Digital Chaotic Systems Examples and Application for Data Transmission

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Abstract

A number of methods have been proposed for synchronizing chaotic systems. The most widely used methods are continuous synchronization schemes. In a continuous synchronization scheme, chaotic systems are coupled to each other continuously such that synchronization errors converge to zero. In this paper, chaos synchronization in coupled discrete-time dynamical systems is presented. Especially, practical impulsive synchronization scheme for 3 discrete time chaotic systems is shown. Simulation results finally demonstrate the effectiveness of the method. Experimental results show that chaotic and hyperchaotic systems can be synchronized by impulses sampled from one or two state variables. The impulsive synchronization can be applied to almost all chaotic and hyperchaotic systems even in the case when continuous synchronization systems fail to work. The example of data transmission based on simple discrete-time chaotic systems is also presented.

1. Introduction

In the past time, synchronization of the chaotic systems has attracted considerable attention due to its great potential applications in secure communication, chemical reactions and biological systems. The first idea of synchronization between two chaotic systems with different initial conditions was introduced by Pecora and Carroll [1]. Since then, many different methods have been applied theoretically and experimentally to synchronize the chaotic systems, such as linear and nonlinear feedback control, adaptive control, back stepping control, variable structure control, impulsive control, etc. Among these methods, the impulsive control provides an efficient method to deal with the dynamical systems. Additionally, in synchronization process, the response system receives the information from the drive system only at the discrete time instants, which reduces the amount of synchronization information transmitted from the drive system to the response system and makes this method more efficient in a great number of applications. Most of the researchers just concern the synchronization between two identical chaotic systems with known parameters or identical unknown parameters. However, in reality there is hardly find the case where the structures of the drive and the response systems can be assumed to be identical. Moreover, the parametric uncertainties of the drive and response systems are always different and time-varying. Therefore, it is significant to investigate the synchronization between two little different or different chaotic systems in the presence of time-varying parametric uncertainties. A chaotic system is extremely sensitive to tiny variations of parameters. But parameters of some systems in practical circumstances cannot be exactly known in advance. The effect of these uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study synchronization in such systems with parametric uncertainties.

Impulsive synchronization had been applied to several chaotic spread spectrum secure communication systems, and had exhibited good performance [2, 3, 4]. Recently, the detailed experiments and performance analysis of impulsive synchronization were carried out for the purpose of applying impulsive synchronization to chaotic communication systems. The following experimental results were reported in [5]:

a) The accuracy of synchronization depends on both the period and the width of the impulse samples.

b) The minimum impulse width for synchronization increases as the impulse period increases.

c) The hyperchaotic systems can be synchronized by transmitting two kinds of samples through a single channel via a time-division scheme.

These experimental results showed that chaotic systems can be impulsively synchronized by using impulse samples derived from some of the state variables of the driven system. This is because some chaotic systems can be decomposed into two parts. One part tends to make the synchronization unstable and the other part tends to make the synchronization stable. If we construct impulsive controllers to stabilize the unstable part we can synchronize chaotic systems by using samples from some state variables of the driven system.

In this paper, we use variational synchronization error systems to study the stability of different impulsive synchronization schemes.

The stability of impulsive synchronization is closely connected to the values of the Lyapunov components of the variational systems. In a unidirectional directional synchronization scheme, during some time periods, the driving signal can cause the synchronization error to increase and during some other time periods it can cause synchronization error to decrease. These two kinds of time periods can be distinguished by monitoring the eigenvalues of the variational synchronization error systems. When the driving signal in a time period is detected to be more likely to increase synchronization errors, we do not use it to drive the driven system. By doing this, we can synchronize two chaotic systems much more efficiently than a continuous synchronization scheme. This kind of synchronization scheme is called selective synchronization. The selective synchronization scheme can synchronize chaotic systems which cannot be synchronized by a continuous synchronization scheme under similar conditions.

