SYNTHESIS OF MIXED LUMPED AND DISTRIBUTED ELEMENT NETWORKS

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ABSTRACT

Synthesis algorithm for mixed lumped and distributed element networks is presented. The algorithm is based on transfer scattering matrix factorization The mixed structure is composed of ladder lumped-elements connected with commensurate transmission lines (Unit elements, UEs). Reflection function expression in two-variable is utilized in the algorithm. First, the type of the element that will be extracted is determined. After obtaining the value of the element, it is extracted, and the reflection function of the remaining network is obtained by using transfer scattering matrix factorization method. Algorithm is given for low-pass structures (similar algorithms have been prepared for high-pass, band-pass and band-stop structures). An example is included to illustrate the implementation of the proposed algorithm.

I. INTRODUCTION

An analytic treatment of the design problem with mixed lumped and distributed elements requires the characterization of the mixed-element structures using transcendental or multivariable functions. The first approach deals with non rational single variable transcendental functions and is based on the classical study of cascaded noncommensurate transmission lines by Kinarivala [1]. First results on the synthesis of a transcendental driving-point impedance function as a cascade of lumped, lossless, two-ports and commensurate transmission lines were given by Riederer and Weinberg [2]. The other approach to describe mixed lumped-distributed two-ports is based on the transformation of Richards, $\lambda = \tanh(p\tau)$ which converts the transcendental functions of a distributed network into rational functions [3]. The attempts to generalize this approach to mixed lumped-distributed networks led to the multivariable synthesis procedures, where the Richards variable λ is used for distributed-elements and the original frequency variable p for lumped-elements. In this way, all the network functions could be written as rational functions of two or more complex variables. After the pioneering work of Ozaki and Kasami [4] on the multivariable positive real functions, the problem of filter design with mixed lumped and distributed elements is attempted to be solved by many researcher especially using the latter multivariable approach. In this context, although there have been valuable contributions for the characterization of some restricted classes of mixed-element structures, a complete theory for the approximation and synthesis problems of mixed lumped and distributed networks is still not available. Thus, the problem is quite challenging from a theoretical as well as the practical point of view.

In the following section, the characterization of two-variable networks is introduced. Subsequently, after giving the synthesis algorithm, an example is presented, to illustrate the utilization of the proposed algorithm.

II. CHARACTERIZATION OF TWO-VARIABLE NETWORKS

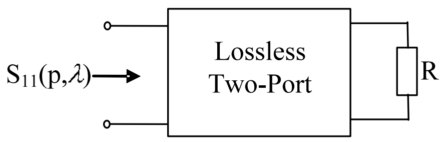

Figure 1. Lossless two-port with input reflectance function $S_{11}(p,\lambda)$.

Let $\{S_{kl}; k,l = 1,2\}$ designate the scattering parameters of a lossless two-port like the one depicted in Fig. 1. For a mixed lumped and distributed element, reciprocal, lossless two-port, the scattering parameters may be expressed in Belevitch form as follows [5-8]

$$S(p,\lambda) = \begin{bmatrix} S_{11}(p,\lambda) & S_{12}(p,\lambda) \\ S_{21}(p,\lambda) & S_{22}(p,\lambda) \end{bmatrix} = \frac{1}{g(p,\lambda)} \begin{bmatrix} h(p,\lambda) & \mu f(-p,-\lambda) \\ f(p,\lambda) & -\mu h(-p,-\lambda) \end{bmatrix} \quad (1a)$$

where $\mu = \dfrac{f(-p,-\lambda)}{f(p,\lambda)}$.

In (1a), $p = \sigma + j\omega$ is the usual complex frequency variable associated with lumped-elements, and

$\lambda = \Sigma + j\Omega$ is the conventional Richards variable associated with equal length transmission lines or so called commensurate transmission lines ($\lambda = \tanh p\tau$, where τ is the commensurate delay of the distributed elements).

$g(p,\lambda)$ is $(n_p + n_\lambda)^{th}$ degree scattering Hurwitz polynomial with real coefficients such that

$$g(p,\lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P}$$

where

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p 0} & \cdots & \cdots & g_{n_p n_\lambda} \end{bmatrix}, \quad \begin{matrix} \mathbf{P}^T = \left[1\ p\ p^2\ \ldots\ p^{n_p}\right] \\ \boldsymbol{\lambda}^T = \left[1\ \lambda\ \lambda^2\ \ldots\ \lambda^{n_\lambda}\right]. \end{matrix} \tag{1b}$$

The partial degrees of two-variable $g(p,\lambda)$ polynomial are defined as the highest power of a variable, whose coefficient is nonzero, i.e. $n_p = \deg_p g(p,\lambda)$, $n_\lambda = \deg_\lambda g(p,\lambda)$.

