{k+5}+i_{k+6}) & i_{k+6}-i_k \\ \frac{\Delta t}{3}(i_{k+6}+4i_{k+7}+2i_{k+8}+4i_{k+9}+2i_{k+10}+4i_{k+11}+i_{k+12}) & i_{k+12}-i_{k+6} \end{bmatrix}\begin{bmatrix} R \\ L \end{bmatrix}=$$
$$\begin{bmatrix} \frac{\Delta t}{3}(V_k+4V_{k+1}+2V_{k+2}+4V_{k+3}+2V_{k+4}+4V_{k+5}+V_{k+6}) \\ \frac{\Delta t}{3}(V_{k+6}+4V_{k+7}+2V_{k+8}+4V_{k+9}+2V_{k+10}+4V_{k+11}+V_{k+12}) \end{bmatrix} \tag{22}$$

4. Performances of the Proposed Algorithms

For the purpose of testing the proposed methods' performances, the single-phase line shown in Fig. 1 is used. To see how the proposed algorithms react in temporary situations, $R_{fault}=0.01\Omega$ serial fault resistance is constructed in the midpoint of the transmission line at the range 0.015 s and 0.018 s. As shown in Fig. 1, the total resistance before and after the fault is 4Ω and the total inductance is 16.9 mH. But at the time of fault, the total fault resistance is seen at the beginning of line is 3.01Ω and the line inductance is 8.466 mH.

Including conventional Differential Equation algorithms, performances of all the proposed estimation algorithms, as stated in the literature is dependent on the switching moments and the sampling frequency. Thus filtering of the calculated line parameters and error analysis is required. A 10^{th} order and one-dimensional median filtering tecnique is implemented on the parameters found by using the proposed algorithms. Fig. 2 and Fig. 3 show not filtered values of R and L parameters before and after the fault respectively. Similarly, Fig. 4 and Fig. 5 show filtered values of R and L parameters before and after the fault respectively.

