THE SPECTRUM EFFICIENCY FOR OPTIMUM COMBINING OF SIGNALS WITH MULTIPLE INTERFERERS AND THERMAL NOISE

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***Abstract:** In this paper we derive the exact bit error rate (BER) for coherent and noncoherent detection of BPSK and BFSK signals with optimum combining in the presence of multiple uncorrelated equal power co-channel interferers and thermal noise in a Rayleigh fading environment.These BER expressions are used to evaluate the spectrum efficiency of cellular systems with adaptive antennas. The influence of various system parameters such as modulation, carried traffic per channel (ac), SNR, cluster size, the number of interferers and antenna elements on the spectrum efficiency and the system performance are analyzed and discussed.*

I. INTRODUCTION

Cellular mobile radio networks capitalize on the concept of frequency reuse to improve the overall spectrum efficiency. However, this introduces co-channel interference (CCI), which ultimately limits the quality of service offered to the users. This trade-off between the spectral efficiency and quality of service has been extensively studied. Recent developments in the area of adaptive antennas has enabled the mitigation of CCI, thereby improving the communication link quality without compromising spectrum efficiency. Adaptive arrays can significantly improve the performance of wireless communication systems by weighting and combining the received signals to reduce fading effects and suppress interference. In particular, with optimum combining the received signals are weighted and combined to maximize the output signal-to-interference-plus-noise power ratio (SINR). This technique provides substantial improvement in performance over maximal ratio combining when interference is present. The performance of an adaptive antenna in the presence of interference and thermal noise was investigated by some writers, but the approximate analitycal results are for the case of single interferer only. For the case of multiple interferers, mostly Monte Carlo simulations were used except some papers with limitations [1-3]. In this paper, starting from the eigenvalues distribution of complex wishart matrices, we first give the exact expression of the bit error probability for coherent and noncoherent detection of BPSK and BFSK using optimum combining in the presence of multiple uncorrelated equal-power interferers, as well as thermal noise, in a Rayleigh fading

II. SYSTEM MODEL

We consider coherent demodulation with optimum combining of multiple received signals in aflat fading environment. The fading rate is assumed to be much slower than the symbol rate. The received signal with vector notation with equal power interferers can be written as;

$$\bar{r} = \sqrt{P_{0d}}\ \bar{u}_0 + \sqrt{P_{ou}}\ \sum_{n=1}^{N} \bar{u}_n + \bar{n}_0 \qquad (1)$$

where $\bar{u}_n = [u_{n1}\ u_{n2} \cdots u_{n\mu}]^T$ is the propagation vector where u_{ni} is the nth interferer signal at the ith branch and complex gaussian random variable (CGRV) with zero mean and unit variance. P_{0d} and P_{ou} are the power of the desired signal and interferers, μ and N denote number of branches and interferers. The random varible (r.v.) $\bar{\gamma}$, which maximizes SINR is given by;

$$SINR_{max} = \bar{\gamma} = P_{od}\,\bar{u}_0^H R^{-1} \bar{u}_0 \quad where \quad R = P_{0u} G + N_0 \Im \qquad (2)$$

Propagation vector $\bar{u}_n$ assumed to be constant over a period then $G = \sum_{n=1}^{N} \bar{u}_n \bar{u}_n^H$ (μ×μ) matrix of rank N. R is a short term autocorrelation matrix which is a random and must be averaged. The thermal noise is independent and identically distributed modelled as white gaussian random vector and N_0 is the thermal noise power, $\Im$ is (μ×μ) identity matrix. Since $\bar{u}_n$ is CGRV the multivariate probability density function (pdf) of $\bar{u}_n$ is;

$$p(\bar{\mathbf{u}}_n) = \frac{1}{\pi^{\mu}\, det(R_n)} exp\left\{-\bar{\mathbf{u}}_n^H \mathbf{R}_n^{-1} \bar{\mathbf{u}}_n\right\} \quad 0 \le n \le N \qquad (3)$$

