Modeling and ARX Identification of a Quadrotor MiniUAV

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Abstract

In this work, a miniature-sized, radio controlled quadrotor is modeled and a black-box model is found using real-time flight data. The quadrotor which is used in this work is equipped with a special telemetry circuit to collect real-time data. Euler angles versus motor speeds data is used to identify the nonlinear rotational subsystem of the quadrotor as a quasi-linear ARX (auto-regressive exogenous) model. The ARX model performances are tested and found quite satisfactory.

1. Introduction

Quadrotors are very popular because of their ability to hover and vertical takeoff and landing (VTOL). As they are commonly remote-controlled vehicles, a modeling and control problem arises. It is possible to say that the first mini quadrotor concept works started to appear in early 2000s and then it turned into a development race. Different modeling techniques including Newton-Euler and Euler-Lagrange are used by researchers. Then the nonlinear dynamics of quadrotor are controlled using several controller approaches. Proportional-integral-derivative (PID and other combinations), integral backstepping, feedback linearization and fuzzy controller techniques are applied.

In the previous work [1], a Crazyflie quadrotor is used. Real-time flight data is collected and processed to obtain a grey-box quadrotor dynamical model to represent the rotational flight dynamics of the quadrotor. The resulting model is compared to the flight data and some conclusions are given.

In this work, the dynamical model for a plus-type quadrotor is given. After that the Crazyflie quadrotor is flown and real-time data is collected. Then this data is analyzed using Matlab to find an appropriate ARX (auto-regressive exogenous) model.

2. Dynamical Model of Quadrotor

A quadrotor model can be evaluated in two parts: the motor and the body dynamics. The body dynamics can be divided into another two: rotational and translational dynamics. See fig. 1.

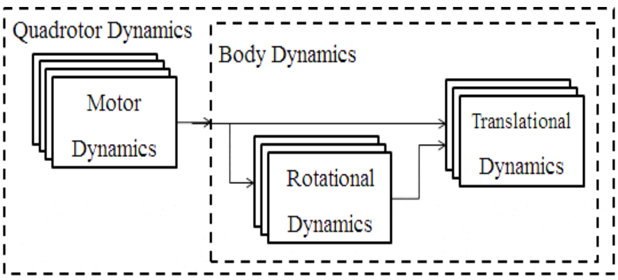

Fig. 1. The sub-blocks of the model

2.1. Reference Coordinate Frames

Before giving the model equations, the coordinate frames and Euler angles have to be described. See fig. 2.

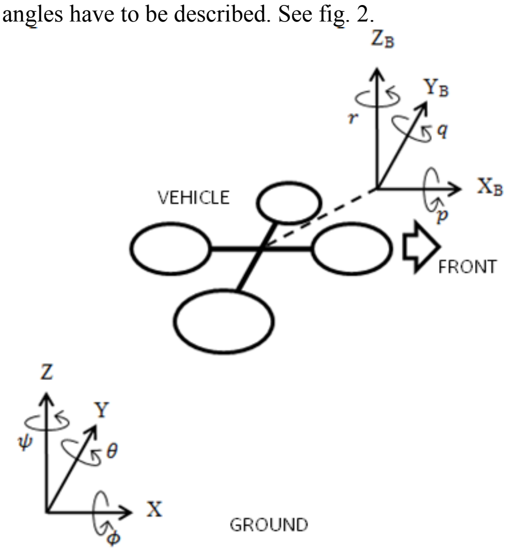

Fig. 2. Reference frames, rotation angles and rotation rates

The position and orientation of the quadrotor will be given relative to a fixed coordinate frame, which is called the inertial frame. The X and Y axes of the frame are placed parallel and coincident to the ground while Z axis is pointing upwards in a right handed configuration. This configuration is also described by East, North, Up (ENU) coordinates.

$$\zeta \triangleq (x, y, z) \in \mathbb{R}^3 \quad (1)$$

The mobile frame placed to the center of gravity of the quadrotor is called the body frame. The X_B axis points the forward direction of the quadrotor, the Y_B axis points to the left and the Z_B axis points up in a right-handed configuration.