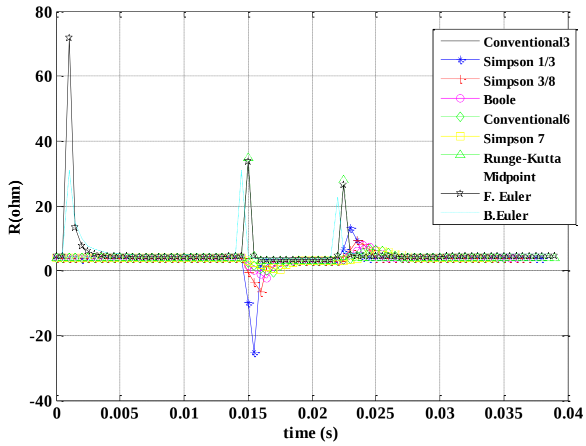

Fig. 2. Not filtered *R* values calculated before and after the fault

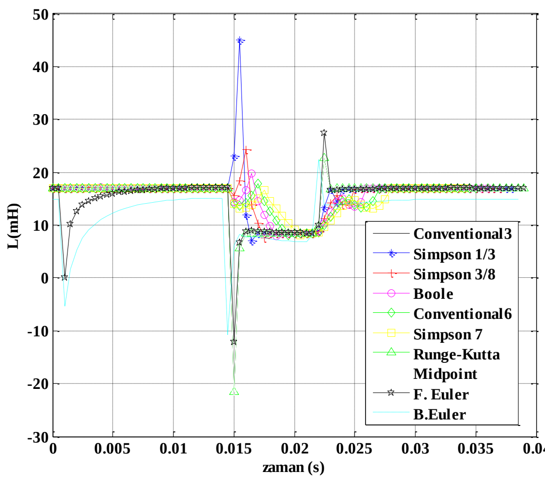

Fig. 3. Not filtered *L* values calculated before and after the fault

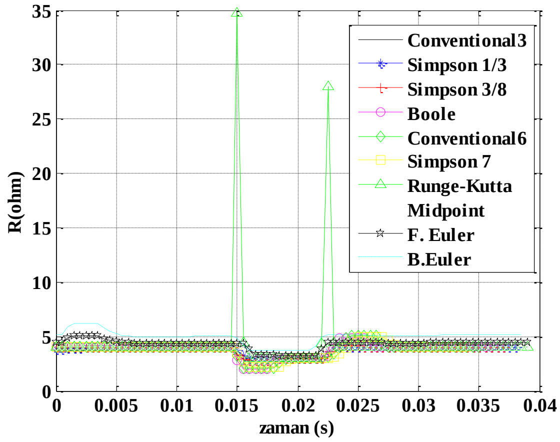

Fig. 4. Filtered *R* values calculated before and after the fault

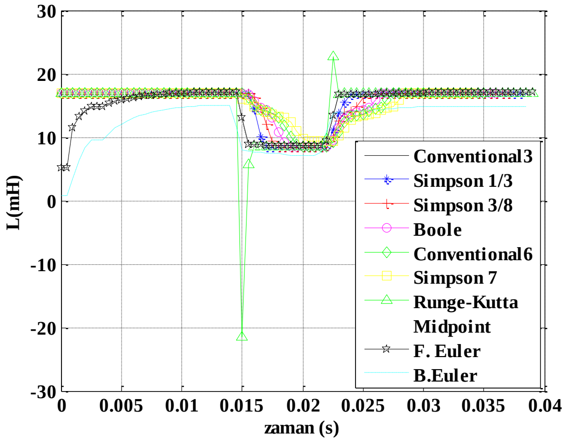

Fig. 5. Filtered *L* values calculated before and after the fault

Table 1 shows the RMS error analysis for the estimation methods both filtered and not-filtered ones. We can see from Table 1 that the best result in not filtered situation is obtained by using Simpson 7 algorithm and the best result in filtered algorithm is obtained by using Simpson 1/3 algorithm.

Table 1. RMS error analysis of the proposed algorithms

Methods	**Not Filtered** ***R* and *L***	**Filtered** ***R* and *L***
Conventional3	4.4922, 16.3969	0.3601, 16.3968
Conventional 6	0.9641, 16.3412	0.5886, 16.3412
Boole	1.203, 16.3562	0.5139, 16.3562
Simpson1/3	3.7514, 16.3837	**0.3468, 16.3839**
Simpson3/8	1.6948, 16.3704	0.3498, 16.3704
Simpson7	**0.8286, 16.3252**	0.6034, 16.3252
Mid-point	4.9222, 16.3968	0.3601, 16.3968
Forward Euler	8.8224, 16.3973	0.5712, 16.3974
Backward Euler	5.1207,16.3936	1.1782, 163937
Runge-Kutta	4.4922,16.3969	0.3601,16.3968

Table 2. The number of voltage and current samples used by the proposed methods

Methods	the number of voltage samples used	the number of current samples used
Forward Euler	2	3
Backward Euler	2	3
Mid-point	3	3
Conventional3	3	3
Runge-Kutta	3	3
Simpson1/3	5	5
Conventional 6	6	6
Simpson3/8	**7**	**7**
Boole	9	9
Simpson7	13	13

Table 2 shows the number of current and voltage samples used in the proposed algorithms. Table 2 can also be regarded as the ranking of the algorithms from the fastest to the slowest.

5. Conclusions

DEA technique is one of the numerical protection techniques which is used for distance protection and finding fault location algorithm. It is a discrete method and this feature provides DEA convenience that it can be implemented on digital circuits and superior to the Fourier-based algorithms. Conventionally, for the estimation of line parameters and finding the fault location, generally trapezoidal-based integration method is used (Short Window and Long Algorithms). In this paper, 6 digital line parameters and fault location estimation techniques have been proposed distinct from conventional techniques and achievements (RMSE analysis) is compared on a single-phase line. Since the obtained data are analyzed, it can be said that the most successful algorithm in the case of not-filtered is Simpson 7 algorithm and in the case of filtered is Simpson 1/3 algorithm. Also, obtaining very similar performances from Conventional3, Mid-point and Runge-Kutta algorithms as seen Table 1 is an issue that needs to be highlighted.

6. References

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