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RULE-BASED QUASI-SLIDING MODE TYPE OF CONTROLLER DESIGN

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Abstract: The main purpose of this study is to design a new rule-based sliding mode controller for general non-linear uncertain dynamical systems. This design procedure has led us to a new controller structure that can be named PID plus QUASI-SLIDING MODE TYPE of controller. The new designed algorithm is tested on a plant with an application of a pulse disturbance and the results are compared with the classical PID controller on optimal basis under the same performance index.

Keywords: PID control, sliding mode control, rule-based control, disturbance' rejection

1. INTRODUCTION

When the system is highly non-linear and exhibits some uncertainties in its parameters, the sliding mode control (SMC) is claimed to result in superb system performance that includes insensitivity to parameter variations and complete rejection of disturbance [4]. However, there exists a chattering phenomenon in SMC as a major obstacle for sliding mode to become one of the most significant discoveries in modern control theory.

On the other hand, various controller design methodologies are presented in literature, PID controllers are still most commonly used controller structure. The main reasons for its widely use in industry are due to its simple control structure, ease of design and inexpensive cost. However, PID-type controllers cannot yield a good control performance if the plant to be controlled is highly non-linear and uncertain. There exist some studies on the design of PID controllers for non-linear plants [1], [2], [3]. In all of these studies the parameters of PID controllers are auto-tuned using various strategies.

In this study, a new algorithm is designed that can be seen as PID plus QUASI-SLIDING MODE TYPE of controller. This new algorithm is then tested on a highly non-linear plant; namely, ball and beam control system with application of a pulse disturbance and the results are compared with the classical PID controller on optimal bases under the same performance index.

2. BASIC PRINCIPLES OF THE METHOD

The basic block diagram used for the proposed design methodologies is illustrated in Figure 1.

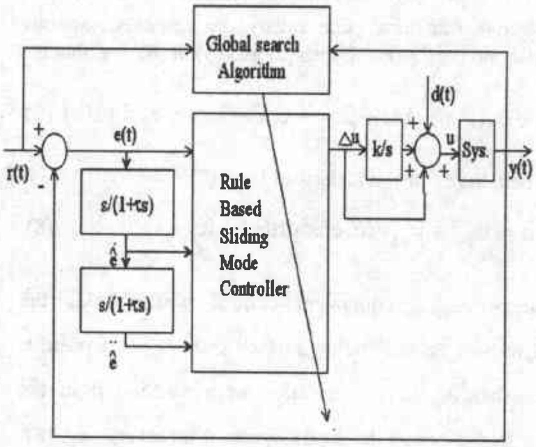

Figure1. Basic blok diagram for the proposed method

In the method the rule-based sliding mode controller uses the error function e defined as

$$e = r - y \tag{1}$$

and the estimated values of the first-order and second order derivatives of e ; namely, $\dot{e}, \ddot{e}$. Here, r is the reference input and y is the output of the system. The system error dynamics is taken to be as follow

$$\ddot{e} = f(e,\dot{e}) - bu + \beta(r,\dot{r},\ddot{r},d,\dot{d}) \tag{2}$$

where f and β is a nonlinear unknown functions , u is the system control input, d is disturbance input and b is a positive real-valued unknown parameter.

The sliding surface switching function is chosen to be in classical form for a second-order error dynamics as follows

$$s = ce + \dot{e} \quad (3)$$

where -c is the slope of the sliding surface. If we consider a candidate Lyapunov function as

$$V = 1/2s^2 \quad (4)$$

then the stability condition in Lyapunov sense gives us the following relation

$$s.\dot{s} < 0 \quad (5)$$

For this purpose, we can choose $\dot{s}$ such that

$$\dot{s} = -\lambda(s) - kg(e,\dot{e})\text{sign}(s) \quad (6)$$

where k is a positive constant, $g(e,\dot{e})$ and $s\lambda(s)$ are positive functions that satisfy the general Lyapunov function conditions. Using (2) and (3) in (6), we obtain

$$c\dot{e} + f(e,\dot{e}) - bu + \beta = -\lambda(s) - kg(e,\dot{e})\text{sign}(s) \quad (7)$$