Similarly, $h(p,\lambda)$ is also a $(n_p + n_\lambda)^{th}$ degree polynomial with real coefficients such that

$$h(p,\lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$$

where

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p 0} & \cdots & \cdots & h_{n_p n_\lambda} \end{bmatrix}, \quad \begin{matrix} \mathbf{P}^T = \left[1\ p\ p^2\ \ldots\ p^{n_p}\right] \\ \boldsymbol{\lambda}^T = \left[1\ \lambda\ \lambda^2\ \ldots\ \lambda^{n_\lambda}\right]. \end{matrix} \tag{1c}$$

$f(p,\lambda)$ is a real polynomial which includes all the transmission zeros of the two-port network. General form of the polynomial $f(p,\lambda)$ is given by

$$f(p,\lambda) = \prod_{i,j} f_i(p) f_j(\lambda); \quad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,m \end{matrix} \tag{1d}$$

where n is the number of transmission zeros of the lumped-element subsection ($n \le n_p$), the difference ($n_p - n$) is the number of transmission zeros at infinity of the lumped-element subsection, m is the number of transmission zeros of the distributed-element subsection ($m \le n_\lambda$), the difference ($n_\lambda - m$) is the number of transmission zeros at infinity of the distributed-element subsection, $f_i(p)$ and $f_j(\lambda)$ define the transmission zeros of lumped and distributed subsections, respectively. Transmission zeros can be located anywhere in p- and λ-planes. From (1d), it can be immediately deduced that the transmission zeros of each subsection have to arise in multiplication form. In other words, it can be assumed that $f(p,\lambda)$ of the entire mixed-element structure is in product separable form as

$$f(p,\lambda) = f_L(p) f_D(\lambda), \tag{1e}$$

where $f_L(p)$ and $f_D(\lambda)$ are the polynomials constructed on the transmission zeros of the lumped- and distributed-element subsections, respectively.

In the lossless two-port network, if one only considers the real frequency transmission zeros formed with lumped-elements, then on the imaginary axis $j\omega$, $f_L(p)$ will be either an even or an odd polynomial in p. Furthermore, due to cascade connection of commensurate transmission lines, $f_D(\lambda)$ will have the following form

$$f_D(\lambda) = (1-\lambda^2)^{n_\lambda/2}. \tag{1f}$$

A practical form of $f(p,\lambda)$ can be obtained by disregarding the finite imaginary axis zeros except those at DC as follows,

$$f(p,\lambda) = p^k (1-\lambda^2)^{n_\lambda/2} \tag{1g}$$

where k designate the total number of transmission zeros at DC.

Since the network is considered as a lossless two-port terminated in a resistance, then energy conversation requires that

$$S(p,\lambda) S^T(-p,-\lambda) = I, \tag{1h}$$

where I is the identity matrix. The open form of (1h) is given as

$$g(p,\lambda) g(-p,-\lambda) = h(p,\lambda) h(-p,-\lambda) + f(p,\lambda) f(-p,-\lambda). \tag{1i}$$

In the following section, the synthesis algorithm, based on scattering transfer matrix factorization [9], for low-pass mixed-element two-port networks is given. The fundamental properties of this mixed-element structure can be found in [6] and [10].

III. MIXED-ELEMENT NETWORK SYNTHESIS ALGORITHM

In this section, synthesis algorithm for low-pass mixed-element structures is explained step by step. Similar algorithms have been prepared for high-pass, band-pass and band-stop structures, but because of space limitations, they are not given here.

3.1. Low-Pass Ladders Connected with Unit Elements

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element low-pass structure are as follows,

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & h_{02} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & h_{12} & \cdots & h_{1n_\lambda} \\ h_{20} & h_{21} & \cdots & \cdots & 0 \\ \vdots & \vdots & 0 & 0 & \vdots \\ h_{n_p 0} & \cdot & 0 & \cdots & 0 \end{bmatrix},$$

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & g_{02} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & g_{12} & \cdots & g_{1n_\lambda} \\ g_{20} & g_{21} & \cdots & \cdots & 0 \\ \vdots & \vdots & 0 & 0 & \vdots \\ g_{n_p 0} & \cdot & 0 & \cdots & 0 \end{bmatrix},$$

$f(p,\lambda) = (1-\lambda^2)^{n_\lambda/2}$.

Step 1: Compute $\alpha_1 = \frac{h(n_p,0)}{g(n_p,0)} = \pm 1$ and $\alpha_2 = \frac{h(n_p,1)}{g(n_p,1)} = \pm 1$. If $h(n_p,1) = 0$ and $g(n_p,1) = 0$, $\alpha_2 = \frac{h(n_p-1,1)}{g(n_p-1,1)} = \pm 1$.

