Solution to Security Constrained Lossy Economic Power Dispatch Problem for a Power System Area Including Limited Energy Supply Thermal Units Using Modified Subgradient Algorithm Based on Feasible Values

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Abstract

In this paper, a security constrained power dispatch problem for a lossy electric power system area including limited energy supply thermal units is modeled in such a way that the modified subgradient algorithm based on feasible values (F-MSG algorithm) can be used to solve it. This model considers bus voltage magnitudes and phase angles, off-nominal tap ratios (once there are off-nominal tap changing transformers in the power system) as independent variables. Load flow equations are added to the model as equality constraints. Power system transmission loss is inserted into the optimization model via those load flow equations. Unit generation constraints, transmission line capacity constraints, bus voltage magnitude constraints, off-nominal tap ratio constraints are added into the optimization problem as inequality constraints. We assume that limited energy supply thermal units are fueled under *take-or-pay* agreement.

The F-MSG algorithm is tested on a fifteen-bus test system. The dispatch problem was also solved by other dispatch techniques that use pseudo spot price algorithm and genetic algorithm. Results obtained from the F-MSG algorithm and the other techniques are compared.

1. Introduction

A specific operation period of a lossy electric power system including limited energy supply thermal units is considered in this paper. During the operation period, system load values and the units that will supply those loads are assumed to be known. The total operation period is divided into subintervals where the system load values remain constant. The minimum value of the total fuel cost for the operation period is determined under some possible electric and fuel constraints.

Under *take-or-pay* (T-O-P) fuel contract, a minimum value of the total fuel amount to be spent by the limited energy supply thermal units during the operation period is determined in advance. The utility company agrees to use at least this minimum amount. If it fails to use the minimum amount, it agrees to pay the cost of the minimum amount [1].

In the literature, the economic dispatch problem for a power system area including limited energy supply thermal units was solved by various solution methods. Some of these methods use the pseudo spot price algorithm (PSPA) [2], the evolutionary programming [3], the Hopfield neural networks [4].

The F-MSG method is a deterministic solution method, which uses deterministic equations at one point to produce the next solution point being closer to the optimum solution in the solution space; whereas the evolutionary methods work on a solution population rather than on a single solution and uses probabilistic tools to produce new solutions. In general, solution times for the evolutionary methods are comparably high with respect to those of deterministic methods for the lossy security constrained economic dispatch problems with convex cost curves.

In the F-MSG algorithm [5], the upper bound for the cost function value is specified in advance and the algorithm tries to find a solution where the cost function is *less than or equal to* the upper bound and all constraints are satisfied. If it finds it (*feasible total cost*), the upper bound is *decreased* a certain amount, otherwise (*infeasible total cost*) the upper bound is *increased* a certain amount. The amount of decrease or increase on the upper bound for the next iteration depends on if any feasible or infeasible total cost value was obtained in the previous iterations. This process continues until absolute value of the change in the upper bound is less than a predefined tolerance value.

2. Problem formulation

In this section, a nonlinear programming model is presented for the economic power dispatch problem considered in this paper.

$$\text{Min } F_{TOT} = \sum_{j=1}^{j_{max}} \left(\sum_{s \in N_s} F_s(P_{Gs,j}) + \sum_{T \in N_T} F_T(P_{GT,j}) \right) t_j \tag{1}$$

Subject to

$$\begin{gathered} P_{Gi,j} - P_{Load\ i,j} - \sum_{k \in N_{Bi}} p_{ik,j} = 0 \\ Q_{Gi,j} - Q_{Load\ i,j} - \sum_{k \in N_{Bi}} q_{ik,j} = 0,\ i = 1,2,..,N,\ j = 1,2,..,j_{max} \end{gathered} \tag{2}$$

$$C_{spent} - C_{tot} = 0, \quad C_{spent} = \sum_{j=1}^{j_{max}} \sum_{T \in \boldsymbol{N_T}} C_T(P_{GT,j})\, t_j \tag{3}$$