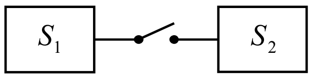

Fig. 1. Block diagram of impulse synchronization between 2 systems

2. Synchronization of chaotic systems

In this section, we only consider the impulsive synchronization between two chaotic systems with unidirectional coupling. In a uni-coupled synchronization scheme (Fig. 1), we transmit impulses sampled from one state variable of the driving system S_1 (master or transmitter) to the driven system S_2 (slave or receiver). To avoid clutter and without loss of generality, we study the case when impulse samples are equidistant. Consider the following general form of a continuous dynamical system whose state variables can be separated into two parts x_{n+1} and y_{n+1} by (1):

$$S_1: \left.\begin{aligned} x_{n+1} &= f(x_n, y_n) \\ y_{n+1} &= g(x_n, y_n) \end{aligned}\right\} \text{ transmitter} \tag{1}$$

Let us assume that samples of the state variable x_n are sent to the driven system. During the time interval when the n-th sample is send from drive system to driven system (Fig. 1, switch on), the driven system can be described by:

$$S_2: \left.\begin{aligned} \bar{x}_{n+1} &= x_n \\ \bar{y}_{n+1} &= g(x_n, \bar{y}_n) \end{aligned}\right\} \text{ receiver} \tag{2}$$

During the time interval when the switch is off, the driven system can be described as:

$$S_2: \left.\begin{aligned} \bar{x}_{n+1} &= f(\bar{x}_n, \bar{y}_n) \\ \bar{y}_{n+1} &= g(\bar{x}_n, \bar{y}_n) \end{aligned}\right\} \text{ receiver} \tag{3}$$

The synchronization error systems are given by:

$$s_{n+1} = \frac{\partial g(x_n, y_n)}{\partial y_n} s_n \tag{4}$$

or by equation (5):

$$\begin{bmatrix} r_{n+1} \\ s_{n+1} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f(x_n, y_n)}{\partial x_n} & \dfrac{\partial f(x_n, y_n)}{\partial y_n} \\ \dfrac{\partial g(x_n, y_n)}{\partial x_n} & \dfrac{\partial g(x_n, y_n)}{\partial y_n} \end{bmatrix} \begin{bmatrix} r_n \\ s_n \end{bmatrix} \tag{5}$$

where $r_n = x_n - \bar{x}_n$ and $s_n = y_n - \bar{y}_n$.

The Lyapunov exponents for a particular driven trajectory are called conditional Lyapunov exponents. Then the discrete version of Pecora and Carroll Theorem is given as follows [6]:

Theorem 1: The systems S1 and S2 will synchronize if the conditional Lyapunov expoments of the difference system (4) are all negative.

The most often way to synchronize the two systems is coupling. There are many ways of coupling. In this paper, the following coupling is used:

$$\begin{aligned} & x_{n+1} = f(x_n, y_n), \quad y_{n+1} = g(x_n, y_n) \\ & q_n = \bar{x}_n + K(x_n - \bar{x}_n) \\ & \bar{x}_{n+1} = f(q_n, \bar{y}_n), \quad \bar{y}_{n+1} = g(q_n, \bar{y}_n) \end{aligned} \tag{6}$$

or equivalently:

$$\begin{aligned} & x_{n+1} = f(x_n, y_n), \quad y_{n+1} = g(x_n, y_n) \\ & \bar{x}_{n+1} = f(\bar{x}_n + K(x_n - \bar{x}_n), \bar{y}_n) \\ & \bar{y}_{n+1} = g(\bar{x}_n + K(x_n - \bar{x}_n), \bar{y}_n) \end{aligned} \tag{7}$$

where K is a constant. The system (6), (7) is x_n coupled and it is important to note that system S_1 is not affected by S_2.

Theorem 2: The systems x_n, y_n and $\bar{x}_n$, $\bar{y}_n$ will synchronize if the conditional Lyapunov expoments of the difference system (4) are all negative and K is sufficiently close to 1.

