

EXPERIMENTAL EVALUATION OF POLYNOMIAL ROBUST H_∞ ADAPTIVE CONTROL FOR A DC MOTOR

İlyas Eker

e-mail: ilyas@gantep.edu.tr

Department of Electrical and Electronic Engineering, University of Gaziantep, 27310 Gaziantep, Turkey

Keywords: Adaptive control, robust control, control implementation, dc motors, recursive estimation.

ABSTRACT

This paper presents experimental application of polynomial robust H_∞ explicit adaptive control for the speed control of a dc motor. Explicit identification of the plant presented with ARMAX is achieved using recursive extended least squares (RELS) estimation method. The robust H_∞ adaptive controller ensures that the system remains asymptotically stable and it provides excellent tracking. Experimental results are compared with the results obtained from adaptive PID control. The results show the significance and feasibility of the approach and the factors involved in proposed robust adaptive control design.

I. INTRODUCTION

Until recently, robust control and adaptive control have been viewed as two techniques that compete with each other for use in controller design in the presence of plant model uncertainty [1]. The robust control deals in general with designing the controller in the presence of bounded plant uncertainties [2]. This simultaneously covers parameter variations that are effective at low- and mid-frequency ranges and unstructured model uncertainties which are often located in the high frequency range. On the other hand, the adaptive control handles the parametric variations, but the problem of handling unstructured model uncertainties remains. It has been recognized by the recent studies that there is a strong interaction between robust control design and adaptive control such that using these two design methods simultaneously in a control problem improves system performance significantly [3]. In addition, combining robust control theory with adaptive control has been an interesting research subject such that adaptive control further reduces the system with bounded uncertainty with robust control [4].

H_∞ design method, first proposed by Zames [5], is a methodology to handle performance and robustness objectives. The method has been improved using state-space approach [6] and polynomial approach [7]. The method has been applied to many industrial problems such

as turbine speed control [2], water level control for steam generators [8], robot arm control [9], gas turbine control [10], but very limited real-time implementations such as induction motor control [11]. The main problem in the implementation of the method is that there are several calculations and steps in the design algorithm. Developments in semiconductor and computer technology have provided fast data processing for on-line and adaptive control applications [12], for example, there has been a considerable development in adaptive control of the servo systems of the dc motors [13], since these machines are more easier to control than some other electrical machines such as induction motor [9] and widely used in several industrial applications[13 - 20].

II. POLYNOMIAL H_∞ CONTROL PROBLEM

The polynomial H_∞ controller design method [10] can be considered for the speed control problems of the electrical machines, since this method has several advantages and applications [2, 21]. The polynomial solution approach is used to solve the problem. Desired system performance is obtained using dynamic weighting functions. The controller is designed to eliminate low frequency disturbances and to attenuate high frequency disturbances and measurement noise. For example, the high frequency roll-off property can be included in the controller to obtain sufficient attenuation of measurement noise. The design algorithm is suitable for on-line adaptive control applications.

The H_∞ cost function to be minimized is [10]:

$$J_\infty = \sup_{|z|=1} \|\Phi_{\phi\phi}(z^{-1})\| \quad (1)$$

where $\Phi_{\phi\phi}(z^{-1}) = P_e \Phi_{ee} P_e^* + F_c \Phi_{uu} F_c^* + P_e \Phi_{eu} F_c^* + F_c \Phi_{ue} P_e^*$, P_e and F_c are minimum phase, strictly Hurwitz error and control dynamic weighting functions, $P_e, F_c \in \mathbb{R}_+(z^{-1})$, Φ_{ee} and Φ_{uu} are the power spectral densities of the error and

control signals, Φ_{eu} and Φ_{ue} are the cross-power spectral densities of the error and control signals and vice versa, respectively. The real rational dynamic weighting functions can be represented in terms of polynomials:

$$P_e(z^{-1}) = \frac{P_{en}(z^{-1})}{P_{ed}(z^{-1})}, \quad F_c(z^{-1}) = \frac{F_{cn}(z^{-1})}{F_{cd}(z^{-1})} \quad (2)$$