Moment generating function (MGF) of $\bar{\gamma}$ can be written in terms of pdf of $\bar{u}_n$, then since R is a random matrix $\Psi_{\bar{\gamma}}(\bar{z}) = E_R\{\Phi_{\bar{\gamma}}(\bar{z})\}$, that is; E{.} denotes expectation.

$$\Phi_{\bar{\gamma}}(\bar{z}) = \int_{\bar{\gamma}} p(\bar{\gamma}) \exp(-\bar{z}\bar{\gamma}) d\bar{\gamma}$$

$$= \int_{\bar{u}_0} p(\bar{u}_0) \exp(-\bar{z}P_{0d}\bar{u}_0^H R^{-1} \bar{u}_0) \times \int_{\substack{\bar{u}_n \\ n\neq 0}} p(\bar{u}_n) d\bar{u}_n$$

$$= \left[det(\Im + \bar{z}P_{0d} R_0 R^{-1}) \right]^{-1} \quad (4)$$

Here R_0R^{-1} is a random matrix and the pdf of R is needed, then MGF of $\bar{\gamma}$ is ;

$$\Psi_{\bar{\gamma}}(\bar{z}) = \left[E_R \left\{ det(\Im + \bar{z}P_{0d} R_0 R^{-1}) \right\} \right]^{-1} \quad (5)$$

Here $\bar{\gamma}$ is r.v. with chi-square distribution of two degrees of freedom and it can be rewritten as;

$$\bar{\gamma} = P_{od}\bar{u}_0^H R^{-1} \bar{u}_0 = P_{0d}\bar{u}_0^H T^H D^{-1} T\bar{u}_0 = P_{0d} \sum_{i=1}^{\mu} \frac{|t_i|^2}{\lambda_i} \quad (6)$$

where λ_i are eigen values of matrix R and T is the matrix of normalized eigenvectors of R, then $\Psi_{\bar{\gamma}}(\bar{z})$ in (5) and $\Phi_{\bar{\gamma}}(\bar{z})$ in (4) is given by;

$$\Phi_{\bar{\gamma}}(z) = \prod_{i=1}^{\mu} \left(1 + z\frac{P_{0d}}{\lambda_i} \right)^{-1} \Big| \bar{z} = z_1 = z_2 = \cdots = z_\mu = z$$

$$\Psi_{\bar{\gamma}}(z) \approx E_{\lambda_i} \left\{ \Phi_{\bar{\gamma}}(z) \right\} = \prod_{i=1}^{\mu} \left(1 + z\frac{P_{0d}}{E(\lambda_i)} \right)^{-1} \quad (7)$$

$R = P_{0u}G + N_0\Im$ is a random and it's eigenvalues are also random. Defining a new matrix Y_u;

$$Y_u = \begin{bmatrix} | & | & | & | \\ \bar{u}_1 & \bar{u}_2 & \cdots & \bar{u}_N \\ | & | & | & | \end{bmatrix}_{\mu\times N} \quad (8)$$

where Y_u is ($\mu\times$N) random matrix and $G = Y_uY_u^H$ is related to complex wishart distribution $G \sim \tilde{W}(\mu,N)$, G is ($\mu\times\mu$) random matrix where the eigenvalues of R can be written in terms of eigenvalues of G ;

$$\lambda_i = P_{0u}\alpha_i + N_0 \qquad i = 1,2,\cdots,\mu$$

$$\Psi_{\bar{\gamma}}(z) = \prod_{i=1}^{\mu} \left(1 + z\frac{P_{0d}}{P_{0u}E(\alpha_i) + N_0} \right)^{-1} \quad (9)$$

Now joint pdf of eigenvalues of G is needed. The joint pdf of eigenvalues of G which are ordered ($0 \le \alpha_{Nmin} \le \ldots \le \alpha_2 \le \alpha_1 \le \infty$) is given by;

$$p(\alpha_1,\alpha_2,\cdots,\alpha_{Nmin}) = \left[\prod_{i=1}^{N_{min}} \Gamma(N_{max} - i + 1)\Gamma(N_{min} - i + 1) \right]^{-1} \times$$
$$\prod_{i=1}^{N_{min}} e^{-\alpha_i} \alpha_i^{N_{max}-N_{min}} \prod_{i<j} (\alpha_i - \alpha_j)^2 \quad (10)$$

