A STUDY ON NUMERICAL COMPUTATION OF POTENTIAL DISTRIBUTION AROUND A GROUNDING ROD DRIVEN IN SOIL

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ABSTRACT

In this study, numerical computation of 3D potential distribution around a grounding rod is presented. The problem is modelled in the cylindrical coordinates as an axisymmetric field problem and easily solved using finite difference method by a computer program based on a different approach.

I. INTRODUCTION

The grounding systems are the most important part of an electrical system from the point of view of the safety of people and equipment. The safety, reliability, and correct operation of electrical power systems depend on the quality of the design and construction of their grounding systems. The grounding systems serve multiple purposes. Not only they do insure a reference potential point for the electric devices but also provide a low resistance path for fault currents into the ground. Such fault currents can arise either from internal sources or from external ones e.g. by short circuits and lightning strokes.

As the simplest and most economical form of grounding electrodes, vertical grounding rods are widely used in practical grounding systems. At different conditions and according to different parameters, behavior of the grounding rods has been investigated for many years. The main objective of these investigations is accurately to calculate grounding resistance, potential and electric field distributions of the grounding rods or electrodes.

The potential distribution around a grounding rod can be easily obtained by using the finite difference method. The potential value that occurs on grounding system depends on kind of materials and dimensions of grounding system and properties of soil. High currents which happened short circuit in results of flashover, breakdown, failure and touch events flow through grounding system into ground. Voltage on the grounding electrode that depends on grounding resistance or generally grounding impedance appeared by means of high currents. Grounding resistance depends on resistivity of soil and geometrical and material properties of grounding electrodes. It is chain of dependence might seen simple but become popular for many researchers [1-4]. Determination of the grounding resistance is very important for theoretical and experimental studies. It depends on geometry, dimension, and buried depth of grounding electrode and soil resistivity [1 - 6].

In the literature i.e. regulations and standards, it is given a lot of empirical, analytical and numerical formulas and methods. In addition, it is explained a lot of measurement method for determination of the grounding resistance [7, 8]. When a current flows through the grounding electrode into ground, a voltage is induced on the electrode. At the end of this event potential distribution occurs between the grounding electrode and the reference ground (Figure 1).

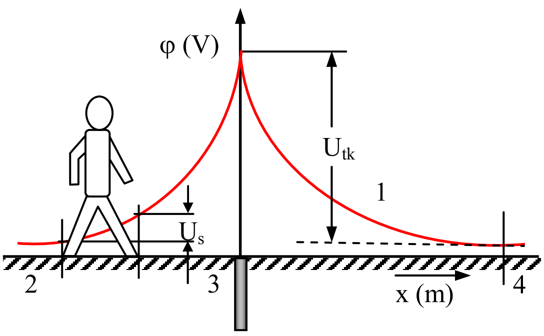

Figure 1. Potential distribution on the soil surface of a vertical grounding rod. φ(V): ground potential, x(m): distance from the rod, Utk: ground potential rise, Us: step voltage, 1: potential distribution, 2: soil, 3: grounding rod, 4: reference ground.

Reference ground is a place where the potential value is accepted zero. In practice, reference ground is accepted as a ground part approximately 20 meters distance from grounding electrode. Potential distribution around a grounding electrode gives touch and step voltages.

Touch voltage is defined as the potential difference between a grounded metallic structure or a grounding electrode and a point on the earth's surface separated by a distance equal to the normal maximum horizontal reach, approximately one meter. Step voltage is defined as the potential difference between two points on the earth's surface, separated by a distance of one pace that will be assumed to be one meter, in the direction of maximum potential gradient. Both of them should be no dangerous values. For this reason, it is needed to known potential distribution around the grounding electrode. In application, it is used a lot of various formulas to obtain this knowledge [7, 8]. In this study, potential distribution around a grounding rod electrode is calculated by using the finite difference method.

II. FINITE DIFFERENCE METHOD

The finite difference method (FDM) is a numerical method for solving partial differential equations. This method can be applied to electrostatic field problems governing Laplace and Poisson equations with different boundary shapes and different kinds of boundary conditions. The basic idea of FDM is to replace the derivatives of an unknown function by the finite difference equivalents of unknown functions at a set of finite discretization points. The form of finite difference equations depends on the form of the domain discretization. In principle domain can be divided into an arbitrary grid. In order to simplify matters the square grid shown in Fig. 2 is adopted.