Then, the control u is derived from (7)

$$u = u_{eq} + k_{eq}g(e,\dot{e})\text{sign}(s) + \lambda_{eq}(s) \quad (8)$$

where u_{eq} is equivalent control which holds the dynamics on the sliding surface s=0, k_{eq} is a positive constant, $\lambda_{eq}(s)$ is a relation or a function from the switching space to input space. Moreover, $\lambda_{eq}(s)$ represents a memory-less non-linear feedback. Since the equivalent control u_{eq} depends on unknown quantities $f(e,\dot{e}), \beta$ and b, these should be estimated to determine the equivalent control and also the total control. Estimation of these quantities is an indirect method for the determination of equivalent control. However, direct estimation of equivalent control is also possible. Using $\hat{u}_{eq}$ for the estimated equivalent control, then (8) can be rewritten as follows

$$u = \hat{u}_{eq} + \Delta u \quad (9)$$

$$\Delta u = \Delta u_s + \lambda_{eq}(s) \quad (10)$$

where Δu_s is the switching term defined as

$$\Delta u_s = k_{eq}g(e,\dot{e})\text{sign}(s) \quad (11)$$

Since on the sliding surface both the switching function s and its time derivative $\dot{s}$ are zero, we are only able to directly estimate equivalent control outside the sliding regime. From this point of view, we can simply assume that the equivalent control is proportional to the estimated switching function that can be considered as an integral of rate of the switching function $\dot{s}$.

Considering equation (6), we can write for the estimated equivalent control with a proportional constant $k_\delta > 0$

$$\hat{u}_{eq} = k_\delta \hat{s} = k_\delta \int_0^t (\lambda(s) + g(e,\dot{e})\text{sign}(s))d\tau \quad (12)$$

In this study, $\lambda(s), \lambda_{eq}(s)$ and $g(e,\dot{e})$ have been taken as

$$\lambda(s) = \gamma_1 s,\ \lambda_{eq}(s) = \gamma_2 s,$$

$$g(e,\dot{e}) = \text{abs}(e) + (1/c).\text{abs}(\dot{e}) \quad (13)$$

where $\gamma_1 > 0$, $\gamma_2 > 0$, $k_\delta > 0$ and $c > 0$.

Considering the equations (9) - (13), our new control law can be expressed as follows

$$u = k_p e + k_i \int e.d\tau + k_d \dot{e} + d_p .g(e,\dot{e}).\text{sign}(s) + d_i \int g(e,\dot{e}).\text{sign}(s)d\tau \quad (14)$$

where $k_p, k_i, k_d d_p$ and d_i are positive parameters to be selected by the designer for the desired performance. It is obvious that this control law is a classical PID plus sliding mode type PI control.

3. THE NEW ALGORITHM

It is desired that the output response of the controlled system should be as fast as possible with minimum overshoot while using minimum amount of control effort with minimum chattering phenomena. The ideal trajectory corresponding to this aim and a sliding surface in $\{e,\dot{e}\}$-space is shown in figure 2. Origin is a stable node.

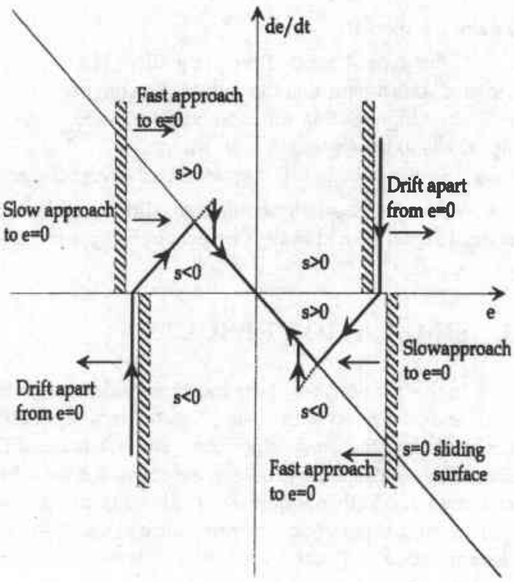

Figure 2. Ideal trajectory of a stable sliding mode control system in {e, de/dt}-space whose origin is a stable node