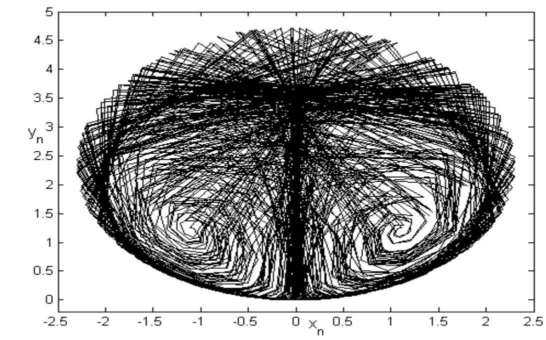

Fig. 2. The projection of state space trajectories for Lorenz digital chaotic system for a=1.2; b=0.8; x_0=0.1; y_0=0.1;

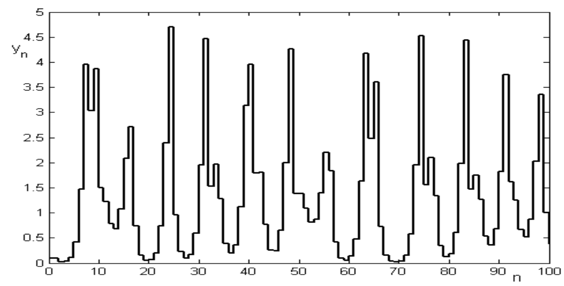

Fig. 3. Evolution of y_n for Lorenz digital chaotic system for a=1.2; b=0.8; x_0=0.1; y_0=0.1;

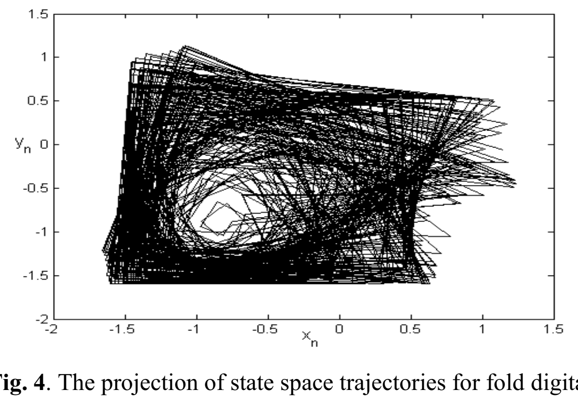

Fig. 4. The projection of state space trajectories for fold digital chaotic system for values: a=-0.09; b=-1.6; x_0=0.1; y_0=0.1;

3. Examples of digital chaotic systems

In this paper we tested the chaos synchronization of three discrete-time chaotic systems: Lorenz, fold and Rössler. The discrete-time chaotic systems have some advantages over the continuous systems. They are chaotic in wider range and are can be easily constructed by computer, microcontroller or by programmable logic. The Lorenz digital chaotic system is given by:

$$\begin{aligned} x_{n+1} &= (1+ab)x_n - bx_n y_n \\ y_{n+1} &= (1-b)y_n + bx_n^2 \\ a &= 1.2, \quad b = 0.8 \end{aligned} \tag{8}$$

The example of state space trajectories are shown in Fig. 2 and evolution of y_n in Fig. 3. Maximal Lyapunov exponent is 0.335.

The second system is "fold" discrete-time chaotic system (sometimes also called fold bifurcation, saddle-node bifurcation or tangent bifurcation). This system is simple for construction and equations of this system are given by:

$$\begin{aligned} x_{n+1} &= y_n + ax_n \\ y_{n+1} &= x_n^2 + b \\ a &= -0.1, \quad b = -1.7 \end{aligned} \tag{9}$$

The state space trajectories are shown in Fig. 3 and evolution of x_n in Fig. 4. Maximal Lyapunov exponent for this system is 0.02.

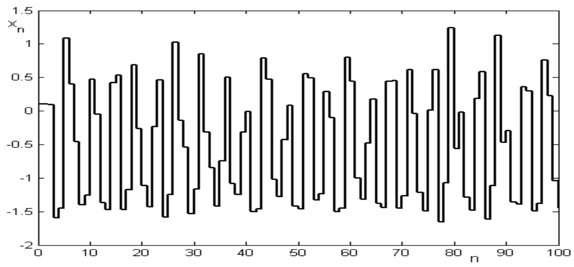

Fig. 5. Evolution of x_n for fold discrete-time chaotic system for a=-0.09; b=-1.6; x_0=0.1; y_0=0.1;