2.2. Euler Angles and Rotations

The most important problem in quadrotor control is its orientation in space. The well-known Euler angles representation is suitable for this purpose. The Euler angles □ (roll), ψ (pitch) and ψ (yaw) are the rotation angles about the axes X, Y and Z respectively (see fig 2).

$$\eta \triangleq (\psi, \theta, \phi) \in \mathbb{R}^3 \quad (2)$$

The yaw-pitch-roll (YPR or ZYX) composite rotation matrix [2] that transforms an orientation from the body frame to the inertial frame is given below.

$$R = R_z R_y R_x \tag{3}$$

The angular velocities (attitude rates) of the quadrotor: p, q and r are shown in fig. 2.

$$\Omega \triangleq (p, q, r) \in \mathbb{R}^3 \tag{4}$$

2.3. Thrusts and Torques

All motor on the quadrotor contribute to the main thrust (lift) relative to their angular velocities. It is possible to say that the motor M_i produces the force f_{Mi}, which is proportional to the square of the angular speed and the total thrust is the sum of the individual thrusts. This is given below where k is the lift constant and ω is the speed of motor Mi.

$$f_{Mi} \triangleq k\omega_{Mi}^2 \tag{5}$$

$$F_B = \begin{bmatrix} 0 \\ 0 \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k\sum\omega_{Mi}^2 \end{bmatrix} \tag{6}$$

The torques that act about the roll and the pitch axes are given below.

$$\tau_\phi = f_{M4} - f_{M2} \tag{7}$$

$$\tau_\theta = f_{M3} - f_{M1} \tag{8}$$

The torque about the yaw axis is different from the ones above, and expressed as follows where b is the drag constant.

$$\tau_\psi = b(-\omega_{M1}^2 + \omega_{M2}^2 - \omega_{M3}^2 + \omega_{M4}^2) \tag{9}$$

$$\tau_B = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} k(\omega_{M4}^2 - \omega_{M2}^2) \\ k(\omega_{M3}^2 - \omega_{M1}^2) \\ b(-\omega_{M1}^2 + \omega_{M2}^2 - \omega_{M3}^2 + \omega_{M4}^2) \end{bmatrix} \tag{10}$$

The torques and thrusts are shown in fig. 3.

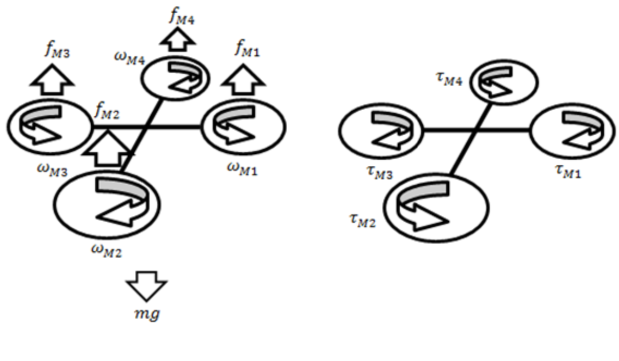

Fig. 3. Torques and thrusts

2.4. Equations of Quadrotor Body

This work only covers the rotational dynamics of quadrotor due to the lack of precise motion capture system.

It can be said that a traditional quadrotor body is symmetrical for X_B, Y_B and Z_B axes. Thus, its moment of inertia matrix will be symmetrical.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{11}$$

The Newton-Euler based model approach is given below, where I_r represents the total moment of inertia of a rotor.

$$\omega \triangleq -\omega_{M1} + \omega_{M2} - \omega_{M3} + \omega_{M4} \tag{12}$$

$$I\dot{\Omega} = \tau_B - \Omega \times I\Omega - I_r\Omega \times \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \triangleq \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \tag{14}$$

$$\dot{p} = \frac{U_1}{I_{xx}} + \frac{I_{yy} - I_{zz}}{I_{xx}} qr - \frac{I_r}{I_{xx}} q\omega \tag{15}$$

$$\dot{q} = \frac{U_2}{I_{yy}} + \frac{I_{zz} - I_{xx}}{I_{yy}} pr + \frac{I_r}{I_{yy}} p\omega \tag{16}$$

$$\dot{r} = \frac{U_3}{I_{zz}} + \frac{I_{xx} - I_{yy}}{I_{zz}} pq \tag{17}$$

If the Euler angles □, θ, ψ assumed to be small, these body frame model can be generalized to the inertial frame. The gyroscopic forces are also considered small and they can be neglected. Finally, the following nonlinear equations of the rotational dynamics are obtained.

$$\ddot{\phi} = \frac{U_1}{I_{xx}} \tag{18}$$

$$\ddot{\theta} = \frac{U_2}{I_{yy}} \tag{19}$$

$$\ddot{\psi} = \frac{U_3}{I_{zz}} \tag{20}$$

2.5. Equations of the Motor Model

A brushed DC motor model will be sufficient for most of the quadrotors. The model is nonlinear because of the nonlinear aerodynamic load of the propeller ($k_L = k + b$). See fig. 4.