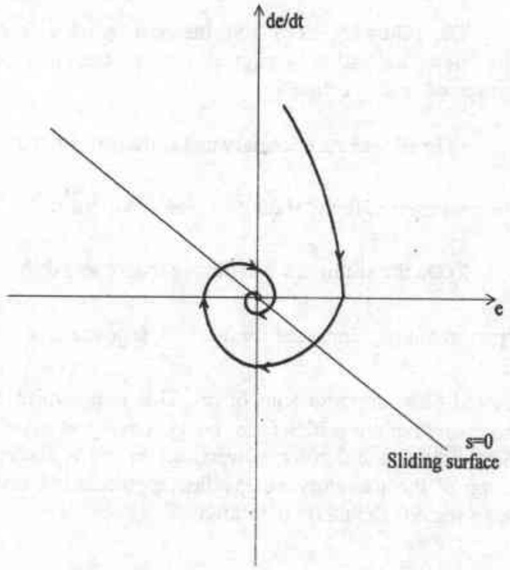

Figure 3. Real trajectory of a stable sliding mode control systems in {e,de/dt}-space whose origin is a stable focus

The arrow on the trajectory points in the direction taken by the trajectory as time increases. The hedged lines with arrows which are parallel to e axis illustrate the regions where error converges to zero and diverges from zero, namely stability and unstability regions of $\{e, \dot{e}\}$ -space.

Sliding surface s=0 divides the stability regions in two parts; i.e. slow approach to e=0 regions and fast approach to e=0 regions.Either if the system is not removed away from fast approach mode cleverly or not prevented to enter into it, then overshoots or undershoots on the system response is unavoidable. In such cases, we have a control system whose real trajectory with stable focus resembles to the shape in figure 3.

In order to have a sufficiently fast responding sliding mode control system with no overshoot, we should set up the control algorithm in such a way that the system has a trajectory with a stable node which resembles to the shape in figure 4.

In order to estimate the best suitable sliding mode control that results in the trajectory with a stable node as in figures 2 and 4, the lower and upper bounds of the slopes of the trajectories which correspond the rates of an admissibly smooth convergence to sliding surface s=0 should be determined in all sectors of $\{e, \dot{e}\}$ -space.

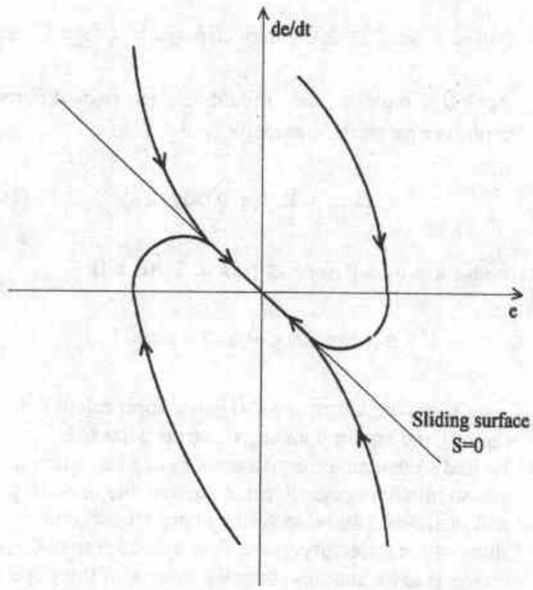

Figure 4. Desired trajectory of a stable sliding mode control system in {e, de/dt}-space whose origin is a stable node

The following steps give the basic principles of the new algorithm designed using the method presented in the section 2.

