THE EFFICIENT APPROACHES ON DOA ESTIMATION

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ABSTRACT

In this study, the efficient approaches based on the image processing techniques for the direction of arrival (DOA) estimation by Sirovich-Kirby method are proposed. ESPRIT (Estimation of Signal Parameters via Rotational Invariant Techniques) is one of the most popular methods for DOA estimation. Although the performance of this method is very reliable, the computational cost is considerably high. In this study, Sirovich-Kirby method is used in order to reduce this computational cost. This method is based on the use of low dimensional inner-product matrix instead of the high dimensional covariance matrix. In addition to that, if the signals have uniform directions, the the inner-product matrix becomes symmetric and Toeplitz matrix. In this case, the basis of discrete cosine transform (DCT) are the eigenvectors of inner-product matrix. Therefore DOA estimation of uniformly directed signals can be achieved by using DCT. Thus the more efficient computation can be obtained.

I. INTRODUCTION

In many signal processing applications, the estimation of unknown parameters of signals from the measurements is the main objective. For example, the direction of arrival estimation by using the signals obtained at the antenna outputs is very important in many sensor systems. There have been several approaches to such a problem [1]. MUSIC (MUltiple SIgnal Classification) and ESPRIT are widely used for DOA estimation. Although often successful, the computational cost is considerably high in these methods, since the dimension of covariance matrix used in DOA estimation is very high. The dimension of covariance matrix depends on the number of sensors. The use of many sensors provides the better DOA estimation.
DOA estimation techniques also find some applications in image processing problems. In [2], one of the most popular applications is presented. The motivation in our study is based on the approach presented in [2]. By considering this approach, we suggest that the image processing techniques can be used to reduce computational cost in ESPRIT and MUSIC and two approaches for DOA estimation problem are presented in this paper. In our first approach, the Sirovich-Kirby method used for face recognition by eigen-faces method proposed in [3] is used. This approach is based on the use of the low dimensional inner-product matrix instead of the high dimensional covariance matrix.
In addition to that if the image consists of the set of uniformly rotated templates, the corresponding inner-product matrix becomes symmetric and Toeplitz matrix [4], [5]. In this case, the basis of DCT are the eigenvectors of inner-product matrix. In our second approach, we use this property for DOA estimation problem under the constraint that the signals are uniformly directed. Thus the computational cost can be much reduced.

II. PROBLEM STATEMENT

Consider the M element sensor array and the d wavefronts, $s_k(t)$ $\{k = 1...d\}$, corrupted by additive sensor noise that is uncorrelated with the emitter signals and has zero mean. The output obtained at the M element sensor array, $\mathbf{z}(t) = [z_1(t), ..., z_M(t)]^T$, can be expressed as:

$$\mathbf{z}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where

$$\begin{aligned} \mathbf{A} &= [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_d)] \\ \mathbf{s}(t) &= [s_1(t), ..., s_d(t)]^T \\ \mathbf{n}(t) &= [n_1(t), ..., n_M(t)]^T \end{aligned} \tag{2}$$

and $\mathbf{A}$ is the steering matrix; $\mathbf{n}(t)$ is the white-noise vector. T represents transpose operator. $\mathbf{a}(\theta_k)$ is a steering vector and expressed as:

$$\mathbf{a}(\theta_k) = [1, e^{-j\frac{\omega\Delta}{c}\sin(\theta_k)}, \ldots, e^{-j\frac{\omega\Delta}{c}(M-1)\sin(\theta_k)}]^T$$

where c and Δ represent the propagation speed and the distance between sensor arrays, respectively.
The objective for DOA estimation problem is to estimate signal parameters, d and θ_k, from $\mathbf{z}(t)$.

III. ESPRIT

ESPRIT is the one of the most important methods proposed for DOA estimation problem presented in Section II. In this method, Eigen-Decomposition (ED) of the covariance matrix of $\mathbf{z}(t)$ is used. The covariance matrix of $\mathbf{z}(t)$ is obtained as:

$$\begin{aligned} \mathbf{R} &= E[\mathbf{z}(t)\mathbf{z}^H(t)] \\ &= \mathbf{A}E[\mathbf{s}(t)\mathbf{s}^H(t)]\mathbf{A}^H + E[\mathbf{n}(t)\mathbf{n}^H(t) \\ &= \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I}_M \end{aligned} \tag{3}$$

where $E[.]$ and H denote the expectation operator and Hermitian transpose, respectively. $\mathbf{S}$ is the covariance matrix; σ^2 is the noise variance; and $\mathbf{I}_M$ represents MxM identity matrix. In general, $\mathbf{R}$ is unknown. Therefore the approximated covariance matrix, $\hat{\mathbf{R}}$, which is obtained from sufficiently large number, say P, snapshots is used instead of $\mathbf{R}$; and its expression is given as:

$$\hat{\mathbf{R}} = \frac{1}{P}\sum_{t=1}^{P}\mathbf{z}(t)\mathbf{z}^H(t) \tag{4}$$