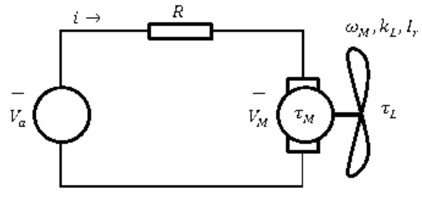

Fig. 4. Motor model

The parameterized model is given below, where u is the armature voltage and the state variable x is the motor speed.

$$\dot{x} = a_0 u - a_1 x - a_2 x^2 \tag{21}$$

The pulse width modulation (PWM) duty can be converted to the armature voltage as given below.

$$0 \leq PWM_{Duty} \leq 1, V_a = V_{Supply} * PWM_{Duty} \tag{22}$$

3. Quadrotor Platform

Crazyflie quadrotor is used in this work. It is a miniature quadrotor that enables the implementation of low-level software to run on its onboard microcontroller. It has a radio communication link to a computer. It can be controlled using a generic USB joystick. See fig. 5.

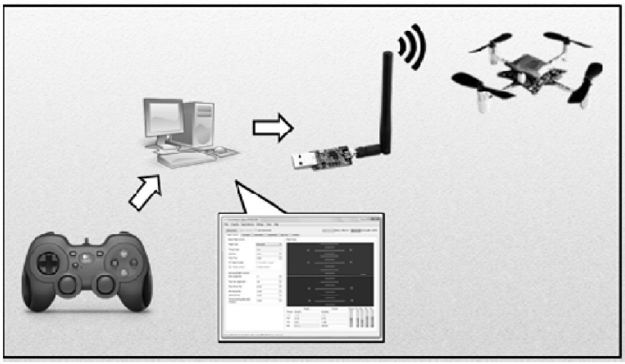

Fig. 5. Motor model

3.1. Motor Model Parameters

To identify the motors, a special speed sensing circuit is implemented to one of the motors. The motor speed, the motor PWM and the battery voltage are recorded on the ground, exciting the motor giving throttle (in open-loop). Then the PWM is converted to the armature voltage using the battery (supply) voltage. Then the motor parameters are estimated using grey-box estimation [1]. The final model is given below.

$$\dot{\omega}_{Mi} = 0.862446V_a - 6.814507\omega_{Mi} - 0.006149\omega_{Mi}^2 \quad (23)$$

This model is used on the collected data to obtain the motor speeds from the PWM values. These speeds are crucial for the rest of this work.

4. Identification

A mathematic model for an unknown system can be obtained using collected input-output data. This is called the black-box modeling approach.

4.1. Input-Mixer and Quasi-Linear Quadrotor Model

To simplify things, a linear model is the best. The quadrotor model is nonlinear but using an input-mixer, it can be taught as a linear system. See fig. 6.

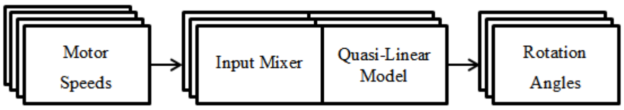

Fig. 6. The input-mixer and the quasi-linear model

The input-mixer equation is given below.

$$u' = \begin{bmatrix} u'_1 \\ u'_2 \\ u'_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{k}U_1 \\ \frac{1}{k}U_2 \\ \frac{1}{b}U_3 \end{bmatrix} \quad (24)$$

Thus, the model below becomes linear when the output of the input-mixer is computed.

$$\ddot{\phi} = c_0 u'_1 \;|\; c_0 = \frac{k}{I_{xx}} \quad (25)$$

$$\ddot{\theta} = c_1 u'_2 \;|\; c_1 = \frac{k}{I_{yy}} \quad (26)$$

$$\ddot{\psi} = c_2 u'_3 \;|\; c_2 = \frac{b}{I_{zz}} \quad (27)$$

4.2. ARX Model Structure

The auto-regressive model that has exogenous input is shown in fig. 7 where $u(t)$ is input, $y(t)$ is output, $w(t)$ is white noise, $B(q)$ and $1/A(q)$ are the output and the noise transfer functions.