where $P_{ed}(z^{-1})$ and $F_{cd}(z^{-1})$ are monic and strictly Hurwitz polynomials, $P_{ed}(0)=1$, $F_{cd}(0)=1$, $P_{en}, P_{ed}, F_{cn}, F_{cd} \in \mathbb{R}_+[z^{-1}]$. The dynamic weighting functions are chosen to obtain desired performance specifications and robustness. The main point is that the dynamic weighting elements may be chosen to penalize the error and control signal peak powers in selected frequency ranges. [2]. The optimal H_∞ cost function in Eq. (1) is minimized to $J_{\infty-\min} = \lambda_{\min}^2$, where λ_{\min} is a positive real constant. The H_∞ optimal controller is

$$C(z^{-1}) = \frac{G(z^{-1})F_{cd}(z^{-1})}{H(z^{-1})P_{ed}(z^{-1})} \quad (3)$$

where $G(z^{-1})$ and $H(z^{-1})$, $G(z^{-1}), H(z^{-1}) \in \mathbb{R}_+[z^{-1}]$, are calculated from a couple of diophantine equations.

ROBUSTNESS

In practice, many components of a plant model are never precisely known. Furthermore, most plants are inherently non-linear, and can be approximated by linear models only in the neighbourhood of the operating point. These and other factors lead to the presence of uncertainty. A controller whose design is based on the nominal model of the plant should provide stability and performance requirements in the presence of uncertainty [18, 22, 23].

In robust control, the true plant (W) is covered by a set of plants Π which can be represented by the nominal plant W_n and a set of stable norm bounded uncertainties Δ , $\Delta \in \mathbb{R}_+(z^{-1})$:

$$\Pi = f(W_n, \Delta) \quad W \in \Pi \quad (4)$$

For robust controller synthesis, the finite-dimensional controller C is designed to stabilise all plants within the set. The set of uncertainties Δ_k , $\Delta_k \in \Delta$, represents uncertainties associated with either a parameter or a component of the plant (e.g. unmodelled dynamics or non-linearity). Norm bounds on these uncertainties are such that $\Delta_k(j\omega) \leq 1$. Nominal system M is assumed to be stable, $M \in \mathbb{R}_+(z^{-1})$. Using the Nyquist D contour the closed loop system in Figure 1 is stable for all uncertainties Δ if the following condition is satisfied [2]:

$$\|M\Delta\|_\infty < 1 \quad \forall \omega \in \mathbb{R} \quad (5)$$

The Eq. (5) is also valid for the open loop unstable plants [23].

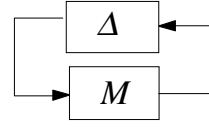


Figure 1. Uncertain system.

III. PLANT IDENTIFICATION

Discrete-time ARMAX model is considered for many plants, since the model is the most often used in the adaptive controller design algorithms:

$$y(k) = -\sum_{i=1}^{n_a} a_i y(k-i) + \sum_{i=1}^{n_b} b_{i-1} u(k-d-i) + \sum_{i=1}^{n_c} c_i e(k-i) + e(k) \quad (6)$$

where $y(k)$ is the plant output, $u(k)$ is the plant input, d is the discrete dead time, z^{-1} is the time shift operator, $e(k)$ is assumed to be zero mean, unmeasurable and statistically independent noise sequence with $E\{e(k)\} = 0$ of variance σ_e^2 . The parameters a_i , b_i and c_i are the real coefficients, $a_i, b_i, c_i \in \mathbb{R}$, a_i and b_i are the plant model parameters and c_i is the noise model parameters. Eq. (6) can be written in terms of the parameters and input-output data:

$$y(k) = \phi^T(k)\theta(k-1) + e(k) \quad (7)$$

where $\phi(k)$ is the data vector that includes the past values of the input and output data, θ is the parameter vector:

$$\phi(k) = \begin{bmatrix} -y(k-1) & -y(k-2) & \dots & -y(k-n_a) \\ u(k-d-1) & u(k-d-2) & \dots & u(k-d-n_b) \\ e(k-1) & e(k-2) & \dots & e(k-n_c) \end{bmatrix}^T \quad (8)$$

$$\theta = \begin{bmatrix} a_1 & a_2 & \dots & a_{n_a} & b_1 & b_2 & \dots & b_{n_b} & c_1 & c_2 & \dots & c_{n_c} \end{bmatrix}^T \quad (9)$$