Firstly, the eigenvalues of $\hat{\mathbf{R}}$, λ_i, are obtained by ED. Then d is determined by k which minimizes the Minimum Description Length (MDL) function defined by

$$\begin{aligned} MDL(k) &= -\log\left\{\frac{\prod_{i=k+1}^{M}\lambda_i^{\frac{1}{M-k}}}{\frac{1}{M-k}\sum_{i=k+1}^{M}\lambda_i}\right\}^{(M-k)P} \\ &\quad +\frac{k}{2}(2M-k)\log P \end{aligned} \tag{5}$$

The eigenvalues are arranged in descending order of magnitude and $\mathbf{\Lambda_s}$ is defined as a diagonal matrix of the d largest eigenvalues; the corresponding eigenvectors are collected in an Mxd matrix $\bar{\mathbf{E}}_\mathbf{s}$. Then $\bar{\mathbf{E}}_\mathbf{1}$ and $\bar{\mathbf{E}}_\mathbf{2}$ are defined as the submatrices of $\bar{\mathbf{E}}_\mathbf{s}$ formed by rows $1:M-1$ and $2:M$, respectively. Next the ED of the $2d$x$2d$ matrix is formed as:

$$\begin{bmatrix} \bar{\mathbf{E}}_1^H \\ \bar{\mathbf{E}}_2^H \end{bmatrix}\begin{bmatrix} \bar{\mathbf{E}}_1 & \bar{\mathbf{E}}_2 \end{bmatrix} = \mathbf{F}\Lambda\mathbf{F}^H \tag{6}$$

Then $\mathbf{F}$ matrix is partitioned into dxd submatrices as follows:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \tag{7}$$

Finally the DOA's are obtained by using the eigenvectors of $-\mathbf{F}_{12}\mathbf{F}_{22}^{-1}$ as [1]:

$$\hat{\theta}_k = \sin^{-1}\left[\frac{c}{\omega\Delta}Im(\ln\frac{\lambda_k}{|\lambda_k|})\right] \tag{8}$$

M has to be a large number to minimize the estimation error in DOA estimation problem. In this case, to choose the covariance matrix whose the size is very large causes the high computational cost.

IV. DOA ESTIMATION BY SIROVICH-KIRBY METHOD

The disadvantage of using ESPRIT is the high computational cost as explained in Section III. The similar problem is met in image processing applications. One of these applications is the face-recognition by using eigen-faces. The problem in this application is to obtain basis images which span the space consist od the faces in the data base. It is well known that the best approximation is obtained by choosing the eigenvectors of covariance matrix as the basis images (Karhunen - Loeve transform). However the size of covariance matrix depends on the number of images used in data base. Therefore it is very large. In [3], this problem can be avoided by using the inner-product matrix instead of the covariance matrix. This approach is used in our study. First we consider the relationship between image model and sensor array model proposed in [2]. In the multi-line-fitting problem represented in [2], the signal, z_l, received at the sensor in front of the l.th row of the NxN image consisting d lines rotated in different angles is expressed as:

$$z_l = \sum_{k=1}^{d} a_l(\theta_k)s_k + n_l, \qquad l = 0,\ldots,N-1 \tag{9}$$

where $a_l(\theta_k) = e^{j\mu l \tan\theta_k}$, $s_k = e^{-j\mu x_{0k}}$, μ is a constant parameter, x_{0k} offset of k. line [2]. (9) can be arranged in vector form as:

$$\begin{aligned} \mathbf{z} &= \sum_{k=1}^{d}\mathbf{a}(\theta_k)s_k + \mathbf{n} \\ &= \mathbf{A}\mathbf{s} + \mathbf{n} \end{aligned} \tag{10}$$

which is exactly analogous to the array processing equation (1). The only difference between (1) and (10) is that there is no time-dependent measurement in (10). In this case, the question arises as to how a covariance matrix in (4) can be obtained. This problem can be overcome when we have only a single snapshot across a very large array; the measurement vector is divided into P subvectors of length M $(d < M < N-d+1)$. In this case, the covariance matrix for an NxN image is

$$\begin{aligned} \mathbf{R}_P &= \mathbf{Z}_P\mathbf{Z}_P^H \\ &= [\mathbf{z}_1,\ldots,\mathbf{z}_P][\mathbf{z}_1,\ldots,\mathbf{z}_P]^H \end{aligned} \tag{11}$$

$$[\mathbf{z}_1,\ldots,\mathbf{z}_P] = \begin{bmatrix} z_0 & z_1 & \cdots & z_{N-M} \\ z_1 & z_2 & \cdots & z_{N-M+1} \\ \vdots & \vdots & \cdots & \vdots \\ z_{M-1} & z_M & \cdots & z_{N-1} \end{bmatrix} \tag{12}$$