$$\begin{gathered} P_{Gs}^{min} \le P_{Gs,j} \le P_{Gs}^{max}, \quad Q_{Gs}^{min} \le Q_{Gs,j} \le Q_{Gs}^{max} \\ s \in N_s,\ \ j = 1,2,\cdots,j_{max} \end{gathered} \tag{4}$$

$$\begin{gathered} P_{GT}^{min} \le P_{GT,j} \le P_{GT}^{max}, \quad Q_{GT}^{min} \le Q_{GT,j} \le Q_{GT}^{max} \\ T \in N_T,\ \ j = 1,2,\cdots,j_{max} \end{gathered} \tag{5}$$

$$p_{l,j} \le p_l^{max}, \qquad l \in L,\ \ j = 1,2,\cdots,j_{max} \tag{6}$$

$$U_i^{min} \le U_{i,j} \le U_i^{max}, i=1,2,\cdots,N, i \ne ref, vc, \ j=1,\cdots,j_{max} \quad (7)$$

$$a_i^{min} \le a_{i,j} \le a_i^{max}, i \in N_{tap}, \ \ j=1,\cdots,j_{max} \quad (8)$$

The meanings of the symbols used in this paper are given in the list of symbols section.

2.1. Determination of Line Flows and Power Generations

In order to express the total cost function in terms of independent variables of our optimization model, line flows should be written in terms of bus voltage magnitudes and phase angles and off-nominal tap ratios (see equations (1) and (2)). The following equations give the active and reactive power flows over the line being connected between buses i and k in the j^{th} subinterval.

$$p_{ik,j} = U_{i,j}^2\left(\frac{g_{ik}}{a_{i,j}^2} + g_{shi}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ik}\cos(\delta_{i,j}-\delta_{k,j}) + b_{ik}\sin(\delta_{i,j}-\delta_{k,j})\right] \quad (9)$$

$$p_{ki,j} = U_{k,j}^2\left(g_{ik} + g_{sh\,k}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ik}\cos(\delta_{k,j}-\delta_{i,j}) + b_{ik}\sin(\delta_{k,j}-\delta_{i,j})\right] \quad (10)$$

$$q_{ik,j} = -U_{i,j}^2\left(\frac{b_{ik}}{a_{i,j}^2} + b_{sh\,i}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ik}\sin(\delta_{i,j}-\delta_{k,j}) - b_{ik}\cos(\delta_{i,j}-\delta_{k,j})\right] \quad (11)$$

$$q_{ki,j} = -U_{k,j}^2\left(b_{ik} + b_{sh\,i}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ij}\sin(\delta_{k,j}-\delta_{i,j}) - b_{ij}\sin(\delta_{k,j}-\delta_{i,j})\right] \quad (12)$$

In the above equations, $U_{i,j}$ and $\delta_{i,j}$ are voltage magnitude and phase angle of bus i in the j^{th} subinterval, respectively, $r_{ik} + jx_{ik}$ is the series impedance of the line between buses i and k, $g_{ik} + jb_{ik}$ is the *series* admittance of the line between buses i and k where $g_{ik} + jb_{ik} = 1/(r_{ik} + jx_{ik})$, $g_{shi} + jb_{shi}$ is the sum of the half line charging admittance and external shunt admittance if any at bus i, and $a_{i,j}$ is the off-nominal tap setting in the j^{th} subinterval with tap setting facility at bus i. $p_{ik,j}$ and $q_{ik,j}$ are the active and reactive power flows going from bus i to bus k at bus i border in the j^{th} subinterval, respectively. $-p_{ki,j}$ and $-q_{ki,j}$ are the active and reactive power flows going from bus i to bus k at bus k border in the j^{th} subinterval, respectively.