N_{max}=max(μ,N) , N_{min}=min(μ,N)

- If μ>N, then G is ($\mu\times\mu$) complex wishart distributed matrix and G has μ eigenvalues with joint pdf given by (10).
- If $\mu\le$N, then $G = Y_u^HY_u$ (N×N) complex wishart distributed matrix and G has N eigenvalues and (μ-N) eignevalues are equal to zero and its pdf is given by (10).

II.1 Special Cases

a) For N=2, two interferer case and N$\le\mu$, N_{min}=2 and $N_{max}=\mu$, then the mean eigenvalues are given below;

$$E\{\alpha_1\} = \int_0^\infty d\alpha_2 \int_{\alpha_2}^\infty d\alpha_1 \alpha_1 p(\alpha_1,\alpha_2)$$

$$= \frac{\Gamma(\mu+2)}{\Gamma(\mu)\Gamma(\mu-1)} \sum_{k=0}^{\mu+1} \frac{\Gamma(\mu+k-1)}{k!2^{\mu+k-1}} - \frac{\Gamma(\mu+1)}{\Gamma(\mu)\Gamma(\mu-1)} \sum_{k=0}^{\mu} \frac{\Gamma(\mu+k)}{k!2^{\mu+k-1}} +$$
$$\frac{\Gamma(\mu)}{\Gamma(\mu)\Gamma(\mu-1)} \sum_{k=0}^{\mu-1} \frac{\Gamma(\mu+k+1)}{k!2^{\mu+k+1}} \quad (11)$$

similarly E{α_2}= 2μ – E{α_1}. Table I shows the mean eigenvalues of the random interference matrix G for specific antenna elements. Here (μ-N) eigenvalues are equal to zero and μ and N are changeable, μ=3, N=2 and μ=2, N=3 have same same mean eigenvalues except the first one has a zero eigenvalue.

Table I

	E{α_1}	E{α_2}
μ=2	3.5	0.5
3	4.875	1.125
4	6.188	1.812
5	7.461	2.539
6	8.707	3.293

b) For N=3, three interferer case and N$\le\mu$. The mean eigenvalues are give in Table II. The results are approximately the same as in [4].

Table II

	E{α_1}	E{α_2}	E{α_3}
μ=3	6.521	2.146	0.333
4	8.056	3.154	0.79
5	9.522	4.159	1.319
6	10.941	5.161	1.898

III. EVALUATION OF BIT ERROR RATE (BER)

III.1 Performance of Noncoherent Detection

For noncoherent detection of binary signals, the conditional BER is given by for γ is;

$$P_e(\gamma) = \frac{1}{2}\exp(-a\gamma) \text{ a=1: DPSK, a=1/2: NCFSK} \quad (12)$$

The avarege BER P_e can be written as;

$$P_e = \int_0^\infty P_e(\gamma)p(\gamma)d\gamma \qquad (13)$$

By defining the MGF for the random variable γ as;

$$\Psi_\gamma(z) = \int_0^\infty p_\gamma(\gamma)\exp(-z\gamma)d\gamma \qquad (14)$$

Using (9), (12) and (14) the average BER for noncoherent detection is;

$$P_{e|\alpha_i} = \frac{1}{2}\Psi_{\bar{\gamma}}(z)\Big|z = a \quad \& \quad P_{e|\alpha_i} = 0.5 \times \prod_{i=1}^{\mu}\left(1 + \frac{a\gamma(1+N\gamma_u)}{1+\gamma_u E\{\alpha_i\}}\right)^{-1}$$

$$\text{where} \quad \gamma_u = \frac{P_{0u}}{N_0}, \quad \gamma_d = \frac{P_{0d}}{N_0}, \quad \gamma = \frac{\gamma_d}{1+N\gamma_u} \qquad (15)$$