The eigenvector- eigenvalue equation for an MxM $\mathbf{R}_P$ matrix is written as:

$$\mathbf{R}_P\mathbf{v}_k = \lambda_k\mathbf{v}_k \tag{13}$$

Let consider the inner-product matrix, PxP $\bar{\mathbf{R}}_P = \mathbf{Z}_P^H\mathbf{Z}_P$, instead of the covariance matrix, $\mathbf{R}_P$, according to Sirovich-Kirby method. In this case, The eigenvector- eigenvalue equation for $\bar{\mathbf{R}}_P$ matrix is expressed as:

$$\begin{aligned} \bar{\mathbf{R}}_P\bar{\mathbf{v}}_k &= \bar{\lambda}_k\bar{\mathbf{v}}_k \\ \mathbf{Z}_P^H\mathbf{Z}_P\bar{\mathbf{v}}_k &= \bar{\lambda}_k\bar{\mathbf{v}}_k \end{aligned} \tag{14}$$

If the both sides of (14) are multiplied by $\mathbf{Z}_P$, the following equation is obtained:

$$\mathbf{Z}_P\mathbf{Z}_P^H\mathbf{Z}_P\bar{\mathbf{v}}_k = \bar{\lambda}_k\mathbf{Z}_P\bar{\mathbf{v}}_k \tag{15}$$

As can be seen from (15), the eigenvalues of $\mathbf{R}_P$ and $\bar{\mathbf{R}}_P$ are the same. The relationship between the eigenvectors of these matrices are given as:

$$\mathbf{v}_k = \mathbf{Z}_P \bar{\mathbf{v}}_k \tag{16}$$

Thus the ED of inner-product matrix whose the size, $(d < P < M)$, is less than the covariance matrix can be used instead of the ED of covariance matrix. After obtaining the eigenvalues of $\bar{\mathbf{R}}_P$ matrix, the number of signals is determined according to MDL test given in (5).The following procedure is the same as the procedure explained in Section III. The DOA's are obtained by using the following equation which is similar to (8) [2]:

$$\hat{\theta}_k = \tan^{-1}\left[\frac{1}{\mu\Delta} Im(\ln \frac{\lambda_k}{|\lambda_k|})\right] \tag{17}$$

where $|\mu| \leq \frac{\pi}{\Delta \tan \theta_k}$ [2].

IV. DOA ESTIMATION BY DCT

As mentioned in Section I., if the signals have uniform directions, the the inner-product matrix becomes symmetric and Toeplitz matrix. Therefore $\bar{\mathbf{R}}_P$ in (14) becomes in the following form:

$$\bar{\mathbf{R}}_P = \begin{bmatrix} q_0 & q_1 & \cdots & q_{p-1} \\ q_{p-1} & q_0 & \cdots & q_{p-2} \\ \vdots & \vdots & \cdots & \vdots \\ q_1 & q_2 & \cdots & q_0 \end{bmatrix} \tag{18}$$

Since $\bar{\mathbf{R}}_P$ is symmetric and Toeplitz, $q_k = q_{p-k}$. Therefore the equation (14) can be written as:

$$\sum_{l=0}^{m-1} q_{p-m+l}\bar{\mathbf{v}}_{\mathbf{kl}} + \sum_{l-m}^{p-l} q_{l-m}\bar{\mathbf{v}}_{\mathbf{kl}} = \bar{\lambda}_k \bar{\mathbf{v}}_{\mathbf{km}} \tag{19}$$

By solving (19) for $m = 0, 1 \cdots, p-1$, the following result is obtained:

$$\bar{\mathbf{v}}_{\mathbf{km}} = \cos\left(2\pi m \left(\frac{k}{p}\right)\right) \tag{20}$$

As can be seen in (20), the basis of discrete cosine transform (DCT) are the eigenvectors of inner-product matrix. The relationship between the eigenvectors of inner-product matrix, $\bar{\mathbf{R}}_{\mathbf{P}}$, and the covariance matrix, $\mathbf{R}_{\mathbf{P}}$, is given in (16).

As a conclusion, the DOA estimation of uniformly directed signals can be achieved by using DCT. Thus the more efficient computation can be obtained.

IV. CONCLUSION

In this study, the new approaches which is based on the Sirovich-Kirby method and the use of DCT for DOA estimation problem are presented. These approaches provide an efficient DOA estimation, since they are based on the use of the inner-product matrix instead of the covariance matrix. We use the assumption that the directions of the signals are uniform for the use of DCT. In Table 1., the estimated directions for the signals which have the directions 0^o, 30^o and 60^o are presented for different Signal-to-Noise Ratios (SNR). As noted in this table, the estimated directions obtained by using our approaches are very closed to the actual values especially for high SNRs.

SNR	0dB	10dB	20 dB	30 dB
ESPRIT	0.0047 29.9995 60.004	0.6037 29.0607 60.0577	0.3825 28.3262 60.4046	-0.006 30.0028 59.9845
Sirovich-Kirby	-1.3556 38.7421 53.3235	-0.1770 39.8666 48.7819	-0.0522 30.1415 59.8955	-0.0009 30.0010 59.9884
DCT	-0.0148 30.0011 60.2806	0.0056 29.9940 60.0067	-0.0004 30.0025 60.0016	-0.0001 30.0002 60.0003

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