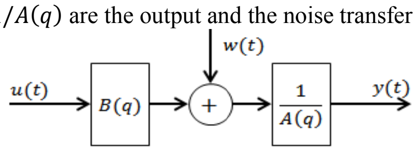

Fig. 7. ARX model

The ARX model can be expressed by the following polynomial where na is the number of poles, nb-1 is the number of zeros and nk is pure time delay.

$$\mathrm{Y(t) + a_1y(t-1) + \cdots + a_{na}y(t-na) =}$$
$$\mathrm{b_1y(t-nk) + \cdots + b_{nb}y(t-nk-nb+1) + w(t)} \quad (28)$$

The model parameters a_i and b_i are found using least-squares method to minimize the variation between the ARX model and the measured input-output data.

4.3. Identification Process

To find a suitable ARX model, a Matlab script is prepared. It selects a limited amount of combinations for na, nb and nk coefficients then trains an ARX model with this coefficients and the collected data, iteratively.

Some clues are inspected and some assumptions are made, to reduce the number of trials.

A priori-knowledge about the model is its order and causality. It is known that the nonlinear model is of second order (for each axis). As a second thought, the model is not related to the past inputs.

From this starting point, it can be said that the order parameter (na) might be 2 and the input dependency parameter (nb) might be 1. Finally, the input delay parameter (nk) is guessed to be non-zero but smaller than 10.

Model orders are given such that ARX (na,nb,nk).

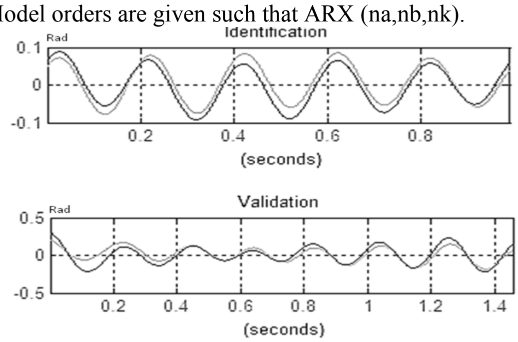

Fig. 8. Roll axis, ARX(2,1,0)

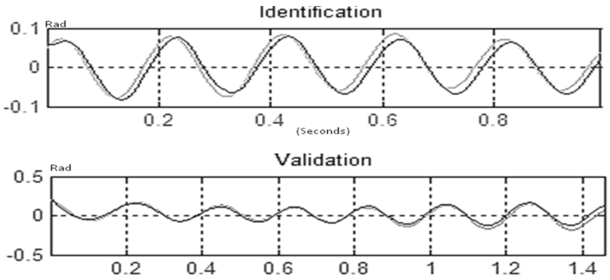

Fig. 9. Roll axis, ARX(2,2,0)

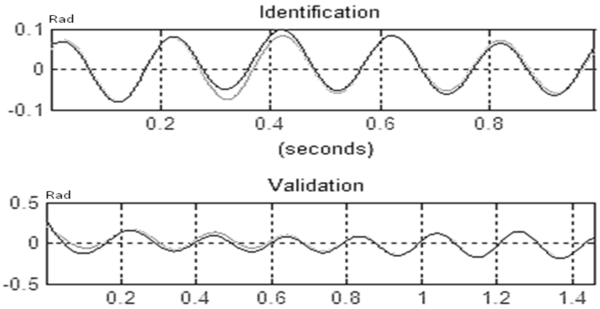

Fig. 10. Roll axis, ARX(2,1,8)

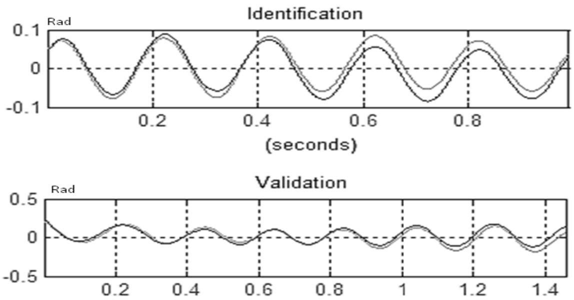

Fig. 11. Roll axis, ARX(4,2,0)

An iterative algorithm is used to test ARX models with different na, nb, nk coefficients. Then the trials are sorted with respect to their RMS error performances.

