# **Comparative Study of Mathematical Methods for Parameters Calculation of Current-Voltage Characteristic of Photovoltaic Module**

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#### Abstract

The photovoltaic module is typically represented by an equivalent circuit whose parameters are calculated using the experimental current voltage characteristic I-V. The precise determination of these parameters remains a challenge for researchers, which led to a diversification in models and numerical methods used for their characterizations. For Sicrystallin module, the parallel resistance  $R_{sh}$  is generally high, and its contribution has a little influence in the model, so for that the model with four parameters is one of the mainly used in literature. Parametric characterization of the four parameters model is the objective of this work. The simulation results for the Shell SP75 module are confronted with those of the manufacturer to develop the different conclusions drawn about the different methods used.

#### Nomenclature

$I_L$ :	Photocurrent.
$I_0$	: Diode saturation current.
q:	Electron charge.
<i>k</i> :	Boltzmann constant,
T :	Cell Température.
A:	Diode factor.
Rs:	Series resistance.
Rsh:	Shunt resistance
Ns:	Number of cells in series.
$V_{oc}$ :	Open circuit voltage.
$I_{sc}$ :	Short circuit current.
$V_m$ :	Voltage at the point of maximal power.
$I_m$ :	Current at the point of maximal power
$E_G$ :	Silicon Gap energy

Dhotocurrent

- Temperature coefficient of open circuit voltage  $V_{oc}$ .  $\mu_{V_{\alpha}}$
- Temperature coefficient of short circuit current  $I_{sc}$ .  $\mu_{I_{sc}}$

#### 1. Introduction

The production of photovoltaic electricity has known in recent years an increasing of interest by a production exceeding 1800 MW throughout the world. This increase was accompanied by a revitalization of researches considered for the optimization of the energy given by solar cells. So the modeling of these cells is a crucial stage in the process of optimization that leds to a diversification in the models proposed by different researchers.

Their differences are mainly in the number of diodes quoted in the model, the shunt resistance infinite or finite, the factor of ideality constant or not...., and the numerical methods used for determining the various unknown parameters [1], [2], [3].

In literature, one can find principally the equivalent model with four parameters associated with mathematical modeling of the current-voltage I-V curve [4]. In this model the effect of the shunt resistance is neglected because its value is particularly important, especially for Si-crystallin modules [5,6]. The model with four parameters as its name indicates has four unknown parameters namely:  $I_L$  (the photocurrent),  $I_0$  (the saturation current), A (ideality factor) and  $R_s$  (the series resistance). These parameters are not usually measurable or included in the manufacturing data. Accordingly, they must be determined from the system of equations I-V at various operating reference points given by the manufacturer or from experimental data [7]. After this stage mathematical modeling is used to estimate both voltage and current of the cell under different temperatures and irradiances.

The objective of the present paper is to compare the numerical methods mainly used in terms of characterization of solar cells made of Si-crystalline, namely the method of the slope; simplified explicit method and iterative method. The simulation results for the module Shell SP75 are confronted with those given by the manufacturer to develop accurate conclusions about the different methods used.

#### 2. Four parameters model

The present model is one of the mainly used for modeling solar cells, from which one can describe the cell current-voltage curve as (Fig. 2) [5,8]

$$I = I_L - I_0 \left[ exp\left(q \left(\frac{(V + IR_s)}{N_s AkT}\right) - 1\right) \right]$$
(1)

The four unknown parameters in this model are  $I_L$  (the photocurrent),  $I_0$  (the saturation current), A (ideality factor) and  $R_s$  (the series resistance). These parameters are determined from measurements of the I-V characteristic at reference values of irradiance and temperature (Eref =1000 W/m<sup>2</sup>, Tref =25 °C, spectrum AM1.5), which is given directly by the manufacturer or from the direct measurement on the module [1]. These measurements are to specify the data necessary for the characterization of the various unknown model parameters (Vco open circuit voltage, Isc short circuit current, Vm and Im are voltage and current at the maximum power point respectively) [9].