III.2 Performance of Coherent Detection

For coherent detection of binary signals, the conditional BER is given by for γ is;

$$P_e(\gamma) = \frac{1}{2}erfc\left(\sqrt{a\gamma}\right) \text{a=1: CPSK, a=1/2:CFSK} \qquad (16)$$

After a large simplification and computational effort (16) can be rewitten as;

$$P_e(\gamma) = \frac{\sqrt{a}}{2\pi}\int_a^\infty \frac{1}{w\sqrt{w-a}}\exp(-\gamma\, w)dw \qquad (17)$$

The average BER is the same as (13), Using (13) and (9) P_e can be written as;

$$P_{e|\alpha_i} = \frac{\sqrt{a}}{2\pi}\int_a^\infty \frac{1}{z\sqrt{z-a}}\prod_{i=1}^{\mu}\left(1 + \frac{z\gamma(1+N\gamma_u)}{1+\gamma_u E\{\alpha_i\}}\right)^{-1} dz \qquad (18)$$

IV. SPECTRUM EFFICIENCY

Spectrum efficiency, E_s, is defined for hexagonal mobile systems as the carried traffic per unit bandwidth and per unit cell area ;

$$E_s = A_c / N_s W C S_c \text{ erlang / MHz / km}^2 \qquad (19)$$

where A_c (erlang) denotes carried traffic per cell, W (MHz) bandwidth per channel , S_c is the cell area and $C = R_u^2 / 3$ is the cluster size. For modelling dual slope path loss model is used;

$$P_{0d} / P_{0u} = R_u^{\,a} \times (g + R_u / g + 1)^b \qquad (20)$$

where R_u is frequency reuse distance, g is called the turning point , $ac = A_c / N_s$ is the carried traffic per channel and N_s defined as the number of channels per cell [6].

V. NUMERICAL RESULTS

Figure 1. shows the BER versus the average received SINR for optimum combining with a single interferer. The results for γ_u=0 corresponds to those for maximal ratio combining. For two interferers, the BER results are shown in Figure 2. The figures show the improvement obtained using optimum combining and the performance increases with the number of antenna elements. For example, for BER $=10^{-3}$ and μ=5, optimum combining requires approximately 4.2 dB less SINR than maximal ratio combining. Figure 3. shows the BER as a function of SINR for optimum combining using DPSK modulation with one, two and three interferers. The figure confirms the performance does not depend significantly on the number of interferers. Figure 4. shows the BER as a function of the average received SINR for BPSK modulation and two interferers. Comparison of Figure 2 and 4 shows that BPSK modulation gives better results than the DPSK modulation. Note that, for μ=2 and μ=4, Figure 5. and 6. show respectively, the effect of cluster size on the BER level for adaptive antennas. It can be observed from the figures that cosiderable improvement in terms of BER can be obtained when the cluster size and the number of antenna elements is increased. Figure 7. shows the effect of the number of interferers and antenna elements on the spectrum efficiency. Note that, increased values of the number of antenna elements and lower values of the number of interferers yield higher spectrum efficiencies. Figure 8. shows the effect of the carried traffic per channel (*ac*) on the spectrum efficiency. Decresaed values of *ac* causes significant improvement in the spectrum efficiency.