The total loss of the network in the j^{th} subinterval can be calculated by the following equations:

$$p_{loss\,ik,j} = p_{ik,j} + p_{ki,j} \quad (13)$$

$$P_{LOSS,j} = \sum_{i\in N}\sum_{k\in N, k\ne i} p_{ik,j}, \quad j=1,2,\cdots,j_{max}. \quad (14)$$

The cost rate function value of the i^{th} unit in the j^{th} subinterval is taken as

$$F_i(P_{Gi,j}) = b_i + c_i P_{Gi,j} + d_i P_{Gi,j}^2, i \in \{N_s, N_T\}, \ j=1,2,\cdots,j_{max} \quad (15)$$

where b_i, c_i, and d_i are constant coefficients. The total cost is also determined as:

$$F_{TOT} = \sum_{j=1}^{j_{max}} \sum_{i\in\{N_s,N_T\}} F_i(P_{Gi,j})\,t_j \qquad (R). \quad (16)$$

2.2. Converting Inequality Constraints into Equality Constraints

Since the F-MSG algorithm requires that all constraints need to be expressed in equality constraint form, the inequality constraints in the optimization model should be converted into the corresponding equality constraints. The following method is used for this purpose, since it does not add any extra independent variable into the optimization model in the conversion process [6]. The double sided inequality $x_i^- \le x_{i,j} \le x_i^+$ in the j^{th} subinterval can be written as the following two inequalities:

$$\begin{aligned} h_{i,j}^+(x_{i,j}) &= (x_{i,j} - x_i^+) \le 0 \\ h_{i,j}^-(x_{i,j}) &= (x_i^- - x_{i,j}) \le 0, \quad j=1,2,\cdots j_{max} \end{aligned} \quad (17)$$

Then, we can rewrite the above inequalities as continuous equality forms by the following:

$$\begin{aligned} h_{i,j}^{eq+}(x_{i,j}) &= \max\left\{0, \ \left(x_{i,j} - x_i^+\right)\right\} \\ h_{i,j}^{eq-}(x_{i,j}) &= \max\left\{0, \ \left(x_i^- - x_{i,j}\right)\right\}, \quad j=1,2,\cdots j_{max} \end{aligned} \quad (18)$$

If $x_i^- \le x_{i,j} \le x_i^+$, it is obvious that $(x_{i,j} - x_i^+) \le 0$, $(x_i^- - x_{i,j}) \le 0$ and, $\max\left\{0, \ \left(x_{i,j} - x_i^+\right)\right\} = 0$, $\max\left\{0, \ \left(x_i^- - x_{i,j}\right)\right\} = 0$. So the inequality constraints in (17) can be represented by the corresponding equality constraints in (18). In this paper the inequality constraints, given in equations (4)-(8), are converted into the corresponding equality constraints in this manner.

3. The Modified Subgradient Algorithm Based on Feasible Values

The nonlinear optimization problem described by equations (1)-(18) can be represented in the standard form given below:

$$\text{Min } F(\boldsymbol{x})$$

$$\textit{Subject to} \begin{cases} \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\ \boldsymbol{x} \in K \end{cases} \quad (19)$$

