NOVEL SWITCHED CAPACITOR HALF DIFFERENTIATOR USING SCHNEIDER OPERATOR

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ABSTRACT

In this paper, a new realization of Fractional-Order Differentiator (FOD) using Switched-Capacitor circuits is presented. Schneider Operator based half-differentiator discretization is suggested. The z-domain discretized models are realized using SC circuits and simulated using OrCAD Pspice. A comparision of the OrCAD and MATLAB results for second and third order half differentiators are presented.

I. INTRODUCTION

Differentiators are useful in the processing of signals in various fields, such as digital control, digital image processing, communication and bio-medical applications [1].

In control systems, it is a common practice to use controllers, to obtain the desired output and to ensure that the system stability is not affected by external disturbances. To further enhance the system control performance, Fractional Order Controllers (FOC) are used [2],[3].

The fractional order system is represented by a fractional differential equation given by eqn. (1).

$$a_n D_t^{\alpha_n} y(t) + ... + a_1 D_t^{\alpha_1} y(t) + a_0 D_t^{\alpha_0} y(t) = b_m D_t^{\beta_m} u(t) + ... + b_1 D_t^{\beta_1} u(t) + b_0 D_t^{\beta_0} u(t) \qquad (1)$$

where β_k, α_k *(k=0,1,2,….)* are real numbers and a_k , b_k, *(k=0,1,…,n)* are arbitrary constants.
For obtaining the discrete model of the fractional–order system of eqn. (1), discrete approximations of the fractional-order operators are used. Then a general expression for the discrete transfer function of the system is obtained as in eqn. (2).

$$G(z) = \frac{b_m(\omega(z^{-1}))^{\beta_m} + ... + b_1(\omega(z^{-1}))^{\beta_1} + b_0(\omega(z^{-1}))^{\beta_0}}{a_m(\omega(z^{-1}))^{\alpha_m} + ... + a_1(\omega(z^{-1}))^{\alpha_1} + a_0(\omega(z^{-1}))^{\alpha_0}} \qquad (2)$$

where $\omega(z^{-1})$ denotes the discrete operator, expressed as a function of the complex variable z or the shift operator z^{-1}. In general the discretization of the fractional-order differentiator $p^{\pm r}$, can be expressed by a generating function $p = (w(z^{-1}))$. This generating function and its expansion determine both the form of the approximation and the coefficients [4].

The fractional order $PI^{\lambda}D^{\delta}$ controller can be described by the fractional order differential equation [4] given in eqn. (3).

$$u(t) = Ke(t) + T_i D_t^{-\lambda} e(t) + T_d D_t^{\delta} e(t) \qquad (3)$$

The discrete approximation of the fractional-order controller is expressed as

$$C(z) = K + T_i(\omega(z^{-1}))^{-\lambda} + T_d(\omega(z^{-1}))^{\delta} \qquad (4)$$

where λ is an integral order, δ is a derivation order, K is a proportional constant, T_i is an integration constant and T_d is a derivation constant.
Taking λ=1 and δ=1, we obtain the classical PID controller.

This paper presents a new realization of half differentiator $s^{1/2}$. The digital implementation of the fractional order differentiator involves discretization of the half differentiator using the Schneider Operator [5],[6]. This new model of the FOD is then realized using Switched Capacitor circuits and simulated using OrCAD Pspice. The proposed discretization scheme exhibits a very good magnitude fit as that of the original continuous fractional order differentiator $s^{1/2}$. The MATLAB implementation of proposed fractional order differentiators of second and third order is also given. It is observed that the magnitude response in both the cases match with each other and also with that of the continuous FOD.

II. PROPOSED SCHNEIDER OPERATOR BASED DISCRETIZATION TECHNIQUE

Schneider [7,8] has proposed higher order mapping functions. These higher order mapping functions exhibit increased accuracy in digitizing linear, time invariant, continuous time filters for real applications.
The s-to-z transformations are based on Adams-Moulton Numerical Integration formulae.
Suppose we have a continuous-time filter as shown in Fig 1, consisting a pure integrator with input u(t), output y(t) and state x(t). The continuous-time transfer function of the resulting filter is:

$$\frac{Y(s)}{U(s)} = \frac{1}{s} \tag{5}$$

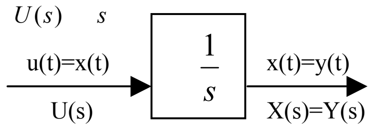

Fig 1: Input-Output relations for a pure integrator

From Fig 1, $\dot{x} = u$ and $x = y$ (6)

This represents an integrator, which can be implemented digitally using the Adams-Moulton Numerical Integration formulae.