The objective is that the parameters, a_i , b_i , c_i are assumed to be unknown and should be estimated recursively. The estimated parameter vector can then be represented as:

$$\hat{\theta} = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \dots & \hat{a}_{n_a} & \hat{b}_1 & \hat{b}_2 & \dots & \hat{b}_{n_b} & \hat{c}_1 & \hat{c}_2 & \dots & \hat{c}_{n_c} \end{bmatrix}^T \quad (10)$$

where the symbol ' $\hat{\cdot}$ ' denotes an estimated parameter. In order to estimate the noise model parameters, $\hat{c}_1, \hat{c}_2, \dots, \hat{c}_{n_c}$, knowledge of $e(k-1), e(k-2), \dots, e(k-n_c)$ is required. However, these are unknown and must be replaced in the data vector by the estimates [24]:

$$\phi(k) = \begin{bmatrix} -y(k-1) & -y(k-2) & \dots & -y(k-n_a) \\ u(k-d-1) & u(k-d-2) & \dots & u(k-d-n_b) \\ \hat{e}(k-1) & \hat{e}(k-2) & \dots & \hat{e}(k-n_c) \end{bmatrix}^T \quad (11)$$

IV. CONTROL SET-UP

Block diagram of the experimental set-up of explicit adaptive control scheme is illustrated in Figure 2. The communication between the computer (Pentium IV, 2 GHz in speed, 256 MB RAM) and the dc motor is achieved using a ADVANTECH-PCL-1800 interface card (130 kHz in speed and A/D conversion in 2.5 μ sec). The card permits user defined program to be used within Matlab communicated with the real-time plant. The dc motor under the experiment operates within the range of ± 10 volt with a permissible speed of 2400 rpm.

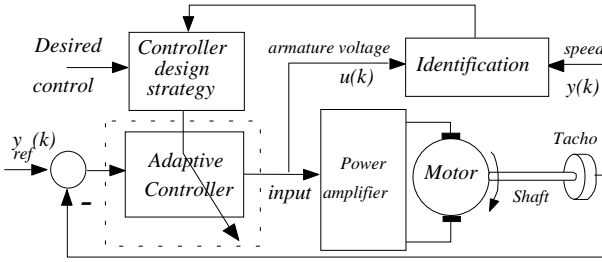


Figure 2. Adaptive control diagram.

Physical plant has been identified over a range of operating points. The identification experiments yield multiple models. Thus, the overall system will be defined by the multiple model representation or family of models. One of the models is chosen as the nominal model for *moment control design* and for choosing of the dynamic weighting functions. An uncertainty bound is developed by using the information contained in the models:

$$\Delta = \frac{0.111z^{-7} + 0.305z^{-6} - 0.122z^{-5} - 0.1275z^{-4} - 0.025z^{-3} + 0.087z^{-2} - 0.02z^{-1} - 0.01}{z^{-8} - 2.092z^{-7} + 2.052z^{-6} - 1.9632z^{-5} + 1.5772z^{-4} - 0.8194z^{-3} + 0.4228z^{-2} - 0.186z^{-1} + 0.028}$$

The tuning parameters of the dynamic weighting functions need to be experienced using the chosen nominal model to obtain the robust controller. The model corresponding to the operating point where the motor runs at a speed of 1200 rpm is chosen as the nominal model. The robust controller design is achieved to satisfy Eq. (5) and design objectives before real-time adaptive experiments are implemented. Therefore, the dynamic weighting functions here are aimed to be used in the adaptive control algorithm.

V. ADAPTIVE CONTROL RESULTS

As the design objectives, overshoot is not allowed in the output speed. High frequency roll-off controller is needed to eliminate high frequency disturbance and measurement noise. Integral action in the controller is also needed to reject low frequency disturbance and to achieve desired steady-state response. The dynamics weighting functions are chosen to satisfy the requirements as:

$$P_e = 0.1 \frac{1 - 0.8z^{-1}}{1 - z^{-1}}, \quad F_c = \frac{(1 - 0.5z^{-1})(1 - 0.85z^{-1})}{1}$$

It was investigated from the pre-test studies of the dc motor that the rise time was about 0.321 s. The sample frequency is chosen to be $f_s = 50 \text{ Hz}$ ($t_s = 20 \text{ ms}$). This satisfies the requirement for the sample frequency for adaptive control applications [25]. The initial values of parameters are also chosen to be zero. The initial covariance matrix diagonal values are $P(0) = 10I$ with a forgetting factor of $\lambda = 0.975$.