The top part of the sorted results is given in table 1.

Table 1. Roll axis models and their performances

Model Configuration	RMS Error Identification	RMS Error Validation
ARX (2 ,2 , 0)	0,0134	0,0277
ARX (2 ,1 , 8)	0,0103	0,0296
ARX (2 ,1 , 9)	0,0099	0,0301
ARX (2 ,1 , 7)	0,0127	0,0304
ARX (2 ,1 , 6)	0,0140	0,0309
ARX (3 ,1 , 4)	0,0184	0,0332
ARX (4 ,2 , 0)	0,0192	0,0340
ARX (2 ,2 , 9)	0,0122	0,0345
ARX (2 ,2 , 6)	0,0157	0,0367
ARX (3 ,1 , 9)	0,0205	0,0388
ARX (3 ,2 , 9)	0,0207	0,0392
ARX (2 ,1 , 5)	0,0232	0,0396

Responses of ARX(2,2,0) and ARX(2,1,8) models to positive and negative step inputs are given in the following figures.

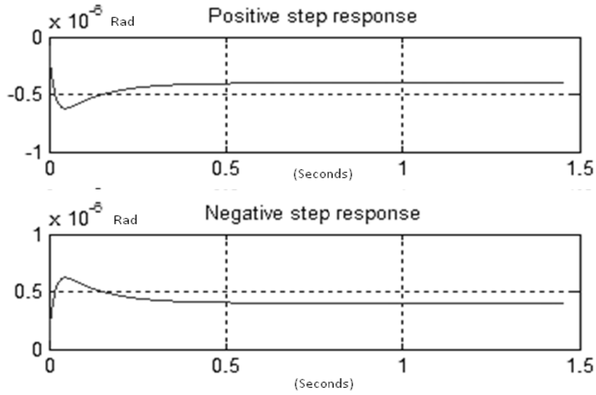

Fig. 12. Roll axis, ARX(2,2,0), step responses

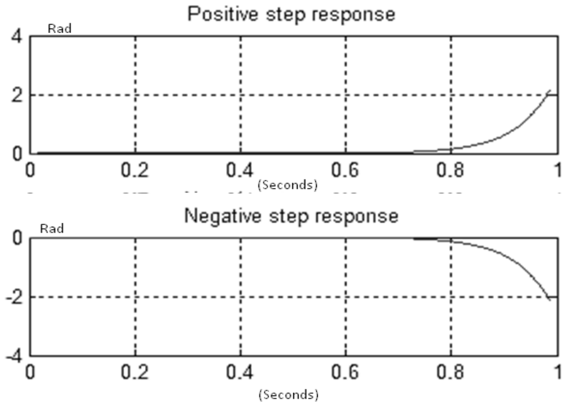

Fig. 13. Roll axis, ARX(2,1,8), step responses

The model that gives the best performance is ARX (2,2,0). But the amplitude of its step response is very small and cannot be true.

It can be clearly seen that the positive step response makes the output of ARX (2,1,8) positive while the negative step response makes it negative. The amplitudes are growing exponentially as expected. This configuration is the choice for this axis. The explicit model is given below.

$$\phi(z) = \frac{4.222\times10^{-8}}{(1-1.959\,z^{-1}+0.9567\,z^{-2})}(\omega_{M4}^2-\omega_{M2}^2)z^{-8} \quad (29)$$

The shape of the quadrotor is symmetric about X and Y axes. Thus, the roll axis model can be used for the pitch axis. Performance of this model with pitch axis data is shown below.

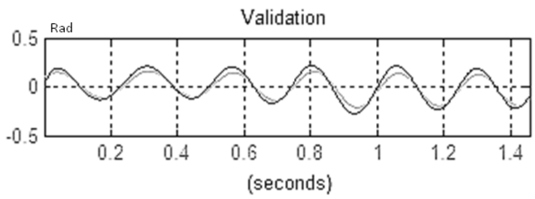

Fig. 14. Roll axis, ARX(4,2,0)

This result proves the assumption.

$$\theta(z) = \frac{4.222\times10^{-8}}{(1-1.959\,z^{-1}+0.9567\,z^{-2})}(\omega_{M3}^2-\omega_{M1}^2)z^{-8} \quad (30)$$

The best configuration for the yaw axis is shown below.

