TREATMENT OF THE ELECTRICAL CIRCUITS BASED ON THE TIME DISCRIMINATION MODELS

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ABSTRACT. A new method of numerical treatment of the electrical circuits is proposed. This method preserves the advantages of the analysis in the state space.

1. INTRODUCTION

The analysis of the time - varying linear or non-linear electrical circuits, based on the determination of the time evolution of the state vector, has the advantage of using a minimal number of variables. The multiport method [1] allows the reduction of the initial problem to the solving an auxiliary resistive circuit.

The calculation of the transition matrix and of the matrix integral, which appear in the state equation solution, needs a remarkable effort. This paper proposes an analysis and calculation method of circuits with or without excess elements [2] using the time discrimination and its associated models [3]. Thus, the above drawbacks are eliminated but the advantages of the analysis with state variables are preserved.

The proposed method allows to trace the time evolution of the state and output variables, as well as the punctual calculation of the state vector.

2. TIME DISCRIMINATION EQUATIONS

The circuit behaviour is analysed during the time $[t_0, wh]$, where t_0 is the initial moment, w is a positive integer and the step h is small enough to be valid the Euler's approximation [3].

Corresponding to the interval $[nh, (n+1)h]$, with $(n+1) < w$, for each ideal normal passive one - port circuit element, models in concordance with the functional equations can be associated.

This paper involves RLC active linear circuits under the following symbols:
R - the resistance matrix
L - the inductance matrix
Γ - the reciprocal inductance matrix
C - the capacitance matrix
S - the elasstance matrix ($\mathbf{S} = \mathbf{C}^{-1}$).

The following matrix equation will describe the circuits elements, corresponding to the whole analysed circuit:

$$\mathbf{u}_R^{n+1} = \mathbf{R}\mathbf{i}_R^{n+1}, \tag{1}$$

$$\mathbf{u}_L^{n+1} = \frac{1}{h}\mathbf{L}\left(\mathbf{I}_L^{n+1} - \mathbf{I}_L^{n}\right), \tag{2}$$

$$\mathbf{i}_L^{n+1} = h\,\boldsymbol{\Gamma}\mathbf{U}_L^{n+1} + \mathbf{i}_L^{n}, \tag{3}$$

$$\mathbf{i}_C^{n+1} = \frac{1}{h}\mathbf{C}\left(\mathbf{U}_C^{n+1} - \mathbf{U}_C^{n}\right), \tag{4}$$

$$\mathbf{u}_C^{n+1} = h\,\mathbf{S}\mathbf{I}_C^{n+1} + \mathbf{u}_C^{n}. \tag{5}$$

The previous symbols have the following significance:
$\mathbf{u}_L$, $\mathbf{I}_L$ - the voltage vector and the current vector of the excess inductor branches;
$\mathbf{i}_L$, $\mathbf{U}_L$ - the current vector and the voltage vector of the essential inductor branches;
$\mathbf{i}_C$, $\mathbf{U}_C$ - the current vector and the voltage vector of the excess capacitor branches;
$\mathbf{u}_C$, $\mathbf{I}_C$ - the voltage vector and current vector of the essential capacitor branches.

The present method uses as state variables the essential capacitor voltages and the essential inductor currents. At the moment $t = nh$, the state vector **x** and the complementary of the state variables vector **X** will be:

$$\mathbf{x}^n = \begin{pmatrix} \mathbf{u}_C^n \\ \mathbf{i}_L^n \end{pmatrix}, \quad \mathbf{X}^n = \begin{pmatrix} \mathbf{I}_C^n \\ \mathbf{U}_L^n \end{pmatrix} \tag{6}$$

At the same moment $t = nh$, the input vector [2] will be:

$$\mathbf{v}^n = \begin{pmatrix} \mathbf{e} \\ \mathbf{j} \end{pmatrix}, \tag{7}$$

with **e** and **j**, the f.e.m. vector of the voltage sources and the corresponding vector of the current sources.

The circuit transition matrix will be $\mathbf{\Phi}(t)$ designed [1, 2].

3. CIRCUITS WITHOUT EXCESS ELEMENTS

An electrical circuit without loops of capacitors and/or sections of inductors [1, 2] can be represented as a resistive multiport connected at the input with the capacitors, inductors, voltage and current sources of the analysed circuit.