where $\boldsymbol{x} = [U_{1,1},\cdots,U_{1,j_{max}},\cdots\cdots,U_{N,1},\cdots,U_{N,j_{max}},\delta_{1,1},\cdots,\delta_{1,j_{max}},\cdots\cdots,\delta_{N,1},\cdots,\delta_{N,j_{max}},\cdots,a_{1,1},\cdots,a_{1,j_{max}},\cdots\cdots,a_{N_{tap},1},\cdots,a_{N_{tap},j_{max}}]$ is the independent variable vector. $F(\boldsymbol{x})$ is the objective function which is given in equation (16), and $\boldsymbol{h}(\boldsymbol{x}) = \left[h_1(\boldsymbol{x}), h_2(\boldsymbol{x}),\cdots\cdots,h_{N_{EQ}}(\boldsymbol{x})\right]$ is the equality constraint vector. It includes all the original equality constraints, which are given in (2)-(3), and the equality constraints, which are obtained from converting all the inequality constraints given in (4) to (8) into the corresponding equality constraints via the method given in Section 2.2 [6]. Note that inequality constraints can also be converted into equality ones using any other approach. K is a sufficiently large compact set containing the potential values of $\boldsymbol{x}$. Region K is bounded by the upper and the lower limits of the voltage magnitudes of the buses and the upper and the lower limits of the tap settings of the transformers, which are given in (7)-(8). Note that the voltage magnitude and phase angle of the reference bus, ($U_{ref,j}$, $\delta_{ref,j}$, $j=1,\cdots,j_{max}$), *are not* included into $\boldsymbol{x}$ since they are not independent variables and remain constant during the solution process. In solving the constrained optimization problem given by equation (19), the first step is to convert it into unconstrained one by constructing the dual problem. This can be done by using various LaGrange functions [7]. LaGrange function must guarantee that the optimal solution of the dual problem be equal to that of the primal constrained problem. Otherwise, there will be a difference between the optimal values of these problems; in other words, a duality gap will occur. The classical LaGrange function guarantees the zero duality gaps for the convex problems. However, if the objective function or some of the constraints are not convex, then the classical LaGrange function cannot guarantee this. Therefore, for the non-convex problems, suitably selected augmented LaGrange functions should be used. Considering the non-convex nature of our problem, we form the dual problem by using the following sharp augmented LaGrange function [8]:

$$\begin{aligned} L(\boldsymbol{x},\boldsymbol{u},c) &= F(\boldsymbol{x}) + c\left\|\boldsymbol{h}(\boldsymbol{x})\right\| - \langle \boldsymbol{u},\boldsymbol{h}(\boldsymbol{x})\rangle \\ &= F(\boldsymbol{x}) + c\left(\left[h_1(\boldsymbol{x})\right]^2 + \left[h_2(\boldsymbol{x})\right]^2 + \cdots + \left[h_{N_{EQ}}(\boldsymbol{x})\right]^2\right)^{1/2} \\ &\quad - \left(u_1 h_1(\boldsymbol{x}) + u_2 h_2(\boldsymbol{x}) + \cdots + u_{N_{EQ}} h_{N_{EQ}}(\boldsymbol{x})\right) \end{aligned} \tag{20}$$

where $u_1, u_2, \cdots, u_{N_{EQ}} \in R$ and $c \ge 0$ are LaGrange multipliers (dual variables). The dual function associated with the constrained problem is defined as

$$H(\boldsymbol{u},c) = \min_{\boldsymbol{x}\in K} \; L(\boldsymbol{x},\boldsymbol{u},c). \tag{21}$$

Then, the dual problem is given by

$$\max_{(\boldsymbol{u},c)\in R^{N_{EQ}}\times R_+} \; H(\boldsymbol{u},c) \tag{22}$$

For the given dual problem, the conditions of guaranteeing zero duality gaps are proven in [8].

3.1. The F-MSG Algorithm

Initialization Step: Select arbitrary active and reactive power generations for all subintervals. Then, perform AC power flow calculations with the corresponding selected active and reactive power generation values in all subintervals to obtain the initial values for the voltage magnitudes and phase angles of the buses in all subintervals. Calculate the initial total cost F_{TOT}.

Step 1) Choose positive numbers $\varepsilon_1, \varepsilon_2, \Delta_1$ and M (upper bound for m). Set $n=1$, $p=0$, $q=0$, and $H_n = F_{TOT}$.

Step 2) Choose $(\boldsymbol{u}_1^n, c_1^n) \in R^{N_{EQ}} \times R_+$ and $\ell(1) > 0$ and set $m=1$, $\boldsymbol{u}_m = \boldsymbol{u}_1^n$, $c_m = c_1^n$,

Step 3) Given $(\boldsymbol{u}_m, c_m)$, solve the following constraint satisfaction problem (CSP)

$$\begin{aligned} &\textit{Find a solution } \boldsymbol{x}_m \in K \textit{ such that} \\ &F(\boldsymbol{x}_m) + c_m\left\|\boldsymbol{h}(\boldsymbol{x}_m)\right\| - \langle \boldsymbol{u}_m, \boldsymbol{h}(\boldsymbol{x}_m)\rangle \le H_n \end{aligned} \tag{23}$$