1) In all sectors, necessary and sufficient condition for convergence to the sliding surface s=0 is $s\dot{s} < 0$

2) On the sector $s\dot{e} \geq 0$ (fast approach and drift-apart regions), convergence to s=0 requires that $\ddot{e}$ should have opposite sign of $\dot{e}$. This requirement is necessary but not sufficient for being convergent to s=0. Sufficiency condition for convergence to s=0 is that the slope of the trajectory on the fast approach and drift apart regions should be as follows:

$$-\infty < (\Delta\dot{e}/\Delta e) = (\ddot{e}/\dot{e}) < -c \qquad (15)$$

To prevent overshoots and undershoots on system output response, this slope should be as large as possible in magnitude. Therefore, in fast approach ($s\dot{s} < 0$ and $s\dot{e} \geq 0$) and drift-apart ($s\dot{s} > 0$ and $s\dot{e} \geq 0$) regions one should assign two different controller parameters, namely d_1, d_2. That is,

$$\Delta u_s = k.g(e,\dot{e}).\text{sign}(s) \qquad (16)$$

where $k = d_1 > 0$ for $s\dot{s} > 0$ and $s\dot{e} > 0$

$k = d_2$ for $s\dot{s} < 0$ and $s\dot{e} > 0$.

3) On the sector $s\dot{e} < 0$ (slow approach to e=0 region), the rate of convergence to s=0 has to be limited so that an excessive amount of overshoot on s=0 should not occur. For this purpose the necessary and sufficient condition is that upper bound of the slope of the trajectory in the slow approach to e=0 region is to be kept less than the inverse of the slope of the sliding surface. Then, the admissible convergence condition to sliding surface in slow approach region becomes

$$-c < (\Delta\dot{e}/\Delta e) = (\ddot{e}/\dot{e}) < 1/c \qquad (17)$$

If the lower bound of this inequality is violated, then it means that the condition $s\dot{s} > 0$ exists. In this situation, sliding mode control signal is applied to with a gain. $k = d_3 > 0$

If the upper bound of the inequality (17) is violated, this means that the rate of the convergence to s=0 is too high. In this situation, sliding mode control signal has to be applied to with a negative gain $k = -d_4$ with $d_4 > 0$ to prevent an overshoot on s=0. When the inequality holds true, sliding mode control signal should always be produced by a positive gain $k = d_5$

3. SIMULATION RESULTS

In this section, we show the simulation results for ball and beam system using controllers optimally designed by the new algorithm and classical PID controller. Optimal controller parameters are searched and used for both of these controller structures. The genetic optimisation search procedure is used for the determination of the controller parameters. The performance index is chosen to be

$$PI = c_1 P.O. + c_2 t_p + c_3 t_s + c_4 e_{ss} \qquad (18)$$

where P.O percent overshoot, t_p peak time, t_s settling time, e_{ss} steady-state error and c_1, c_2, c_3, c_4 are weighing and scaling factors. This performance index is chosen so that it has a practical and physical meaning to the designer. Moreover, the control signal is limited a value which is equal to two times of the set value

SYSTEM: The ball and beam system shown in figure 5 is taken into consideration.

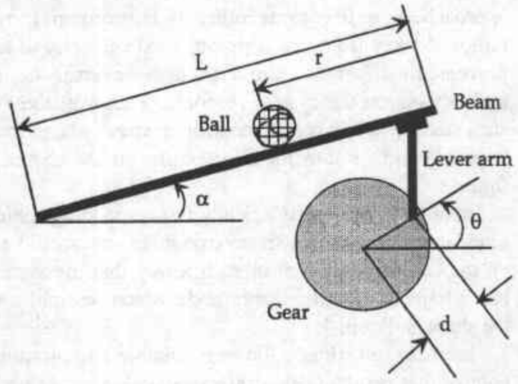

Figure 5. The Ball and Beam System

The Lagrangian equation of motion for the ball is given by the following equation