Step 2:

α_1	α_2	First component	Next component
+1	-1	UE	Series inductor (L)
-1	+1	UE	Parallel capacitor (C)
+1	+1	Series inductor (L)	UE
-1	-1	Parallel capacitor (C)	UE

Step 3: a) If the component that will be extracted is a UE, then characteristic impedance is $Z = \frac{1+S_{11}^\lambda}{1-S_{11}^\lambda}$, where $S_{11}^\lambda = \frac{\sum_{i=0}^{n_\lambda} h(0,i)}{\sum_{i=0}^{n_\lambda} g(0,i)}$. The polynomials $g^{(UE)}(\lambda)$, $h^{(UE)}(\lambda)$ and $f^{(UE)}(\lambda)$ are $g^{(UE)}(\lambda) = \frac{Z^2+1}{2Z}\lambda + 1$, $h^{(UE)}(\lambda) = \frac{Z^2-1}{2Z}\lambda$, $f^{(UE)}(\lambda) = (1-\lambda^2)^{1/2}$. The polynomials $g^{(RN)}(p,\lambda), h^{(RN)}(p,\lambda)$ and $f^{(RN)}(p,\lambda)$ of the remaining network can be computed as $g^{(RN)}(p,\lambda) = -h^{(UE)}(-\lambda)h(p,\lambda) + g^{(UE)}(-\lambda)g(p,\lambda)$, $h^{(RN)}(p,\lambda) = g^{(UE)}(\lambda)h(p,\lambda) - h^{(UE)}(\lambda)g(p,\lambda)$, $f^{(RN)}(p,\lambda) = (1-\lambda^2)^{(n_\lambda-1)/2}$.

b) If the component that will be extracted is a lumped element (L or C), then the element value is $EV = \frac{g(n_p,0)+\alpha_1 h(n_p,0)}{g(n_p-1,0)-\alpha_1 h(n_p-1,0)}$. If the lumped element is an inductor, the polynomials $g^{(L)}(p), h^{(L)}(p)$ and $f^{(L)}(p)$ are $g^{(L)}(p) = \frac{EV}{2}p + 1$, $h^{(L)}(p) = \frac{EV}{2}p$, $f^{(L)}(p) = 1$. The polynomials $g^{(RN)}(p,\lambda), h^{(RN)}(p,\lambda)$ and $f^{(RN)}(p,\lambda)$ of the remaining network can be computed as $g^{(RN)}(p,\lambda) = -h^{(L)}(-p)h(p,\lambda) + g^{(L)}(-p)g(p,\lambda)$, $h^{(RN)}(p,\lambda) = g^{(L)}(p)h(p,\lambda) - h^{(L)}(p)g(p,\lambda)$, $f^{(RN)}(p,\lambda) = (1-\lambda^2)^{n_\lambda/2}$. If the lumped element is a capacitor, the polynomials $g^{(C)}(p), h^{(C)}(p)$ and $f^{(C)}(p)$ are $g^{(C)}(p) = \frac{EV}{2}p + 1$, $h^{(C)}(p) = -\frac{EV}{2}p$, $f^{(C)}(p) = 1$. The polynomials $g^{(RN)}(p,\lambda), h^{(RN)}(p,\lambda)$ and $f^{(RN)}(p,\lambda)$ of the remaining network can be computed as $g^{(RN)}(p,\lambda) = -h^{(C)}(-p)h(p,\lambda) + g^{(C)}(-p)g(p,\lambda)$, $h^{(RN)}(p,\lambda) = g^{(C)}(p)h(p,\lambda) - h^{(C)}(p)g(p,\lambda)$, $f^{(RN)}(p,\lambda) = (1-\lambda^2)^{n_\lambda/2}$.

Step 4: Set new $h(p,\lambda), g(p,\lambda)$ and $f(p,\lambda)$ two-variable polynomials as $h(p,\lambda) = h^{(RN)}(p,\lambda)$, $g(p,\lambda) = g^{(RN)}(p,\lambda)$, $f(p,\lambda) = f^{(RN)}(p,\lambda)$, and go to Step 1.

IV. EXAMPLE

The coefficient matrices of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, and polynomial $f(p,\lambda)$ describing the mixed-element low-pass structure are given as,

$$\Lambda_h = \begin{bmatrix} 0 & 3.15 & -1.05 \\ 3.5 & -3.8 & 23.3 \\ 3 & 65.4 & 0 \\ 36 & 0 & 0 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 3.85 & 1.45 \\ 6.5 & 14 & 23.3 \\ 15 & 65.4 & 0 \\ 36 & 0 & 0 \end{bmatrix}$$

$, f(p,\lambda) = (1-\lambda^2)$.