If a solution to (23) does not exist or $\ell(m) > M$, then go to Step 6; otherwise, if a solution $\boldsymbol{x}_m$ exists then check whether $\boldsymbol{h}(\boldsymbol{x}_m) = \boldsymbol{0}$. If $\boldsymbol{h}(\boldsymbol{x}_m) = \boldsymbol{0}$ (or if $\left\|\boldsymbol{h}(\boldsymbol{x}_m)\right\| \le \varepsilon_1$) then go to step5, otherwise go to step 4.

Step 4). Update dual variables as

$$\boldsymbol{u}_{m+1} = \boldsymbol{u}_m - \alpha\, s_m \boldsymbol{h}(\boldsymbol{x}_m) \tag{24}$$

$$c_{m+1} = c_m + (1+\alpha)\, s_m \left\|\boldsymbol{h}(\boldsymbol{x}_m)\right\| \tag{25}$$

where s_m is a positive step size parameter defined as

$$0 < s_m = \frac{\lambda\,\alpha\left(H_n - L(\boldsymbol{x}_m, \boldsymbol{u}_m, c_m)\right)}{\left[\alpha^2 + (1+\alpha)^2\right]\left\|\boldsymbol{h}(\boldsymbol{x}_m)\right\|^2} \tag{26}$$

where α and λ are constant parameters with $\alpha > 0$ and $0 < \lambda < 2$. Step size s_m corresponding to the dual variables $(\boldsymbol{u}_m, c_m)$ should also satisfy the following property:

$$\left(s_m \left\|\boldsymbol{h}(\boldsymbol{x}_m)\right\| + c_m - \left\|\boldsymbol{u}_m\right\|\right) > \ell(m). \tag{27}$$

Set $m = m+1$, update $\ell(m)$ in such a way that $\ell(m) \to +\infty$ as $m \to +\infty$, and go to step 3.

Step 5) If $p = 0$, it means that any infeasible total cost rate value has not been chosen yet, then set $\Delta_{n+1} = \Delta_n$, otherwise set $\Delta_{n+1} = (1/2)\Delta_n$. If $\Delta_{n+1} < \varepsilon_2$, then stop, $\boldsymbol{x}_m$ is an approximate optimal primal solution, and $(\boldsymbol{u}_m, c_m)$ is an approximate dual solution; otherwise set $H_{n+1} = \min\left\{F(\boldsymbol{x}_m),\ H_n - \Delta_{n+1}\right\}$, $q = q+1$, $n = n+1$, and go to step 2.

Step 6) If $q = 0$, it means that any feasible cost rate value has not been chosen yet, then set $\Delta_{n+1} = \Delta_n$; otherwise, set $\Delta_{n+1} = (1/2)\Delta_n$. If $\Delta_{n+1} < \varepsilon_2$ then stop, and in this case, the last calculated feasible $\boldsymbol{x}_m$ is an approximate optimal primal solution, and $(\boldsymbol{u}_m, c_m)$ is an approximate dual solution; otherwise, set $H_{n+1} = H_n + \Delta_{n+1}$, $p = p+1$, $n = n+1$ and go to step-2.

In this algorithm, steps 3 and 4 can be considered as the inner loop, and steps 2, 5 and 6 can be considered as the outer loop. We call any outer loop, in which a feasible cost rate value is generated by the algorithm, as a *feasible state*, n_f. The following problem is solved by using GAMS® solver:

$$\begin{aligned} &\textit{Minimize} \quad f = 0 \\ &\textit{Subject to} \begin{cases} L(\boldsymbol{x},\boldsymbol{u},c) - H_n \le 0 \\ \boldsymbol{x} \in K \end{cases} \end{aligned} \tag{28}$$

where f is a 'fictitious' objective function which is identically zero, or can be taken as any constant value [5].