The Rössler discrete-time hyperchaotic system is described by:

$$\begin{aligned} x_{n+1} &= ax_n(1-x_n) - b(z_n + c)(1-2y_n) \\ y_{n+1} &= dy_n(1-y_n) + ez_n \\ z_{n+1} &= f\left[(z_n + c)(1-2y_n) - 1\right](1-gx_n) \\ a &= 3.8; \quad b = 0.05; \quad c = 0.35; \quad d = 3.78; \\ e &= 0.2; \quad f = 0.1; \quad g = 1.9; \end{aligned} \tag{10}$$

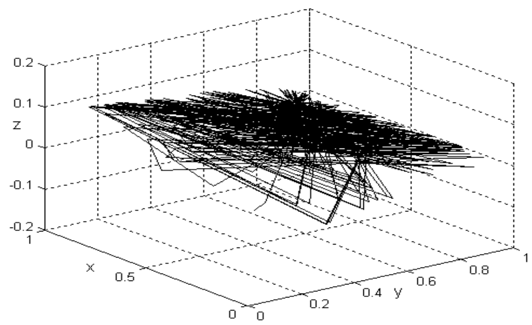

Fig. 6. The projection of state space trajectories of Rössler discrete-time hyperchaotic system for values: a=3.8; b=0.05; c=0.35; d=3.78; e=0.2; f=0.1; g=1.9; x_0=0.1; y_0=0.1; z_0=0;

The state space trajectories of this system are shown in Fig. 6, evolution of x_n, is shown in Fig. 7. Maximal Lyapunov exponent for this system is 0.41 (This system has 2 positive exponents).

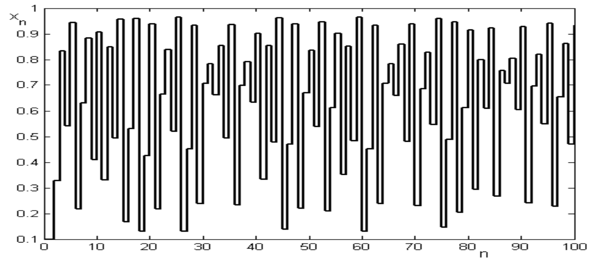

Fig. 7. Evolution of x_n for Rössler discrete-time hyperchaotic system. Constants and initial values are the same as in Fig. 6.

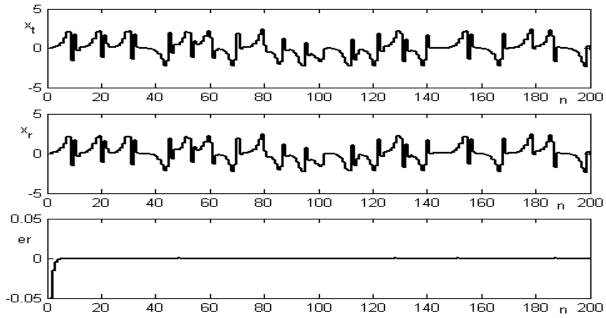

Fig. 8. Synchronization of 2 discrete-time Lorenz systems with different parameters and initial values. Transmitter state evolution x_t (top), receiver state evolution x_r (middle) and $error = x_t - x_r$ (bottom).

4. Systems synchronization

In this section, simulation examples of discrete/time chaotic systems are presented. Only unidirectional coupling cases are shown. The minimal number of controlled variables has to be equal to the number of positive Lyapunov exponents of the system. If the hyperchaotic systems have two positive Lyapunov exponents, then, at least two driving signals are needed to synchronize them [7, 8, 9, 10, 11, 12, 13, 14].