For integration of third order, Adams-Moulton Numerical Integration formulae is

$$y_k = y_{k-1} + (T/12) * (5\dot{x}_k + 8\dot{x}_{k-1} - \dot{x}_{k-2}) \tag{7}$$

Replacing $\dot{x} = u$ we get,

$$y_k = y_{k-1} + (T/12) * (5u_k + 8u_{k-1} - u_{k-2}) \tag{8}$$

Taking z-transforms of both sides we get

$$Y(z)(1 - z^{-1}) = \left(T/12\right)(5 + 8z^{-1} - z^{-2})U(z) \tag{9}$$

Multiplying by z^2 on both sides we get

$$Y(z)(z^2 - z) = \left(T/12\right)(5z^2 + 8z - 1)U(z) \tag{10}$$

So the resulting discrete-time transfer function is

$$\frac{Y(z)}{U(z)} = \left(\frac{T}{12}\right)\frac{(5z^2 + 8z - 1)}{(z^2 - z)} \tag{11}$$

Comparing the discrete-time transfer function of eqn. (11) with the continuous-time transfer function of eqn. (5), the s-to-z mapping function representing Schneider rule is obtained as

$$s = \left(\frac{12}{T}\right)\frac{(z^2 - z)}{(5z^2 + 8z - 1)} \tag{12}$$

For a half differentiator

$$s^{1/2} = \left[\left(\frac{12}{T}\right)\frac{(z^2 - z)}{(5z^2 + 8z - 1)}\right]^{1/2} \tag{13}$$

With T=0.001s, the approximate mathematical models for half differentiator are obtained as:

$$G_2(z) = 48.989 * \left(\frac{z^2 - 0.5z - 0.125}{z^2 + 0.8z - 0.42}\right) \quad (n = 2) \tag{14}$$

$$G_3(z) = 48.989 * \left(\frac{z^3 - 0.5z^2 - 0.125z - 0.0625}{z^3 + 0.8z^2 - 0.42z + 0.336}\right) \quad (n = 3) \tag{15}$$

where n is the order of the expansion.

III. CONTINUED FRACTION EXPANSION (CFE) TECHNIQUE [9]

A digital transfer function of order n is expressed as

$$G(z) = \frac{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0}{b_n z^n + b_{n-1} z^{n-1} + \cdots + b_0} \tag{16}$$

where a_i's & b_i's are the coefficients of numerator and denominator polynomials for i=0,1,….n.
This transfer function $G_n(z)$ can be expanded in the general form given in eqn. (17).

$$G_{n1}(z) = -1 + \cfrac{\pm 1}{g_2(z) \pm \cfrac{\pm 1}{g_3(z) \pm \cfrac{\pm 1}{\ddots \pm \cfrac{\pm 1}{g_n(z)}}}}$$

$$G_n(z) = A_0 * G_{n1}(z) \qquad (17)$$

where A_0 is a constant and

$g_p(z) = B_p z \pm A_p$

where A_p & B_p are coefficients of CFE for $p=1,2\ldots n$ and $A_0=a_n/b_n$.

Each ladder of the transfer function of eqn. (17) can be represented by any one of the two eqns. (18a,b)

$$G_{1,p}(z) = \pm \frac{1}{A_p + B_p z^{-1} + G_{m,p+1}(z)} \qquad (18a)$$

$$G_{2,p}(z) = \pm \frac{1}{A_p + B_p z + G_{m,p+1}(z)} \qquad (18b)$$

These are transfer functions of leaky inverting and non-inverting integrators and hence can be realized using Switched Capacitor circuits.

IV. REALIZATIONS OF SCHNEIDER OPERATOR BASED HALF DIFFERENTIATOR USING SWITCHED CAPACITOR CIRCUITS [10]

Based on the second order and third order approximate models of half differentiator developed in Section II, the switched capacitor realizations are as follows:

a) The Continued Fraction Expansion of the Schneider based second order model for half differentiator of eqn. (14) is:

$$G_2(z) = 48.989 * \left(\frac{z^2 - 0.5z - 0.125}{z^2 + 0.8z - 0.42}\right)$$

$$= (-48.989) * \left[(-1) + \cfrac{1}{g_1(z) - \cfrac{-1}{g_2(z)}}\right] \qquad (19)$$

where

$g_1(z) = 0.769z + 0.78988$

$g_2(z) = 6.95z - 1.577548.989$

The Switched Capacitor realization [11] of equation (19) is shown in Fig 2.

The symbols e and o represent non-overlapping clocks as shown in Fig 3.

b) Similarly the CFE expansion of the Schneider based third order model for half differentiator of eqn. (15) is:

$$G(z) = -1 + \cfrac{1}{g_1(z) - \cfrac{1}{g_2(z) - \cfrac{-1}{g_3(z)}}} \qquad (20)$$