The way of updating the dual variables $(\boldsymbol{u}_m, c_m)$ in step 4 will force the solution in Step 3 to converge to the feasible solution (see Theorems in [5]).

4. Numeric Example

The proposed dispatch technique was tested on a fifteen-bus test system. Please refer to reference [2] for all necessary data for the test system. The initial parameters, explained in section 3.1, are chosen as $\alpha = 100$, $\lambda = 1$, $\varepsilon_1 = 1\times10^{-4}$, $\Delta_1 = 2000\ R$, $M = 5000$, $c_1^1 = 10$ and $\boldsymbol{u}_1^1 = [0,0,...0,0]_{(1\times511)}$. Also we chose the function $\ell(m)$ as $\ell(m) = m$. The maximum active power transmission capacity limit for all transmission lines is taken as 100 *MW*. The common reactive power generation limits of all units are taken as $Q_{Gi}^{max} = 2.5\ pu$, $Q_{Gi}^{min} = -1.0\ pu$, $i \in \{N_S, N_T\}$. The simulation program was coded in MATLAB.

The dispatch problem considered in this paper is previously solved by using the PSPA [2] and genetic algorithm [9]. The selected *pu* initial active and reactive generations in each subinterval are given in Table 1. The initial bus voltage magnitudes and phase angles in each subinterval are found by performing load flow solutions with the selected active and reactive power generations. No more load flow calculation is carried out in the subsequent stages of the solution technique. In the following three cases, the same dispatch problem is solved by using the F-MSG algorithm.

4.1. Case 1: The Fuel Constraint is Not Considered

To show the effect of T-O-P fuel contract, first we solved the dispatch problem with the assumption that the fuel constraint does not exist. Therefore, we did not consider the fuel constraint in equation (3) and we applied the F-MSG algorithm to the dispatch problem with the calculated initial bus voltage magnitudes and phase angles. In 20 outer loops and 15 feasible states, the solution point is reached. The consumed gas amount and the total cost are found as $C_{spent} = 17494.9543$ *ccf* and, $F_T = 182572.7874 + 2\times50000 = 282572.7874\ (R)$ When the same problem is solved by means of the PSPA, the consumed gas amount by the limited energy supply thermal units and the total cost were found to be $C_{spent} = 17497.069\ ccf$ and $F_{TOT} = 185638.566 + 2\times50000 = 285638.566\ R$. Also, from the solution, which is produced by the method based on genetic algorithm, the consumed gas amount by the limited energy supply thermal units and the total cost were found to be $C_{spent} = 17706.323\ ccf$ and $F_T = 184806.204 + 2.0\times50000$ $= 284806.204\ R$. We see from the figures given in the above that the solution technique based on the ***F-MSG method gives the lowest total cost*** when the fuel constraint is not considered.

4.1. Case 2: The Fuel Constraint is Considered

In this case, the fuel constraint is added into the dispatch problem and it is solved by means of the F-MSG algorithm by using the initial bus voltage magnitudes and phase angles. The amount of gas, spent by the limited energy supply thermal units at the initial point, is found to be $C_{spent} = 44437.0011$ *ccf*. Therefore, the initial total cost value is calculated as $F_{TOT}^0 = 152014.0190 + 2\times50000 = 252014.019\ R$. In 18 outer loops and 7 feasible states the solution point total cost is found. The solution-point active and reactive power generations for the current case are given in Table 2. By using the active power generations in Table 2, the total consumed gas amount and the cost are calculated as $C_{spent} = 50000.0041$ *ccf* and $F_{TOT} = 243826.4140\ R$, respectively. The same dispatch problem