The example of synchronization of 2 Lorenz systems (S_1 and S_2) with different parameters and initial value is shown in Fig. 8. Systems parameters are:

S_1: a=1.25; b=0.75; x_0=0.05; y_0=0.05;
S_2: a=1.22; b=0.73; x_0=0.10; y_0=0.01;

Driving signal for Lorenz systems synchronization is given by d_n (generated by driving system):

$$d_n = (1+ab)x_n - bx_n y_n - \varepsilon x_n \tag{11}$$

where ε is positive constant (0<ε<1). In presented example ε=0.3. Slave system equations are:

$$\begin{aligned} \overline{x}_{n+1} &= d_n + \varepsilon \overline{x}_n \\ \overline{y}_{n+1} &= (1-b)\overline{y}_n + b\overline{x}_n^2 \end{aligned} \tag{12}$$

The example of synchronization of 2 fold systems (S_1 and S_2) with different parameters and initial value is shown in Fig. 9. Systems parameters are:

S_1: a=-0.10; b=-1.70; x_0=0.10; y_0=0.10;
S_2: a=-0.11; b=-1.71; x_0=0.05; y_0=0.05;

Driving signal for fold systems synchronization is given by d_n (generated by driving system):

$$d_n = x_n^2 + b - \varepsilon y_n \tag{13}$$

where ε is positive constant (0<ε<1). In presented example ε=0.2. Slave system equations are:

$$\begin{aligned} \overline{x}_{n+1} &= \overline{y}_n + a\overline{x}_n \\ \overline{y}_{n+1} &= d_n + \varepsilon \overline{y}_n \end{aligned} \tag{14}$$

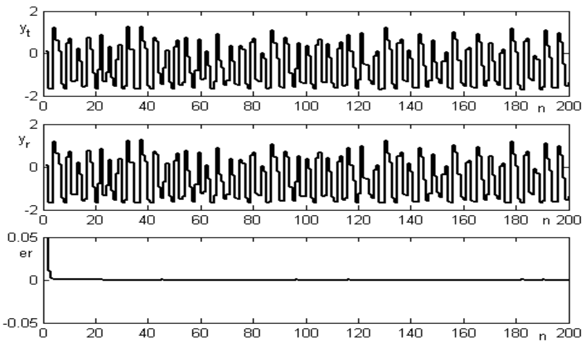

Fig. 9. Synchronization of 2 discrete-time fold systems with different parameters and initial values. Transmitter state evolution y_t (top), receiver state evolution y_r (middle) and $error = y_t - y_r$ (bottom).

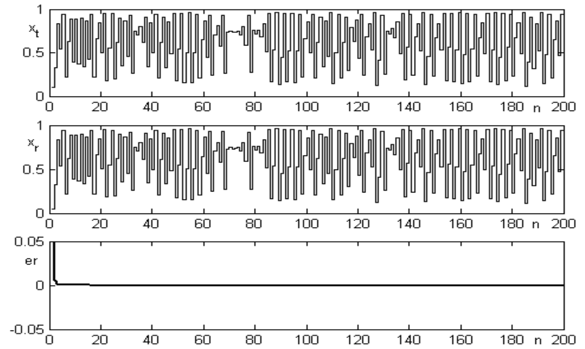

Fig. 10. Synchronization of 2 discrete-time Rössler systems with different parameters and initial values. Transmitter state evolution x_t (top), receiver state evolution x_r (middle) and $error = x_t - x_r$ (bottom).

The example of synchronization of 2 Rössler discrete-time hyperchaotic system (S_1 and S_2) with different parameters and initial value is shown in Fig. 10. Systems parameters are:

S_1: a=3.8; b=0.05; c=0.35; d=3.78; e=0.2; f=0.10; g=1.9; x_0=0.10; y_0=0.10; z_0=0;

S_2: a=3.7; b=0.07; c=0.34; d=3.80; e=0.18; f=0.12; g=2.0; x_0=0.05; y_0=0.05; z_0=0;

Driving signals for Rössler discrete-time hyperchaotic system synchronization are given by ${}^1d_n$ and ${}^2d_n$ (generated by driving system):

$$\begin{aligned} {}^1d_n &= ax_n(1-x_n) - b(z_n+c)(1-2y_n) - \varepsilon x_n \\ {}^2d_n &= dy_n(1-y_n) + ez_n - \varepsilon y_n \end{aligned} \tag{15}$$

where ε is positive constant (0<ε<1). In presented example ε=0.1. Slave system equations are:

$$\begin{aligned} \overline{x}_{n+1} &= {}^1d_n + \varepsilon \overline{x}_n \\ \overline{y}_{n+1} &= {}^2d_n + \varepsilon \overline{y}_n \\ \overline{z}_{n+1} &= f\left[(\overline{z}_n + c)(1-2\overline{y}_n) - 1\right](1 - g\overline{x}_n) \end{aligned} \tag{16}$$