$$0 = ((J/R^2) + m)\ddot{r} + mg\sin\alpha - mr(\alpha)^2 \qquad (19)$$

The equation, which relates the beam angle to the angle of the gear can be approximated as follows

$$\alpha = (d/L)\theta \qquad (20)$$

The constants and variables are defined as

m :	The mass of the ball	0.11 kg
R :	The radius of the ball	0.015m
d :	The lever arm offset	0.03m
g :	The gravitational acceleration	$9.8m/s^2$
L :	The length of the beam	1.0m
J :	The ball's moment of inertia	9.99.e-6 kgm^2
r :	The ball position coordinate	
α :	The beam angle coordinate	
θ :	The servo gear angle	

A controller will be designed so that the ball's position can be manipulated. Figure 6, 7 and 8 show the responses of classical PID- controller, of the new rule-based quasi-sliding mode type of controller and of the PID plus rule-based quasi-sliding mode type of controller with optimum parameters due to a step input of .25 m, and a pulse disturbance of 0.25 m with duration of 0.5 seconds at time t=20sec, respectively.

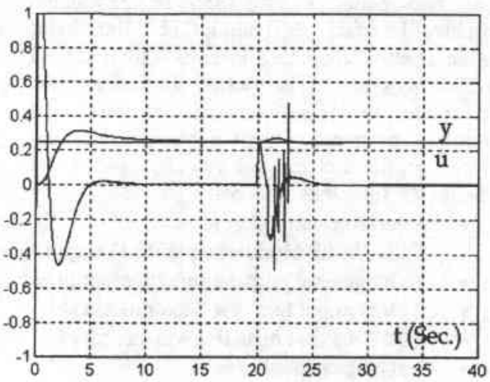

Figure 6. PID- Controller Response for Ball and Beam System

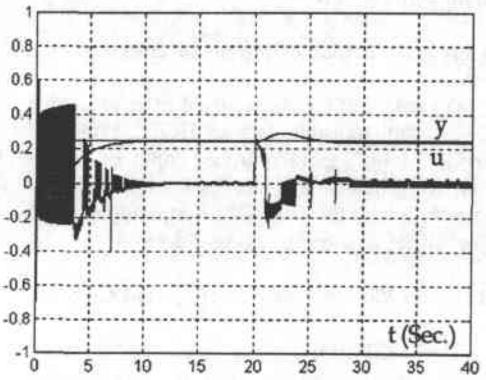

Figure 7. Quasi-Sliding Mode Type of Controller Response for Ball and Beam System.

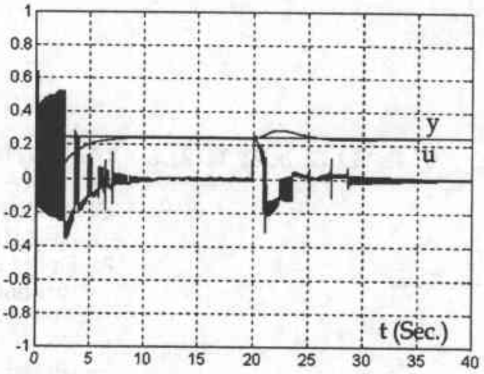

Figure 8. PID plus Rule-Based Quasi-Sliding Mode Type of Controller Response for Ball and Beam System.

It can be seen from the graphs that the new controller forces the system output to reach the reference point without any overshoot in much lesser time than the classical PID controller. Moreover, disturbance rejection and robustness of the new controller is much better than the classical PID controller

4. CONCLUSION

set(y,'Linewidth',3);In this study, a new rule-based control algorithm is presented. The algorithm is tested on the special ball and beam system and the results are compared with classical PID controller. The optimum controller parameters are searched for both PID controller and the new rule-based controller using a genetic optimisation procedure to be fair in comparison. It is seen from the results given in Section 3 that the new algorithm provided much better results than the classical PID controller based on the defined performance index.

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