VI. REFERENCES

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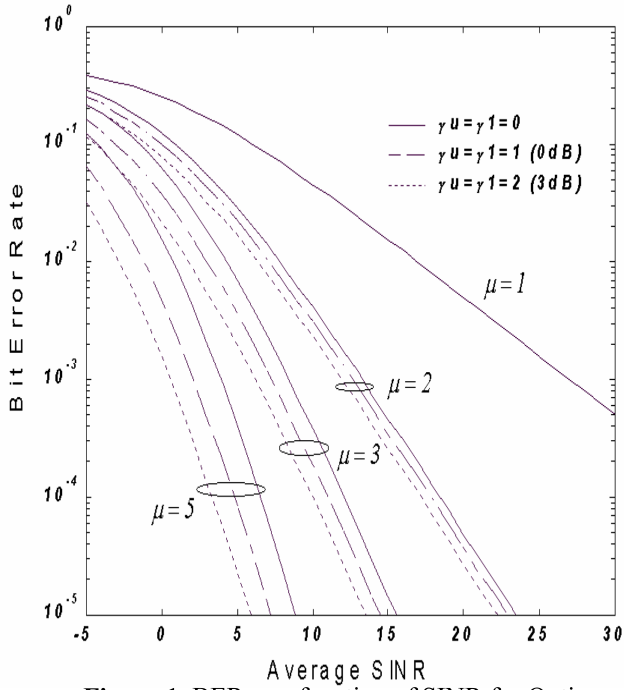

Figure 1. BER as a function of SINR for Optimum Combining with Rayleigh Fading and DPSK modulation for N=1

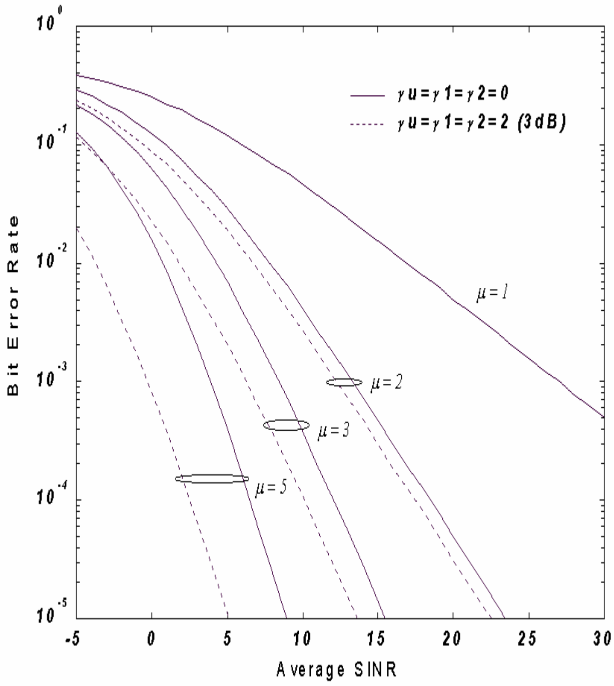

Figure 2. BER as a function of SINR for Optimum Combining with Rayleigh fading and DPSK modulation for N=2

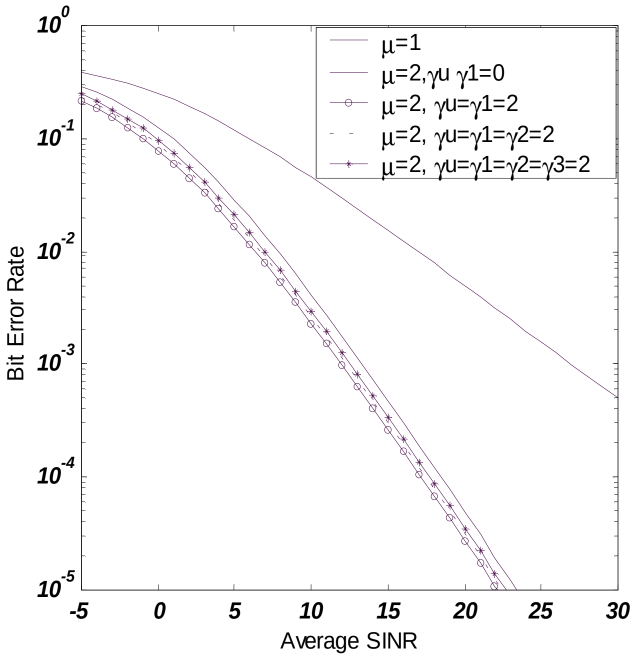

Figure 3. BER as a function of SINR for Optimum Combining with Rayleigh Fading and DPSK modulation for N=1,2,3