Several experiments were performed under constant load conditions. The results of one of the experiments were given in the present paper. A wide range of operating point was chosen such that the motor runs with a variable speed of between 240 rpm and 2040 rpm. Such speed variations were achieved by applying a square wave signal with periods of 3 s from the reference. The test is performed 80 s such that a set of 4000 data is processed. The set point speed variations and the output speed responses for adaptive H_∞ control and adaptive PID control at between 30 s and 47 s after starting are illustrated in Figure 3. The method given in [26] is used for PID adaptive control. The responses of the PID control are not acceptable, since a significant overshoot in speed and oscillatory responses are obtained. Such overshoot and slower response is not desired, especially in precision control applications of robotic systems. On the other hand, significant performance and excellent tracking is obtained with the proposed control. The corresponding control signals (armature voltage) are shown in Figure 4. The minimum cost value, λ_{min} is illustrated in Figure 5 for the moment of control that satisfies the recommendation $J_{\infty-min} = \lambda_{min}^2 < 1$ [10]. The estimated parameters for a fourth-order model are shown in Figure 6, Figure 7 and Figure 8 and these converge after a certain sample. The speed of parameter convergence depends on the forgetting factor used. More faster parameter convergence can be obtained if the value of the forgetting factor is reduced, but noise amplification.

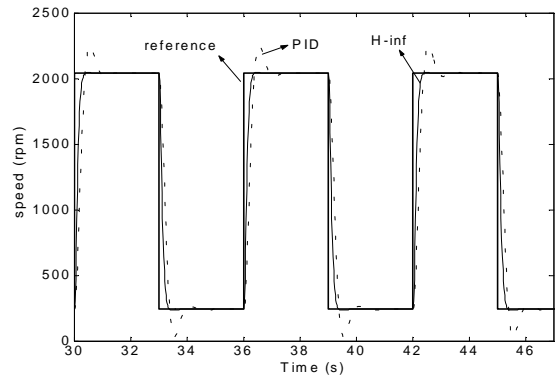


Figure 3. Tracking of H_∞ and PID control designs.

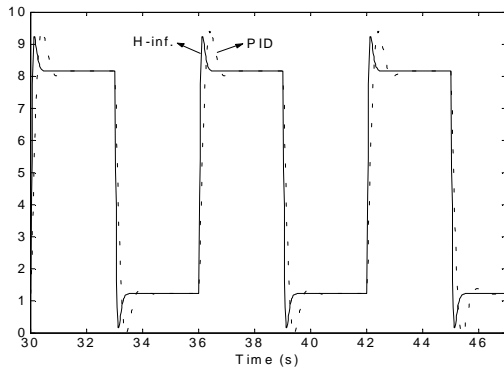


Figure 4. Control input signal (armature voltage) of H_∞ control design.

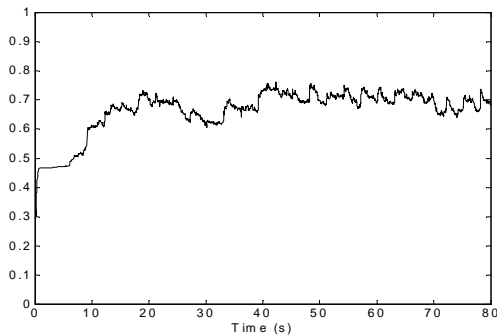


Figure 5. Minimum cost value, λ_{min} .

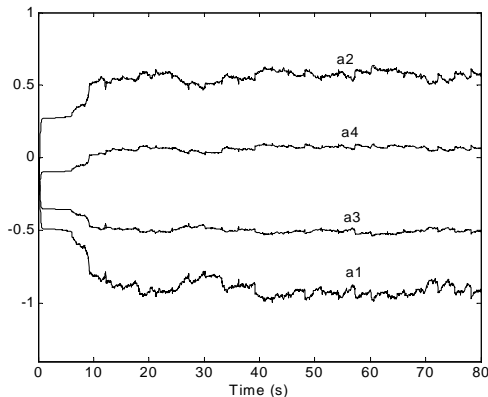


Figure 6. Parameters of $\hat{a}_i(z^{-1})$.