Fig.1. Four parameters equivalent model for solar cell.

Three remarkable couples of points from the I-V curve (0, Isc), (Voc, 0) and (Vm, Im), can be employed in order to determinate the unknown parameters, as

$$I_{sc} = I_L - I_0 \left[ exp \left( q \frac{I_{sc} R_s}{N_s A k T} \right) - 1 \right]$$
(2)

$$0 = I_L - I_0 \left[ exp\left(q \frac{V_{oc}}{N_s AkT}\right) - 1 \right]$$
(3)

$$I_m = I_L - I_0 \left[ exp\left(q \frac{V_m + I_m R_s}{N_s A k T}\right) - 1 \right]$$
(4)

#### 3. Identification methods for different parameters

By observing the equations (2,3,4), it is clear that we are in front of a problem of four equations and three unknowns, which shouted diversification in the choice of the equation to add. What brought that we are in front of a several methods of resolution cited in the literature [10]-[14], with a variable accuracy from a method to another. Our choice was on three different methods. The first one is the explicit simplified method which is based on a purely mathematical resolution with some simplifications. The second one is the method of the slope which is based in part of its algorithm on a geometry calculation, and the third one is the iterative method which is based in part of its algorithm on a numerical resolution.

#### 3.1. Simplified explicit method

This method considers as a first approximation  $I_L = I_{sc}$ , after simplification of equations (2), (3) and (4) we obtain [5], [10], [15]

$$I_{sc} = I_L \tag{5}$$

$$0 = I_L - I_O \left[ exp \left( q \frac{V_{OC}}{N_s A k T} \right) \right]$$
(6)

$$I_m = I_L - I_0 \left[ exp \left( q \frac{V_m + I_m R_s}{N_s A k t} \right) \right]$$
(7)

From (5) and (6) one can deduce the saturation current

$$I_0 = I_{sc} \exp\left(-\frac{q}{N_s A k T} V_{oc}\right)$$
(8)

and from that (1) becomes

$$I = I_{sc} \left[ 1 - exp \left( q \frac{V - V_{oc} + IR_s}{N_s A k T} \right) \right]$$
(9)

The equation at the point of maximum power at is turn becomes

$$I_m = I_{sc} \left[ 1 - exp \left( q \frac{V_m - V_{oc} + I_m R_s}{N_s A k T} \right) \right]$$
(10)

From this equation, we can deduce the value of series resistance

$$R_{s} = \frac{\frac{N_{s}AkT}{q}ln\left(1 - \frac{I_{m}}{I_{sc}}\right) + V_{oc} - V_{m}}{I_{m}}$$
(11)

The last parameter to be determined is the ideality factor A, by exploiting the fact that the derivative of the maximum power is zero:

$$\frac{dP}{dV} = 0 = \frac{\partial I}{\partial V} V + I \frac{\partial V}{\partial V}$$
(12)

and using equation (1) one can find:

$$A = \frac{q(2V_m - V_{OC})}{N_S kT \left[ \frac{I_{SC}}{I_{SC} - I_m} + ln \left( 1 - \frac{I_m}{I_{SC}} \right) \right]}$$
(13)

The substitution of different parameters with their respective formulas in equation (1) gives a simple equation linking the current I to the voltage V for different temperatures and irradiances.

## 3.2. Method of the slope at point

The difference given by this method in comparison of the previous method is in the manner of calculating the series resistance [16],[17]. It is based on the fact that the series resistance influences remarkably the slope of the characteristic curve I-V in the vicinity of the point  $(V_{oc}, 0)$ . So, in order to calculate  $R_s$  one uses the derivative of current described in equation (1) as

$$\frac{dI}{dV} = -I_0 \left[ exp \left( q \frac{V + IR_s}{N_s A k T} \right) - 1 \right] \frac{q}{N_s A k T} \left( 1 + R_s \frac{dI}{dV} \right) \quad (14)$$

which gives at the point  $(V_{oc}, 0)$ 

$$R_{s} = -\frac{dV}{dI}\Big|_{Voc} - \frac{1}{\frac{I_{o}q}{N_{s}AkT}} \exp\left(\frac{qV_{oc}}{N_{s}AkT}\right)$$
(15)

The slope  $M = \frac{dV}{dI}(I=0)$  at the point  $(V_{co},0)$  is deduced geometrically from experimental data (Fig.2).