Using the symbols (6), from the relations (5) and (3), it result the matrix equation:

$$\mathbf{x}^{n+1} = \mathbf{x}^{n} + h\begin{pmatrix} S & 0 \\ 0 & \Gamma \end{pmatrix}\mathbf{X}^{n+1} \tag{8}$$

The calculation of the vector **X** is made relative easy, from the above mentioned multiport as outputs of the sources involved by the time discrimination models [1, 3] which have replaced the capacitors and the inductors. It results:

$$\mathbf{X}^{n+1} = \mathbf{E}\,\mathbf{x}^{n} + \mathbf{F}\,\mathbf{v}^{n+1}, \tag{9}$$

matrices **E** and **F** containing transmitances [3].

The equations (8) and (9) lead to

$$\mathbf{x}^{n+1} = \mathbf{M}\,\mathbf{x}^{n} + \mathbf{N}\,\mathbf{v}^{n+1}, \tag{10}$$

with

$$\mathbf{M} = \mathbf{1} + h\begin{pmatrix} S & 0 \\ 0 & \Gamma \end{pmatrix}\mathbf{E}, \tag{11}$$

1 being the unit matrix, and

$$\mathbf{N} = h\begin{pmatrix} S & 0 \\ 0 & \Gamma \end{pmatrix}\mathbf{F}. \tag{12}$$

Using (10), the state vector evolution, between the initial value $\mathbf{x}^0 = \mathbf{x}(0)$ and the final one $\mathbf{x}^w = \mathbf{x}(wh)$, can be obtained. Its knowledge in all discreet moments, resulted by choosing the step h, allows the direct determination of the evolution of all wished outputs [5, 6].

From (10), it follows by induction:

$$\mathbf{x}^{n} = \mathbf{M}^{n}\mathbf{x}^{0} + \sum_{k=1}^{n}\mathbf{M}^{n-k}\mathbf{N}\mathbf{v}^{k}. \tag{13}$$

This formula allows the direct calculation of the state vector for any moment t_n = nh, when the initial conditions and the excitations vector are known.

The comparative analysis of the initial condition component of the general state equation solution [1]

$$\mathbf{x}(t) = \Phi(x)\,\mathbf{x}(0) + \int_0^t \Phi(t-\tau)\,\mathbf{B}\mathbf{v}(\tau)\,d\tau \tag{14}$$

and of the relations (8) and (13), leads to:

$$\Phi(h) = \mathbf{M} \text{ and } \Phi(nh) = \mathbf{M}^{n}, \tag{15}$$

therefore to a calculation modality of the transition matrix discrete values by successive multiplication of the matrix **M** by itself.

If $\mathbf{x}_p(t)$ is a particular solution of the state equation, we have [1, 2]:

$$\mathbf{x}(t) = \Phi(t)\,[\mathbf{x}(0) - \mathbf{x}_p(0)] + \mathbf{x}_p(t) \tag{16}$$

At t = h, from (10) and (16), it results that:

$$\mathbf{x}^{1} = \Phi(h)\,\mathbf{x}^{0} + \mathbf{x}_p^{1} - \Phi(h)\,\mathbf{x}_p^{0}, \tag{17}$$

and

$$\mathbf{x}^{1} = \mathbf{M}\mathbf{x}^{0} + \mathbf{N}\mathbf{v}^{1}, \tag{18}$$

also

$$\mathbf{N}\mathbf{v}^{1} = \mathbf{x}_p^{1} - \mathbf{M}\mathbf{x}_p^{0}. \tag{19}$$

By induction, from the last relation follows

$$\mathbf{N}\mathbf{v}^{n+1} = \mathbf{x}_p^{n+1} - \mathbf{M}\mathbf{x}_p^{0} \tag{20}$$

With (10) and (20) results in

$$\mathbf{x}^{n+1} = \mathbf{M}(\mathbf{x}^{n} - \mathbf{x}_p^{n}) + \mathbf{x}_p^{n+1} \tag{21}$$

It can be observed that (21) considerably facilitates state vector computation, as needs only the previous determination of the matrix **M**, so according to (11) the matrix **E** determination. To take into account (9), the matrix **E** results from the formal passived multipole.