Table 1. Selected initial *pu* generations, ($S_{base} = 100\ MVA$ **).**

	Time interval number, (*j*)					
	1	2	3	4	5	6
$P_{G1,j}$	1.303	1.040	1.282	1.625	2.032	2.567
$Q_{G1,j}$	0.610	0.724	0.763	0.760	0.600	0.500
$P_{G3,j}$	1.100	1.200	1.200	1.200	1.200	1.200
$Q_{G3,j}$	0.375	0.445	0.575	0.680	0.620	0.640
$P_{G8,j}$	1.100	1.200	1.200	1.200	1.200	1.200
$Q_{G8,j}$	2.185	2.225	2.370	2.475	2.410	2.400
$P_{G10,j}$	1.000	1.100	1.200	1.200	1.200	1.200
$Q_{G10,j}$	-0.145	-0.135	-0.116	-0.082	-0.093	-0.096
$P_{G11,j}$	1.000	1.100	1.100	1.200	1.200	1.100
$Q_{G11,j}$	0.180	0.210	0.220	0.205	0.210	0.257
$P_{G12,j}$	1.000	1.100	1.200	1.200	1.200	1.200
$Q_{G12,j}$	0.720	0.797	0.817	0.857	0.851	0.875
$P_{G14,j}$	1.000	1.200	1.200	1.200	1.200	1.200
$Q_{G14,j}$	0.590	0.655	0.723	0.780	0.747	0.788
$P_{LOSS,j}$	0.303	0.340	0.382	0.425	0.432	0.467

was also solved by means of the PSPA [2] and, the genetic algorithm [9]. From the solution by the PSPA, the total consumed gas and the total cost were found to be $C_{spent} = 50018.8\ ccf$ and $F_{TOT} = 244669.0\ R$. The same values from the solution by the genetic algorithm were obtained as $C_{spent} = 49999.747\ ccf$ and $F_{TOT} = 244898.621\ R$. It is seen from the presented figures that ***the F-MSG algorithm gives the lowest total cost and the most accurate gas consumption values***.

4.3. Case-3: Active Power Generation and Transmission Line Constraints are Hit in the Solution-point

In this section, in addition to consideration of the fuel constraint, P_{G3}^{max} and p_{12}^{max} are taken as equal to 145 *MW* and 75 *MW*, respectively just to create both a generation and a transmission line constraint hits in the solution-point since $P_{G3,6}$ and $p_{12,6}$ were found to be 147.7100 *MW* and 80.2611 *MW*, respectively in the solution-point of case-2. We used the same initial complex bus voltages and, in eighteen outer loops and four feasible states; the total active generation cost is converged to $F_{TOT} = 244385.2371\ R$. The total gas consumption of the limited energy supply thermal units is found to be $C_{spent} = 49999.9867$ *ccf*. P_{G2} and p_{12} are obtained as 145.0024 *MW* and 75.0184 *MW* which are very close to P_{G2}^{max} and p_{12}^{max} , respectively

5. Discussion and Conclusion

In this paper, we propose a security constrained power dispatch technique using the F-MSG algorithm for a power system area including limited energy supply thermal units. The dispatch technique is tested on a fifteen-bus test system, which was solved by means of the PSPA and the genetic algorithm

Table 2. Optimal *pu* generations for *case-2*.

	Time interval number, (j)					
	1	2	3	4	5	6
$P_{G1,j}$	1.51144	1.29622	1.49889	1.78941	2.15239	2.63281
$Q_{G1,j}$	0.62294	0.73012	0.76802	0.76619	0.60944	0.51872
$P_{G3,j}$	1.14911	1.24392	1.31685	1.37339	1.41781	1.47710
$Q_{G3,j}$	0.81219	0.89307	1.03806	1.15535	1.09156	1.09483
$P_{G8,j}$	0.46178	0.48031	0.49425	0.51190	0.52046	0.53177
$Q_{G8,j}$	1.25221	1.28552	1.40387	1.50582	1.43794	1.40916
$P_{G10,j}$	0.61910	0.70314	0.75200	0.76510	0.75940	0.74693
$Q_{G10,j}$	0.21325	0.22393	0.25320	0.29095	0.27676	0.27157
$P_{G11,j}$	1.18974	1.28971	1.30407	1.32115	1.31274	1.27014
$Q_{G11,j}$	0.19438	0.22803	0.23522	0.22218	0.22982	0.27534
$P_{G12,j}$	1.35401	1.55035	1.60204	1.62530	1.62189	1.57880
$Q_{G12,j}$	0.67818	0.76883	0.79669	0.83031	0.82041	0.83522
$P_{G14,j}$	1.20704	1.36859	1.40540	1.42784	1.43316	1.41105
$Q_{G14,j}$	0.7167	0.78767	0.854413	0.908547	0.865878	0.907311
$P_{LOSS,j}$	0.2922	0.3323	0.3735	0.4142	0.4178	0.4487