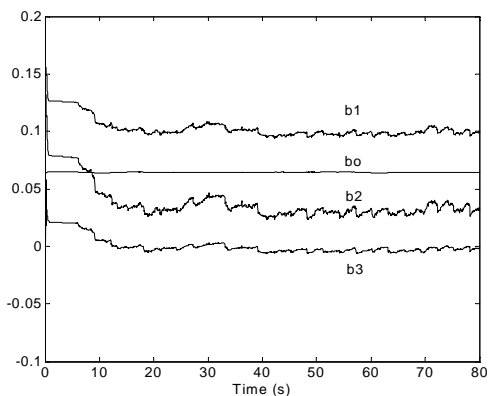


Figure 7. Parameters of $\hat{b}_i(z^{-1})$.

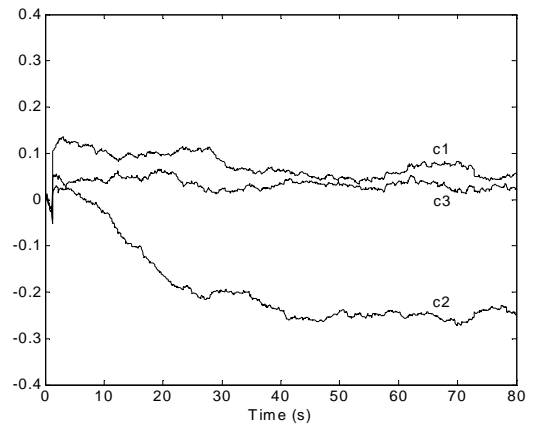


Figure 8. Parameters of $\hat{c}_i(z^{-1})$.

VI. CONCLUSIONS

In this paper adaptive polynomial H_∞ control theory was successfully applied to the control of the speed of a dc motor. The dynamic weighting functions of the control design theory were used to achieve certain control actions and closed loop performance. Experimental results demonstrated good performance and closed loop stability over the whole range of plant operation tested. The results also showed that the adaptive controller gives very good command tracking performance.

From a practical performance point of view, experimental tests that were conducted successfully for the dc motor running in closed loop conditions demonstrated that the computer based real-time H_∞ adaptive control is practical and easy to use. The measured input/output data obtained experimentally from the real-time set-up were used by a developed software program to generate the optimal control input signal (armature voltage). A fourth-order discrete-time plant model was shown to be flexible enough for the optimal control.

REFERENCES

1. I. D. Landau, From robust control to adaptive control, Control Eng. Prac., Vol. 7, pp. 1113-1124, 1989.
2. İ. Eker, M. Tumay, Robust multivariable-cascade governors for hydro turbine controls, Electrical Eng., Vol. 84, No. 4, pp. 229-327, 2002.
3. İ. Eker, M. A. Johnson, New aspects of cascade and multi-loop process control, Trans. of the IChemE Part-A Chemical Eng. Res. and Design, Vol. 74, No. 1, pp. 38-54, 1996.
4. N. A. Fairbairn, M. J. Grimble, H_∞ robust controller for self-tuning control applications Part.3: Self-tuning controller implementation, Int. J. Control, Vol. 52, No. 1, pp. 15-36, 1990.
5. G. Zames, Feedback and optimal sensitivity: Model reference transformation, multiplicative seminorms, and approximate inverses, IEEE Trans., Vol. AC-26, No. 2, pp. 301-320, 1981.