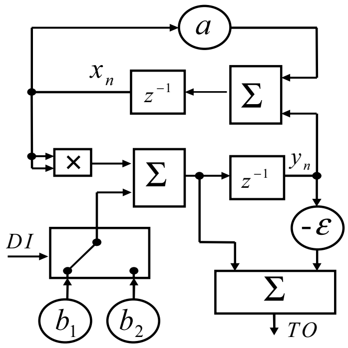

Fig. 11. Transmitter block diagram. DI – digital input, TO – transmitter output.

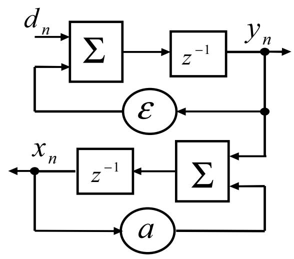

Fig. 12. Receiver block diagram.

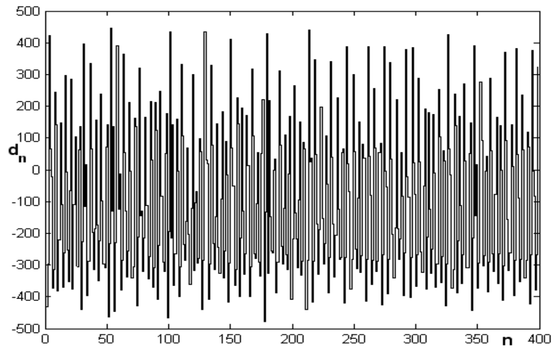

Fig. 13. Transmitted samples (10 bit word).

5. Data transmission based on chaotic systems

This part is devoted to application of discrete-time chaotic systems synchronization for digital data transmission. The two fold chaotic systems (9) are used for this purpose. The principle of transmission is based on transmitter chaotic system parameter b changing according logical signal H and L:

$$b=-1.7 \text{ for H}, \quad b=-1.65 \text{ for L} \tag{17}$$

State space variable y_n is multiplied by constant and rounds the value to the nearest integers (converted to n-bit data word) and transmitted to receiver. In receiver, first of all received signal is divided by constant and used for synchronization. On the end, the digital signal is recovered by signal processing. In this example, digital data word is transmitted:

$w =$ [0 1 0 1 0 0 0 0 0 1 0 1 1 1 0 1 0 1 0 1]

Block diagram of the transmitter and receiver are shown in Fig. 11 and Fig. 12. The transmitted signal (binary coded in 10 bit word) is shown in Fig. 13, autocorrelation value of transmitted signal in Fig. 14 and raw received signal, low-pass filtered signal and recovered digital signal are in Fig, 15.