At the moment t = nh, the state vector computation can directly be made with:

$$\mathbf{x}^{n} = \mathbf{M}^{n}(\mathbf{x}^{0} - \mathbf{x}_p^{0}) + \mathbf{x}_p^{n}, \tag{22}$$

expression which is obtained from (21).

4. CIRCUITS WITH EXCESS ELEMENTS

The analysed circuit can contain loops formed only by capacitors and independent ideal voltage sources, as well sections of inductors and independent ideal current sources. The voltages of excess capacitors and the currents of excess inductors will be expressed depending on the state variables and inputs, corresponding to the moment t = nh:

$$\begin{pmatrix} \mathbf{U_C^n} \\ \mathbf{I_L^n} \end{pmatrix} = \begin{pmatrix} \mathbf{K_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_2} \end{pmatrix} \begin{pmatrix} \mathbf{u_C^n} \\ \mathbf{i_L^n} \end{pmatrix} + \begin{pmatrix} \mathbf{K_1^1} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_2^1} \end{pmatrix} \begin{pmatrix} \mathbf{e^n} \\ \mathbf{j^n} \end{pmatrix} \tag{23}$$

in which the matrix $\mathbf{K_1}$ and $\mathbf{K_1^1}$ contain the voltage transfer factors, and $\mathbf{K_2}$ and $\mathbf{K_2^1}$ contain current transfer factors [2, 4].

The essential elements will be described by (3) and (5) respectively, but the excess elements will be described by the (2) and (4) relationships.

Using the Euler's associated models [3] corresponding to the above mentioned relationships, the initial circuit is transformed into a resistive active circuit. The computation of the **X** vector is made in this auxiliary circuit one. It results:

$$\mathbf{X^{n+1}} = \mathbf{Ex^n} + \mathbf{E_1} \begin{pmatrix} \mathbf{U_C^n} \\ \mathbf{I_L^n} \end{pmatrix} + \mathbf{Fv^n}, \tag{24}$$

E, $\mathbf{E_1}$ and **F** being matrices which contain transmitances.

From (8), taking into account (23) and (24), it follows the relation which allows the determination of the state vector evolution:

$$\mathbf{x^{n+1}} = \mathbf{Mx^n} + \mathbf{Nv^{n+1}} + \mathbf{N_1 v^n} \tag{25}$$

with

$$\mathbf{M} = \mathbf{1} + \mathbf{h} \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma} \end{pmatrix} (\mathbf{E} + \mathbf{E_1 K}) , \tag{26}$$

$$\mathbf{N} = \mathbf{h} \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma} \end{pmatrix} \mathbf{F} , \tag{27}$$

$$\mathbf{N_1} = \mathbf{h} \begin{pmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma} \end{pmatrix} \mathbf{E_1 K^1} , \tag{28}$$

$$\mathbf{K} = \begin{pmatrix} \mathbf{K_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_2} \end{pmatrix} \text{ and } \mathbf{K^1} = \begin{pmatrix} \mathbf{K_1^1} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_1^2} \end{pmatrix} \tag{29}$$

If $\mathbf{x_p}$ is a particular solution of the state equation, from (17) and (25) follows that

$$\mathbf{Nv^{n+1}} + \mathbf{N_1 v^n} = \mathbf{x_p^{n+1}} - \mathbf{Mx_p^n} , \tag{30}$$

equality which allows the transformation of (25) into the (21) form, therefore the common form for circuits with and without excess elements.

5. CONCLUSIONS

The proposed method deals with a minimal number of variables - the state variables - that leads to minimal dimensions of the matrices which appear in the computation process.

In comparison with analysis and calculation techniques based on the normal form of the state equation and on its solution, containing the transition matrix, the proposed method has substantial advantages, since it does not suppose matrix function or matrix function integral computation.

Essential and excess elements with memory modelling is extremely easy, and after the introduction of the models corresponding to the discrimination, the problem is reduced to the analysis of a resistive active circuit.

The discrimination step can be choose so that the solution can be optimally reached, concerning the computation time and accuracy.

Based on the method presented in this paper, an iterative computation method of the non-linear electrical networks can be elaborated.

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