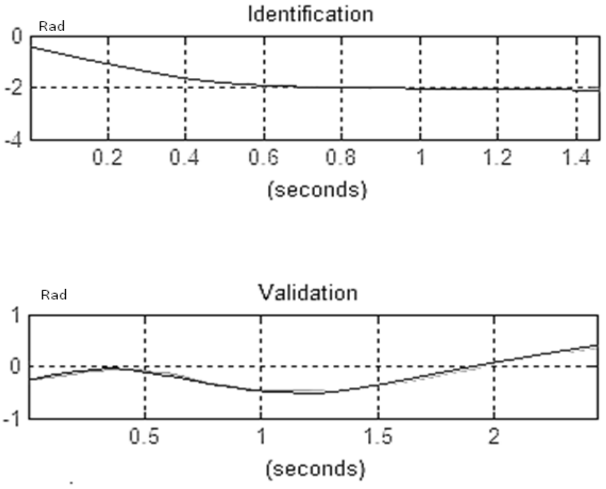

Fig. 15. Yaw axis, ARX(2,1,8)

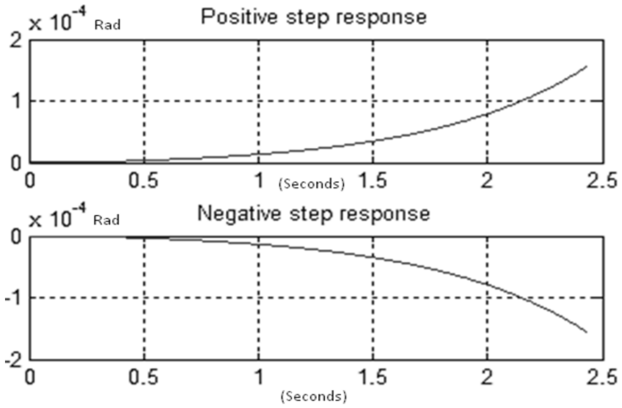

Fig. 16. Yaw axis, ARX(2,1,8), step responses

Table 2. Yaw axis models and their performances

Model Configuration	RMS Error Identification	RMS Error Validation
ARX(2 ,1 , 8)	0,0129	0,0140
ARX(3 ,2 , 8)	0,0124	0,0172
ARX(2 ,1 , 7)	0,0142	0,0277
ARX(2 ,2 , 8)	0,0096	0,0419
ARX(2 ,2 , 7)	0,0115	0,0461
ARX(3 ,2 , 7)	0,0113	0,0573
ARX(2 ,1 , 9)	0,0091	0,0694
ARX(2 ,2 , 6)	0,0107	0,0746
ARX(3 ,1 , 9)	0,0089	0,0795
ARX(4 ,2 , 9)	0,0103	0,0849
ARX(2 ,1 , 8)	0,0129	0,0140
ARX(3 ,2 , 8)	0,0124	0,0172

ARX(2,1,8) model gives the best performance and its step responses are reasonable. Thus, it is the chosen model.

The explicit form of the yaw axis model is given in the equation below.

$$\psi(z) = \frac{1.382\times 10^{-9}}{(1 - 1.995\, z^{-1} + 0.995\, z^{-2})} * (-\omega_{M1}^2 + \omega_{M2}^2 - \omega_{M3}^2 + \omega_{M4}^2) z^{-8} \qquad (31)$$

5. Conclusions

In this work, a quadrotor is modeled using Newton-Euler approach. And also the motor of the quadrotor is modeled.

The Crazyflie quadrotor has no speed sensing circuitry for the motors. The estimated motor model is used to obtain the motor speeds from the collected flight data.

Motor speeds versus Euler angles data is used to identify ARX models. A different approach is used to obtain a quasi-linear model that simplifies the ARX modeling of the nonlinear quadrotor model.

The final ARX models have nk=8 showing that the quadrotor software has a relatively big measurement delay. This delay may be prevented using more effective sensor fusion algorithms.

Validation RMS error performances of the models are satisfactory, meaning that the work has achieved its goal.

6. References

[1] Sarıoğlu, Atakan "Modeling and Parameter Estimation of a Quadrotor MiniUAV" EUCASS-2015, Krakow, Poland.

[2] Schilling, Robert J. Fundamentals of robotics: analysis and control. Vol. 13. New Jersey: Prentice Hall, 1990.