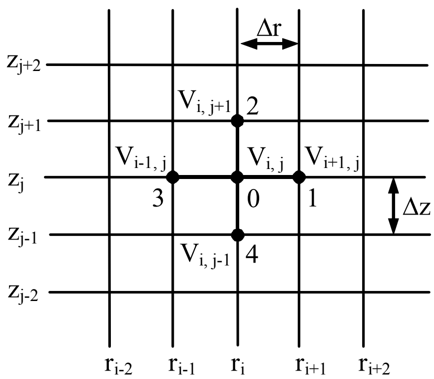

Figure 2. Grid nodes in r-z plane subdivided by square meshes in the cylindrical coordinates

In Fig. 2 the distances between grid lines are called steps or mesh lengths while each intersection of grid lines is called a node. After the domain is subdivided into grid the continuous function is replaced by a great number of discretized values at these nodes. The original partial differential equation is then transformed in to a set of algebraic equations. The solution of these simultaneous equations is the approximate solution of the original boundary value problem [9-11].

FDM is used in both two dimensional (2D) and three dimensional (3D) cases in cartesian, polar and cylindrical coordinates. The low frequency fault current is known as the main part of the fault current. The performance of the grounding electrodes under low frequency currents is almost the same as it is under the direct current current. So, potential and electric field distribution can be analyzed in static electric field. Thus, problem in this study has been considered as an axisymmetric Laplacian field problem in the cylindrical coordinates.

At the solution of axisymmetric fields in the cylindrical coordinates (r, z, θ) it can be assumed that the z-axis of the coordinates coincides with the axis of symmetry. The potential distribution is independent of the coordinate θ, i.e. $\partial V/\partial\theta$. Thus the expression of Laplace's equation in cylindrical coordinates becomes

$$\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{\partial^2 V}{\partial z^2}=0 \tag{1}$$

Where, r and z are the cylindrical coordinates, V = V(r, z) is electric potential as a function of r and z coordinates. In the case of Δr = Δz = h, the central finite difference approximations of the partial derivatives of V at the node (i, j) are

$$\frac{d^2V}{dr^2}\cong\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{h^2} \tag{2}$$

$$\frac{dV}{dr}\cong\frac{V_{i+1,j}-V_{i-1,j}}{2h} \tag{3}$$

$$\frac{d^2V}{dz^2}\cong\frac{V_{i,j+1}-2V_{i,j}+V_{i,j-1}}{h^2} \tag{4}$$

Introducing these approximations into Eq. (1) the following equation is obtained

$$\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{h^2}+\frac{1}{r_{i,j}}\frac{V_{i+1,j}-V_{i-1,j}}{2h}+\frac{V_{i,j+1}-2V_{i,j}+V_{i,j-1}}{h^2}=0 \tag{5}$$

This equation can be rewritten the following form

$$(1+\frac{h}{2r_{i,j}})V_{i+1,j}+(1-\frac{h}{2r_{i,j}})V_{i-1,j}+V_{i,j+1}+V_{i,j-1}-4V_{i,j}=0 \tag{6}$$

The general potential formula of the node (i, j) is then

$$V_{i,j} = \frac{1}{4}\left[\left((1+\frac{h}{2r_{i,j}})V_{i+1,j} + (1-\frac{h}{2r_{i,j}})V_{i-1,j} + V_{i,j+1} + V_{i,j-1}\right)\right] \tag{7}$$

This equation shows the potential value of any point as a relationship with the potentials of its four neighboring points.