6. J. C. Doyle, K. Glover, P. P. Khargonekar, B. A. Francis, State-space solutions to H_2 and H_∞ control problems, *IEEE Trans.*, Vol. AC- 8, pp. 831-848, 1989.
7. M. J. Grimble, H_∞ robust controller for self-tuning control applications Part.2. Self-tuning and robustness, *Int. J. Control*, Vol. 46, No. 5, pp. 1819-1840, 1987.
8. A. G. Parlos, O. T. Rais, Nonlinear control of U-tube steam generators via H_∞ control, *Control Eng. Prac.*, Vol. 8, pp. 921-936, 2002.
9. S. B. Choi, S. S. Han, H. K. Kim, C. C. Cheong, H_∞ control of a flexible gantry robot arm using smart actuators, *Mechatronics*, Vol. 9, pp. 271-286, 1999.
10. M. J. Grimble, *Robust Industrial Control: Optimal Design Approach for Polynomial Systems*, Prentice Hall International UK., 1994.
11. E. Prempain, I. Postlethwaite, A. Benchaib, A linear parameter variant H_∞ control design for an induction motor, *Control Eng. Prac.*, Vol. 10, pp. 633-644, 2002.
12. W. Holderbaum, K. J. Hunt, H. Gollee, H_∞ robust control design for unsupported paraplegic standing: experimental evaluation, *Control Eng. Prac.*, Vol. 10, No. 11, pp. 1211-1222, 2002.
13. J. H. Horng, Neural adaptive tracking control of a DC motor, *Information Sciences*, Vol. 118, pp. 1-13, 1999.
14. Y. Yavin, P. D. Kemp, Modelling and control of the motion of a rolling disk: effect of the motor dynamics on the dynamical model, *Computer Methods in Applied Mechanics and Eng.*, Vol. 188, pp. 613-624, 2000.
15. V. C. Mummadi, Steady-state and dynamic performance analysis of PV supplied DC motors fed from intermediate power converter, *Solar Energy Materials and Solar Cells*, Vol. 61, pp. 365-381, 2000.
16. S. E. Lyshevski, Nonlinear control of mechatronic systems with permanent-magnet DC motors, *Mechatronics*, Vol. 9, pp. 539-552, 1999.
17. J. Santana, J. L. Naredo, F. Sandoval, I. Grout, O. J. Argueta, Simulation and construction of a speed control for a DC series motor, *Mechatronics*, Vol. 2, No. 9/10, 1145-1156, 2002.
18. K. Ziaei, N. Sepehri, Modelling and identification of electrohydraulic servos, *Mechatronics*, Vol. 10, pp. 761-772, 2000.
19. M. J. Chung, D. G. Gweon, Modelling of the armature slotting effect in the magnetic field distribution of a linear permanent magnet motors, *Electrical Eng.*, Vol. 84, No. 2, pp. 101-108, 2002.
20. J. O. Jang, G. J. Jeon, A parallel neuro-controller for DC motors containing nonlinear friction, *Neurocomputing*, Vol. 30, pp. 233-248, 2000.
21. İ. Eker, T. Kara, Operation and control of a water supply systems, *Isa Transactions (Article in Press)*, 2003.
22. A. Forrai, T. Tanoi, S. Hashimoto, H. Funato, K. Kamiyama, Robust controller design with hard constraints on the control signal, *Electrical Eng.*, Vol. 83, No. 4, pp. 179-186, 2001.
23. M. Morari, E. Zafiriou, *Robust process control*, Prentice-Hall Inc., USA, 1989.
24. R. Iserman, K. H. Lachman, D. Matko, *Adaptive control systems*, Prentice Hall, UK, 1992.
25. P. E. Wellstead, M. B. Zarrop, *Self-tuning Systems: Control and Signal Processing*, John Wiley and Sons, U.K, 1991.
26. C. C. Hang, T. H. Lee, K. H. Weng, *Adaptive Control*, Instrument Society of America, USA, 1993.

Notation

In the polynomial notation employed, all the polynomials are assumed to be functions of the unit delay operator, z^{-1} or complex s-plane. For simplicity, the arguments of the polynomials are sometimes omitted so that $X(z^{-1})$ is denoted by X . The adjoint of $X(z^{-1})=X^*(z^{-1})$ is denoted by X^* .

R	Set of all real numbers
$R(z^{-1})$	Set of all real rational functions in z^{-1} .
$R(s)$	Set of all real rational functions in complex s-plane.
$R_+(z^{-1})$	Set of all real rational functions whose poles lie within the unit circle on the z-plane (strictly stable).
$R[z^{-1}]$	Set of all polynomials of finite degree in z^{-1} whose coefficients satisfy $\{r_i \in R; i = 1, \dots, n\}$
$R_+[z^{-1}]$	Set of all polynomials whose zeros lie within the unit circle on the z-plane (strictly stable polynomials)



İlyas Eker (1965) received B.Sc. in Electrical and Electronic Engineering (EEE), Middle East Technical University (METU)/Turkey in 1988. From 1988 to 1992 he completed his M.Sc., and he worked as research assistant in EEE, Gaziantep University, Turkey. He joined Industrial Control Centre, University of Strathclyde, Glasgow, UK in 1992, where he received his Ph.D degree in 1995. At present, he currently works as assistant professor in EEE, Gaziantep University/Turkey. His current research interests are self-tuning control, robustness, fuzzy control, linear and nonlinear control, and their applications.