Fig.2. The slope calculation at the open circuit voltage point

# 3.3. Iterative Method

This method also is distinguishing from the two previous methods in the manner of calculating of series resistance, where the temperature coefficient of the open circuit voltage given by the manufacturer may be used to provide an additional equation for determining the series resistance.

The temperature coefficient is the derivative of the open circuit voltage versus the temperature:

$$\mu_{V_{OC}} = \frac{\partial V_{OC}}{\partial T} = \frac{N_s A k}{q} \left[ ln \left( \frac{I_{sC}}{I_0} \right) + \frac{T \mu_{I_{sC}}}{I_{sC}} - \left( 3 + \frac{q E_G}{A k T} \right) \right]$$
(16)

By giving a value to  $R_s$ , the other three parameters are calculated in the same manner as the previous sections. Using iterative methods as the bisection in the interval  $[0, R_{smax}]$ , where  $R_{smax}$  is the maximum possible value of series resistance [5], [7].

The value of ideality factor is close to 1 for  $R_{s \max}$ , so to determine its value, one just replace A = 1 in (11), that yield to:

$$R_{smax} = \frac{1}{I_m} \left[ \frac{N_s kT}{q} ln \left( 1 - \frac{I_m}{I_{sc}} \right) + V_{oc} - V_m \right]$$
(17)

The variations of different parameters in versus of the variation of irradiance or temperature for the different method are expressed as

For the short circuit current and open circuit voltage:

$$I_{sc} = I_{sc,ref} \frac{E}{E_{ref}} + \mu_{I_{sc}} \left( T - T_{ref} \right)$$
(18)

$$V_{oc} = V_{oc \ ref} + V_t \ln\left(\frac{E}{E_{ref}}\right) + \mu_{Voc}\left(T - T_{ref}\right) \quad (19)$$

where 
$$V_t = \frac{N_s \cdot A \cdot k \cdot T}{q}$$

The variation in the ideality factor is given by

$$\frac{A}{A_{ref}} = \frac{T}{T_{ref}}$$
(20)

and  $I_0$ :

$$I_0 = I_{0,ref} \left(\frac{T}{T_{ref}}\right)^3 exp\left[\frac{q E_{gap}}{k A} \left(\frac{1}{T_{ref}} - \frac{1}{T}\right)\right]$$
(21)

At the last the variations of the current and voltage at the maximum power point are described by

$$V_{m} = V_{mref} + V_{t} ln \left(\frac{E}{E_{ref}}\right) + \mu_{Voc} \left(T - T_{ref}\right)$$
(22)  
$$I_{m} = I_{m,ref} \frac{E}{E_{ref}} + \mu_{I_{sc}} \left(T - T_{ref}\right)$$
(23)

#### 4. Results and discussion

The equations of the previous sections for the test model Shell SP75 were simulated in the Matlab environment, where the electrical characteristic of Shell SP75 module are summarized in Table 1 [18].

Firstly, the unknown parameters are deduced by the different methods using data provided by the manufacturer in standard test conditions (E=1000w/m2, T=25°C).

Secondly the authors have attempted from results previously deduced at standard test conditions to estimate the behavior of solar panel for different temperatures and irradiances. The results derived by simulation are compared with those of the experiment.

One can see in Figures 3 and 4 that the results given by the different methods are in concordance with the experimental results of the current-voltage I-V and power-voltage characteristics in the standard test conditions. At the contrary the exactness of estimated results for the different temperatures and irradiances is variable from one method to another, where the explicit method is strongly influenced by the variation of irradiance and the iterative method is influenced by the variation of both irradiance and temperature. On the other hand the slope method keeps its performances regardless of variations in meteorological data.