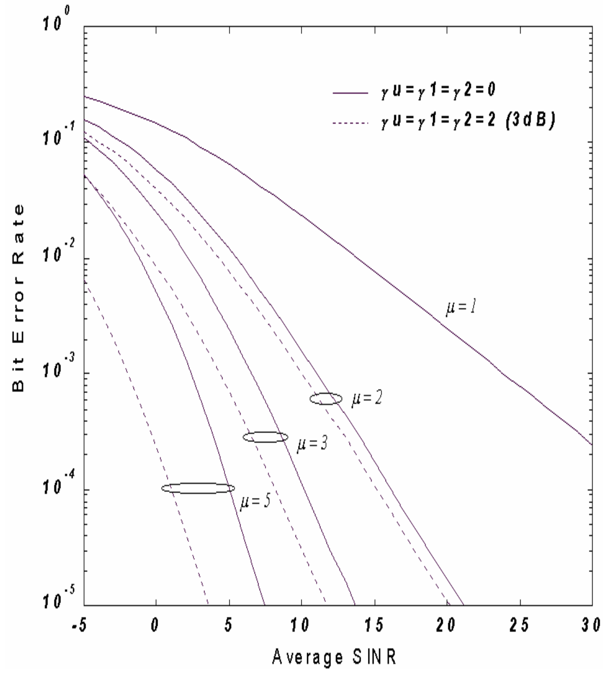

Figure 4. BER as a function of SINR for Optimum Combining with Rayleigh Fading and BPSK modulation for N=2

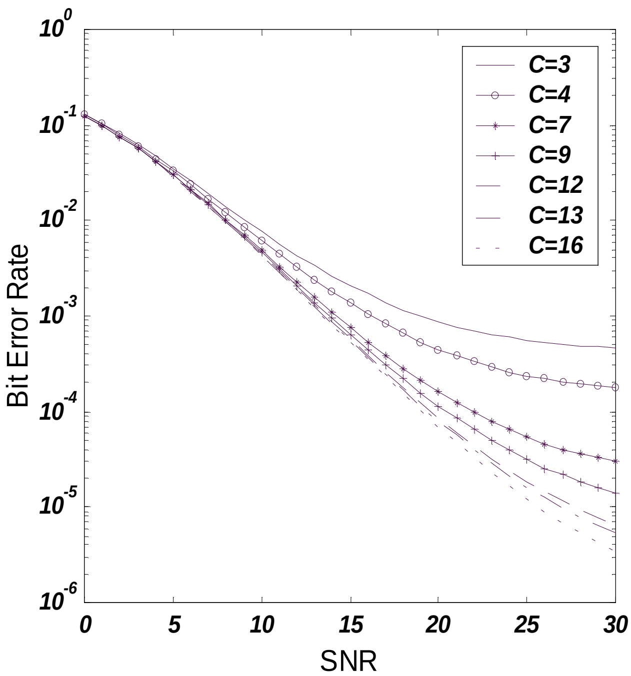

Figure 5. BER as a function of SNR for Optimum Combining with Rayleigh Fading and DPSK modulation for ac=0.6792, N=2, μ=2, g=0.67