previously. Among the results obtained from the above techniques, the proposed technique provides *the lowest total cost and the most accurate gas consumption values*. The fuel constraint, which can take place, due to T-O-P fuel agreement can also be handled by using the effect of the *scaling factor* on the total fuel consumption by the limited energy supply thermal units [2]. Usage of the scaling factor *decreases the number of independent variables* that is used in the solution of considered dispatch problem as well. We are currently performing research on application of the F-MSG method to some other economic power dispatch problems with *non-convex total cost curves.*

To our knowledge, the proposed solution technique has not been applied to the problem considered in this paper.

6. List of Symbols

R : a fictitious monetary unit

N : number of buses in the network.

N_T , N_S : sets that contain all limited energy supply thermal and normal thermal units in the network, respectively.

N_{Bi} : set that contains all buses *directly* connected to bus *i*.

N_{tap} , L : sets that contains all tap changing transformers and lines in the network, respectively.

t_j : length of time interval *j*, (*h*).

$p_{l,j}$: active power flow on line *l* in the j^{th} subinterval, (*pu* or *MW*).

$P_{Gi,j}$, $Q_{Gi,j}$: active and reactive power generations of the i^{th} unit in the j^{th} subinterval, respectively, (*pu* or *MW, MVar*).

$P_{Load\,i,j}$, $Q_{Load\,i,j}$: active/reactive loads of the i^{th} bus in the j^{th} subinterval, respectively ,(*pu* or *MW, MVar*).

$P_{LOSS,j}$: total active loss in the j^{th} subinterval, (*pu* or *MW*).

$C_T(P_{GT,j})$: fuel consumption rate for the T^{th} limited energy supply thermal unit in the j^{th} subinterval, *(ton/h, ccf/h, etc.)*.

C_{tot} : minimum total fuel amount that should be spent by all limited energy supply thermal units during the operation period according to T-O-P fuel contract *(ton, m³, ccf, etc.)*.

C_{spent} : amount of the total gas spent by the all limited energy supply thermal units during the operation period, *(ton, m³, ccf, etc)*, ($1ccf = 1000\,ft^3 = 27.317\,m^3$)

$P_{Gi}^{min}, Q_{Gi}^{min}; P_{Gi}^{max}, Q_{Gi}^{max}$: lower/upper active/reactive generation limits of the i^{th} generation unit, respectively, $i \in \{N_S, N_T\}$, (*pu* or *MW, MVar*).

p_l^{max} : maximum active transmission capacity of transmission line l , (*pu* or *MW*).

N_{EQ} , N_{VAR} : number of equality constraints and independent variables, respectively

$\boldsymbol{x}_m^n$: independent variable vector obtained at the m^{th} iteration of the inner loop of the n^{th} outer loop iteration.

$\boldsymbol{u}_m^n$, c_m^n : dual variables calculated at the m^{th} iteration of the inner loop of the n^{th} iteration of the outer loop.

s_m : positive step size parameter calculated at the m^{th} iteration of the inner loop.

F_T^n : total cost value which will be checked in the n^{th} outer loop, (*R*).

Δ_{n+1} : decrement/increment on F_n value, at the end of n^{th} outer loop iteration, according to whether F_n is feasible or not, (*R*).

ε_1, ε_2 : tolerance values for $\|h(\boldsymbol{x})\|$ and Δ_n , respectively.

7. References

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