Tables 3,4,5 and 6 show that the different methods give distant values of the unknown parameters. That gives an idea

about the existence of various solutions, where all the parameters interact in a non-linear manner between them. So, the obtained results lose the physical meaning which is the source of their definition.

 
 Table 1. Electrical characteristic of photovoltaic cell Shell SP75 at standard condition test.

Standard irradiance	$E_{ref}$	$1000 W/m^2$
Standard temperature	$T_{ref}$	25°C
maximal Power	$P_{Mref}$	75W
Voltage at maximal power point	V <sub>m ref</sub>	17V
Current at maximal power point	I <sub>m ref</sub>	4.4A
Open circuit voltage	V <sub>oc ref</sub>	21.7V
Short circuit current	I <sub>sc ref</sub>	4.8A
Temperature coefficient of short circuit current $I_{sc}$	$\mu_{I_{sc}}$	2 mA/°C
Temperature coefficient of open circuit voltage $V_{oc}$	$\mu_{V_{oc}}$	-76 mV/°C
Cell number	$N_s$	36

**Table 2.** Unknown parameters at standard testconditions (E=1000w/m², T=25°C)

	Explicit method	Slope method	Iterative method
Iph	4.8	4.8	4.8
Á	1.3978	1.5	1.5091
Rs	0.3381	0.2860	0.28
I <sub>0</sub>	2.4594e-7	7.7196e-7	8.4836e-7

Table 3. Shell SP75 Cell parameters values for E=800w/m<sup>2</sup> and T=25°C.

	Explicit method	Slope method	Iterative method
Isc	3.84	3.84	3.84
Voc	21.4115	21.3905	21.3886
Im	3.52	3.52	3.52
Vm	16.7115	16.6905	16.6886
I <sub>ph</sub>	3.84	3.84	3.84
À	1.3650	1.5	1.4172
Rs	0.4441	0.2137	0.41
L	1.6523e-7	7.7196e-7	3.1402e-7

Table 4. Shell SP75 Cell parameters values for  $E=400 \text{w/m}^2$  and  $T=25^{\circ}\text{C}$ .

	Explicit method	Slope method	Iterative method
Isc	1.92	1.92	1.92
Voc	20.5155	20.4289	20.4212
Im	1.76	1.76	1.76
Vm	15.8155	15.7289	15.7212
I <sub>ph</sub>	1.92	1.92	1.92
Α	1.2632	1.5	1.2028
Rs	1.0211	-0.1475	1.1
I <sub>0</sub>	4.5328e-8	7.7196e-7	2.0418e-8

Table 5. Shell SP75 Cell parameters values for E=1000w/m<sup>2</sup> and T=40°C.

	Explicit	Slope	Iterative
	method	method	method
Iph	4.83	4.83	4.83
A	1.5755	1.4681	1.4443
Rs	0.1632	0.2590	0.35
I <sub>0</sub>	7.0691e-6	2.6468e-6	2.0859e-6

Table 6. Shell SP75 Cell parameters values for E=1000w/m<sup>2</sup> and T=60°C.

	Explicit method	Slope method	Iterative method
Iph	4.87	4.87	4.87
A	1.6762	1.5620	1.3654
Rs	0.1243	0.1489	0.395
I <sub>0</sub>	8.2006e-5	3.6709e-5	6.7202e-6



**Fig. 3.** I(V) and P(V) characteristic for Shell SP75 module using the different methods and for various irradiances.



**Fig. 4.** I(V) and P(V) characteristic for Shell SP75 module using the different methods and for various temperatures.

# 5. Conclusion

The objective of the solar panels modeling is of the estimation of their behavior under different meteorological conditions. Three different mathematical methods have been presented in order to estimate the parameters of the four parameters model and to simulate its current-voltage and powervoltage characteristics, the slope method based in part of its algorithm on a geometric calculation, the explicit method based on an analytical solution and the iterative method based on a numerical resolution. By comparing their results with those of experimentation, we concluded that the slope method is the less influenced by the meteorological data variations and the most accurate.

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