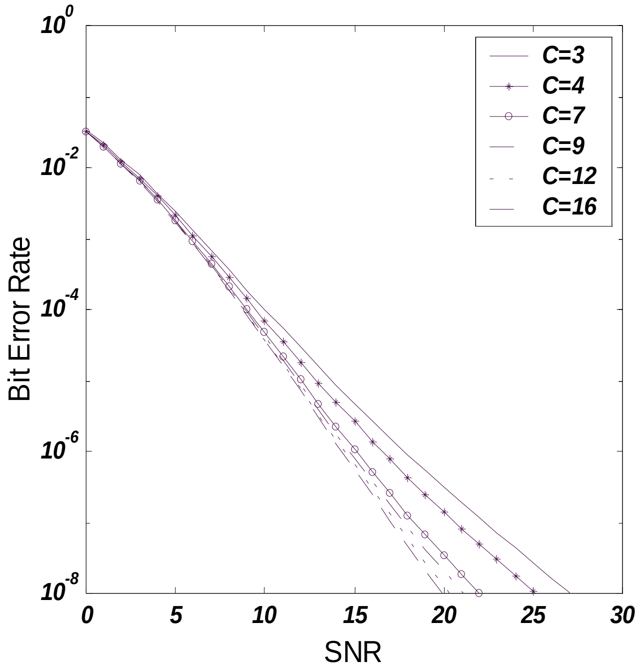

Figure 6. BER as a function of SNR for Optimum Combining with Rayleigh Fading and DPSK modulation for ac=0.6792, N=2, μ=4, g=0.67

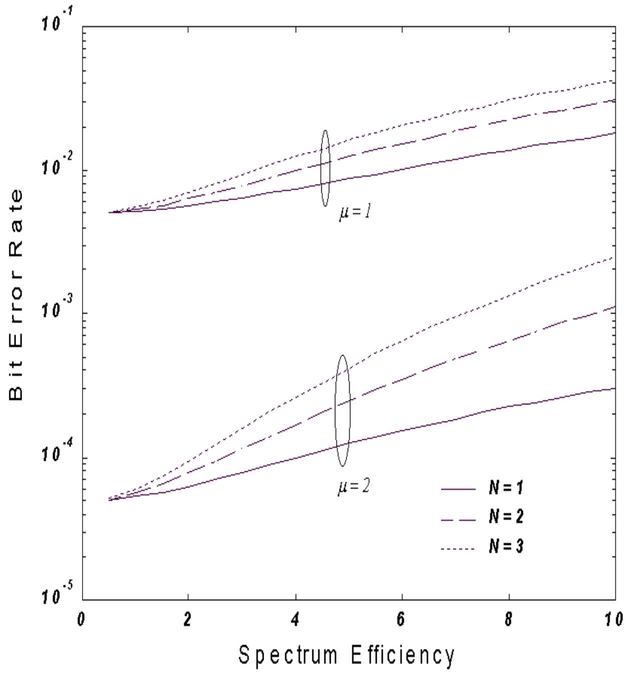

Figure 7. BER as a function of Spectrum efficiency for Optimum Combining with Rayleigh Fading and DPSK modulation for ac=0.6792, SNR=20 dB, g=0.67

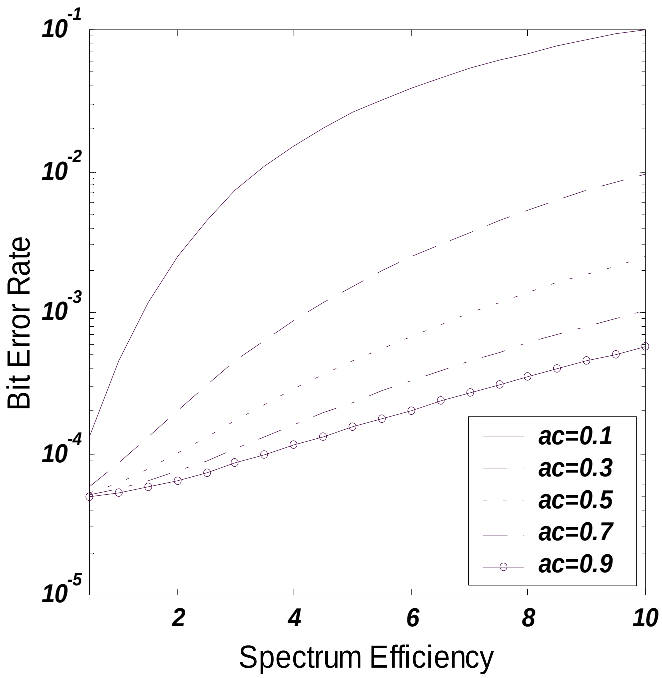

Figure 8. BER as a function of Spectrum efficiency for Optimum Combining with Rayleigh Fading and DPSK modulation for ac=0.6792, N=2, μ=2, g=0.67