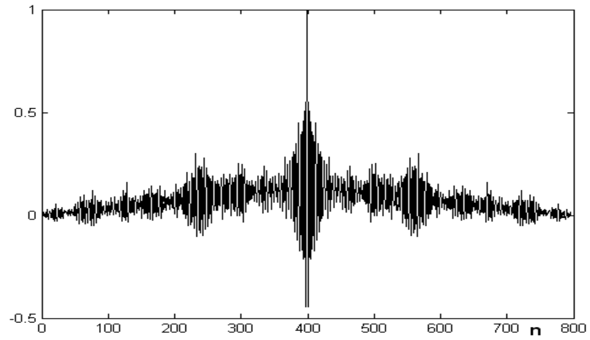

Fig. 14. Autocorrelation of transmitted signal

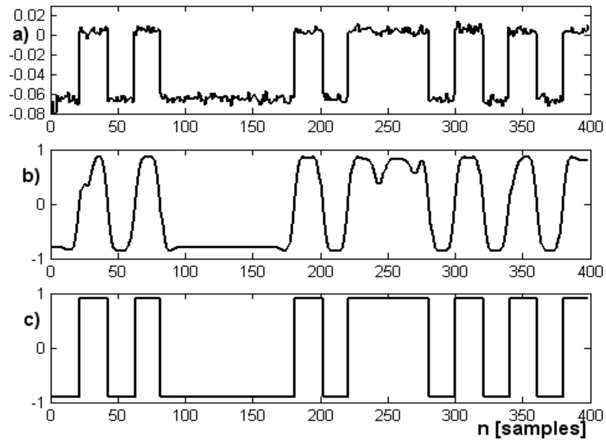

Fig. 15. Data transmission ex.: a) Raw recovered signal, b) Low-pass filtered signal, c) Recovered digital signal: $w =$ [0 1 0 1 0 0 0 0 0 1 0 1 1 1 0 1 0 1 0 1]

6. Conclusion

In this paper example of 3 discrete-time chaotic systems synchronization was shown. Also the data transmission based on chaotic system was simulated. All presented systems can be more easily realized by means of microcontroller or programmable array than continuous chaotic systems. Moreover, the discrete-time systems aren't so sensitive for chaotic behavior area as continuous systems. In future, more sophistic system for data transfer, based on chaotic systems will be developed.

7. Acknowledgment

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8. References

[1] L. M. Pecora, T. L. Carol, "Synchronization in chaotic systems", *Phy. Rev. Lett.*, vol. 64, no. 8, pp. 821-824, 1990.

[2] M. Brucoli, D. Cafanga, L. Camimeo, "Synchronization method of hyperchaotic circuits based on a scalar transmitted signal", *Proc. European Conf. Circuit Theory and Design, Stresa,* pp. 365 368, 1999.

[3] L. O. Chua, L. Kocarev, K. Eckert, M. Itoh, "Experimental chaos synchronization in Chua's circuit", *Int. J. Bifurcation and Chaos* 2(3), pp. 705-708, 1992.

[4] L. O. Chua, M. Itoh, L. Kocarev, K. Eckert, "Chaos synchronization in Chua's circuit", *J. Circuits Syst. Comput.* 3(1), pp. 93-108, 1993.

[5] M. Itoh, T. Yang, L. O. Chua, "Experimental study of impulsive synchronization of chaotic and hyperchaotic circuits", Int. *J. Bifurcation and Chaos* 9(1), pp. 1393-1424, 1999.

[6] K. Pyragas, "Predictable chaos in slightly perturbed unpredictable chaotic systems", *Phys. Lett. A181*, pp. 203 210, 1993.

[7] M. Itoh, "Spread spectrum communication via chaos", *Int. J. Bifurcation and Chaos* 9(1), pp. 153-213, 1999.

[8] A. Panas, I. Yang, L. O. Chua, "Experimental results of impulsive synchronization between two Chua's circuits", *Int. J. Bifurcation and Chaos* 8(3), pp. 639-644, 1998.

[9] T. Stjanovski, L. Kocarev, U. Parlitz, "Digital coding via chaotic systems", *IEEE Trans. Circuit Syst.* 44(6), pp. 562-565, 1997.

[10] C. W. Wu, T. Yang, L. O. Chua, "On adaptive synchronization and control of nonlinear dynamical systems", *Int. J. Bifurcation and Chaos* 6(3), pp. 455-471, 1996.

[11] Li. Chang-Pin, G. Chen, "A note on Hopf bifurcation in Chen's system", *Int. J. Bifur. Chaos*, no.13, pp.1609–1615, 2003.

[12] Li. Chang-Pin, G. Chen, "Chaos in the Chen's system with a fractional order", *Journal: Chaos, Solitons and Fractals*, 2004, 22, pp. 443–450.

[13] Y. Yan,. Li. Chang-Pin, "On synchronization of three chaotic systems", *Journal: Chaos, Solitons and Fractals*, no. 23, pp.1683–1688, 2005.

[14] Z. Peng, Y. Li, X. Liao, C. Li, "Generalized projective synchronization for the chaotic Lorenz system and the chaotic Chen system", *Journal of Shanghai University (English Edition),* Shanghai University Press, ISSN 1007-6417 (Print) 1863-236X (Online), Volume 10, No: 4, August, 2006.