Step 1: $\alpha_1 = \frac{h(n_p,0)}{g(n_p,0)} = \frac{h(3,0)}{g(3,0)} = \frac{36}{36} = +1$. $h(3,1) = 0$ and $g(3,1) = 0 \Rightarrow$ $\alpha_2 = \frac{h(n_p-1,1)}{g(n_p-1,1)} = \frac{h(2,1)}{g(2,1)} = \frac{65.4}{65.4} = +1$.

Step 2: $\alpha_1 = +1$ and $\alpha_2 = +1$, so the first component that will be extracted is an inductor, and the element value is

$$EV = \frac{g(n_p,0)+\alpha_1 h(n_p,0)}{g(n_p-1,0)-\alpha_1 h(n_p-1,0)} = \frac{g(3,0)+h(3,0)}{g(2,0)-h(2,0)},$$
$$= \frac{36+36}{15-3} = 6$$

$L_1 = 6$. The polynomials $g^{(L)}(p), h^{(L)}(p)$ and $f^{(L)}(p)$ of the inductor are $g^{(L)}(p) = \frac{EV}{2}p + 1 = 3p + 1$, $h^{(L)}(p) = \frac{EV}{2}p = 3p$, $f^{(L)}(p) = 1$. The polynomial $f^{(RN)}(p,\lambda)$ and coefficient matrices Λ_h and Λ_g of the remaining network are $f^{(RN)}(p,\lambda) = (1-\lambda^2)$,

$$\Lambda_h = \begin{bmatrix} 0 & 3.15 & -1.05 \\ 0.5 & -5.9 & 15.8 \\ -6 & 12 & 0 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 3.85 & 1.45 \\ 3.5 & 11.9 & 15.8 \\ 6 & 12 & 0 \end{bmatrix}.$$

If the same algorithm is used, the extracted element values and the remaining network coefficient matrices and polynomial $f^{(RN)}(p,\lambda)$ are as follows,

Z_1 of UE_1=2 and

$$\Lambda_h = \begin{bmatrix} 0 & 2.4 \\ 0.5 & 7.9 \\ -6 & 0 \end{bmatrix}, \quad \Lambda_g = \begin{bmatrix} 1 & 2.6 \\ 3.5 & 7.9 \\ 6 & 0 \end{bmatrix},$$

$, f^{(RN)}(p,\lambda) = (1-\lambda^2)^{1/2}$,

$C_1 = 3$ and $\Lambda_h = \begin{bmatrix} 0 & 2.4 \\ 2 & -0.4 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 1 & 2.6 \\ 2 & 0.4 \end{bmatrix},$

$f^{(RN)}(p,\lambda) = (1-\lambda^2)^{1/2}$,

Z_2 of UE_2=5 and

$$\Lambda_h = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \Lambda_g = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad f^{(RN)}(p,\lambda) = 1,$$

$L_2 = 4$ and $\Lambda_h = 0$, $\Lambda_g = 1$, $f^{(RN)}(p,\lambda) = 1$, and the termination resistor $R = 1$. The obtained network is given in Fig. 2.

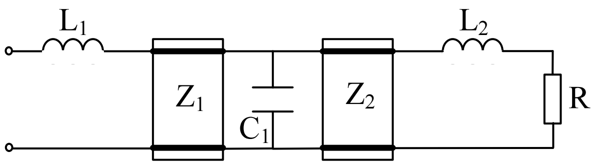

Figure 2. Synthesized low-pass mixed-element network, normalized element values: $L_1 = 6, L_2 = 4, C_1 = 3, Z_1 = 2, Z_2 = 5, R = 1$.

V. CONCLUSION

The unavoidable connections between lumped-elements destroy the performance of the lumped-element networks at high frequencies. But these connection lines can be used as circuit components. In this case, the circuits must be composed of mixed lumped and distributed elements. But a complete theory for the approximation and synthesis problems of mixed-element networks is still not available. So in this paper, synthesis of practically important class of mixed-element networks, namely ladder lumped-elements connected with UEs, is examined. The synthesis of mixed-element networks may be realized by using single variable boundary polynomials, namely the polynomials $h(p,0), g(p,0), f(p,0)$ for lumped-element section, and $h(0,\lambda), g(0,\lambda), f(0,\lambda)$ for distributed-element section. In this case, synthesis is carried out for lumped and distributed sections separately. Then, the components are mixed, to construct the mixed-element network. But in the proposed algorithms, the synthesis of mixed-element network is carried out by using the two-variable reflection function of the mixed-element network, and components are extracted according to the connection order in the mixed-structure. Synthesis algorithm, based on transfer scattering matrix factorization, for low-pass networks is given. The implementation of the proposed algorithm is utilized by the given example. As a result, a simple to use mixed-element network synthesis algorithm is presented, which is necessary for the applications using this important class of mixed-element networks, e.g. design of filters, broadband matching networks and amplifiers.

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