$G_3(z) = -48.989 * G(z)$

where

$g_1(z) = 0.769z + 0.7898$

$g_2(z) = 2.63z - 0.4842$

$g_3(z) = 1.27z - 0.05486$

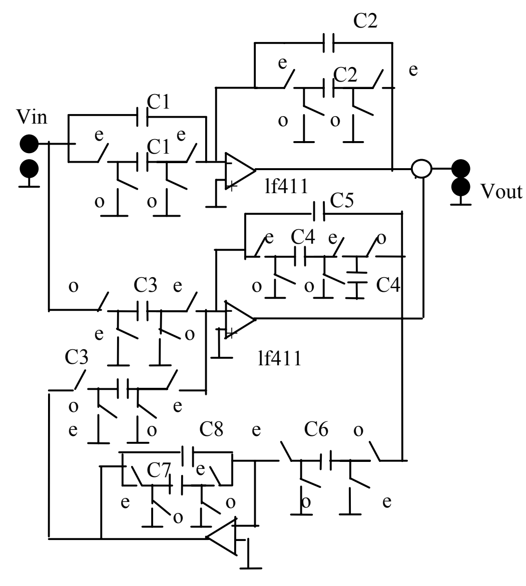

Fig 2: Second Order Schneider Based SC Half Differentiator

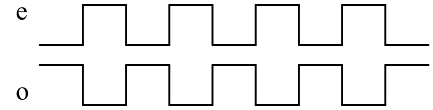

Fig 3: Non-overlapping even (e) and odd (o) clocks

The circuits are implemented using OrCAD 9.1 with transmission gates as switches. Level – 3 Pspice models of MOSFETS with 2μ technology have been used for implementing transmission gates. The op-amps used are LF 411. The clock frequency for the circuits simulated using OrCAD and MATLAB is chosen as 1 kHz.

V. RESULTS AND CONCLUSIONS

This paper proposes a novel discretization method using Schneider operator, which is a second order mapping function for the half-differentiator $s^{1/2}$. The proposed discretization method is then implemented using Switched Capacitor circuits and simulated on OrCAD Pspice and MATLAB. The simulation results reveal a very good magnitude fit response for the second and third order Schneider Operator based half differentiator. Fig 4 (a) shows the OrCAD results and 4 (b) shows the MATLAB results for these differentiators. The Pspice simulation results and the MATLAB results for the discretization method presented in this paper match with each other and also with the theoretical result of half differentiator in continuous-time domain.
It is observed that a phase compensation G(z) is required for achieving a phase of 45˚ as that of the continuous-time domain.

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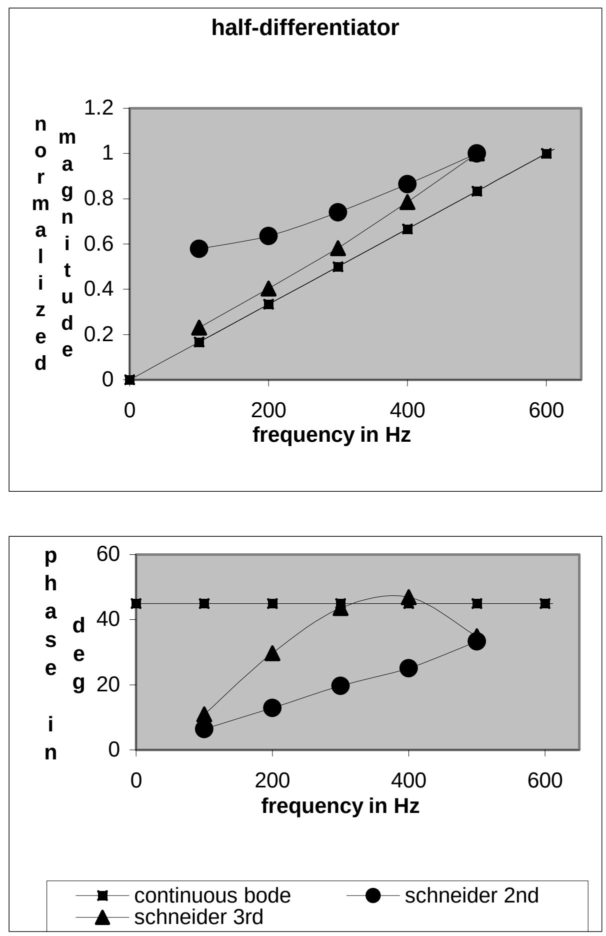
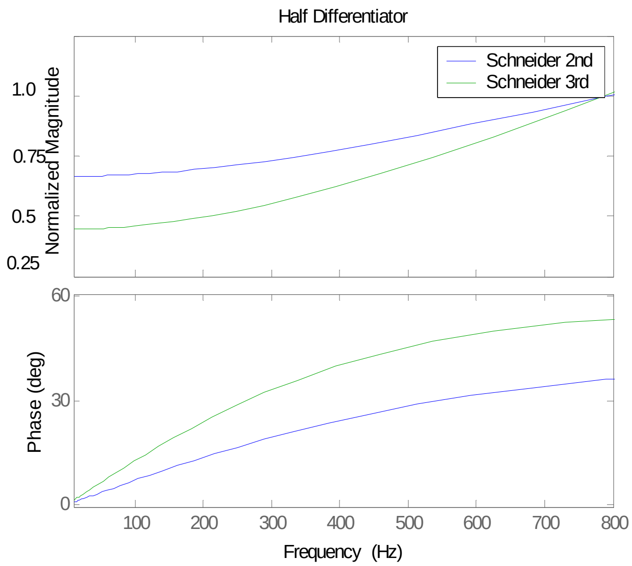


4(a)



4(b)

Fig 4: Simulation Results of Schneider Based Second and Third Order Half Differentiator using (a) OrCAD Pspice and (b) MATLAB.