Concerning the points located on the axis of symmetry, i.e. r = 0, the term $\frac{1}{r}\frac{\partial V}{\partial r} = 0$ then becomes indefinite. By using the extremum principle (L'Hopital's rule),

$$\lim_{r\to 0}\frac{1}{r}\frac{\partial V}{\partial r} = \left(\frac{\partial^2 V}{\partial r^2}\right)_{r=0} \tag{8}$$

Laplace's equation reduces to

$$2\frac{\partial^2 V}{\partial r^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{9}$$

or

$$2\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{h^2} + \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{h^2} = 0 \tag{10}$$

Since the field is symmetric to the axis $(V_{i+1,j} = V_{i-1,j})$, the difference formula of the points located on the axis is

$$V_{i,j} = \frac{1}{6}\left[4V_{i+1,j} + V_{i,j+1} + V_{i,j-1}\right] \tag{11}$$

III. FINITE DIFFERENCE MODEL OF THE ROD ELECTRODE

It is studied on a model in order to calculate potential distribution around a grounding rod electrode by using FDM. In the model a rod electrode having diameter of 16 mm and length of 2 m is used as a grounding rod. The grounding rod is placed vertically from the earth surface into the uniform soil. A half of hemispherical region with 20 m radius is considered as a solution region of the problem due to it has axisymmetric geometry. The solution region is divided into grid by square meshes. Mesh length, h is selected 2 m. The FDM model of the problem is showed in Figure 3. In this figure, there are 90 square meshes and 112 nodes. Furthermore, on same figure, boundary conditions of the problem used in computation are indicated.

Boundary conditions in Fig. 3 denote
B1: V = 100 Volt, on the rod electrode,
B2: r = 0, Eq. (11) at the points located on the axis of symmetry under the rod electrode is valid,
B3: V = 0 Volt, at the reference ground
B4: $\partial V / \partial n = 0$, Neumann boundary condition for the points located on the ground surface is as follows

$$V_{i,j} = \frac{1}{4}\left[\left((1+\frac{h}{2r_{i,j}})V_{i+1,j} + (1-\frac{h}{2r_{i,j}})V_{i-1,j} + 2V_{i,j-1}\right)\right]$$

The node numbers used in the model are shown in Figure 3. Nodes 11 and 102 are on the grounding rod. The potential values of these nodes are assumed as 100 Volt for normalization. Potentials for all of the nodes at 20 meters far from the grounding electrode are assumed zero volt. Therefore, potential values of boundary nodes 1, 12, 23, 34, 35, 45, 55, 56, 65, 74, 83, 84, 91, 92, 93, 94, 98, 99, 100, 101 and 112 have been taken zero. Potentials of the other nodes in the region are unknown potentials.

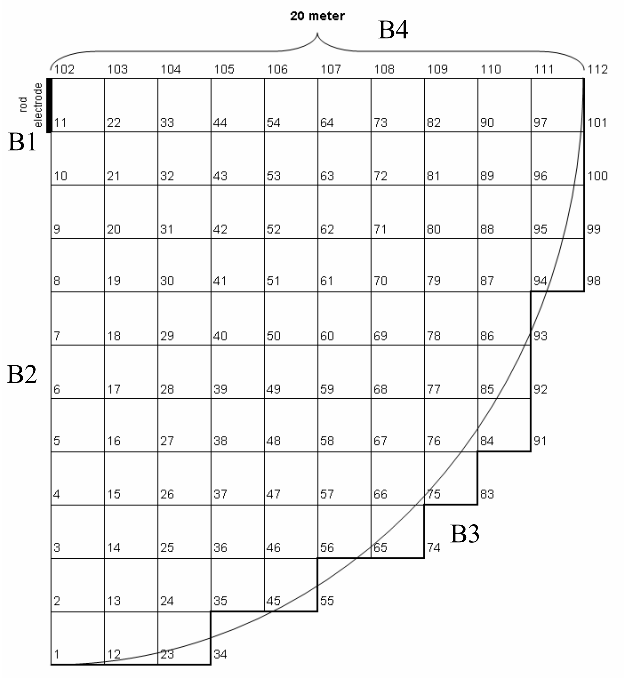

Figure 3. Grid of the problem and node numbers

Unknown node potentials are calculated via matrix equation [A].[x] = [B] written by using FDM. In this equation, [A] is coefficients matrix, [x] is unknown node potentials matrix and [B] is a matrix of known node potentials. This equation defines a set of linear algebraic equations. In our problem, number of unknown potentials is 88 and size of the coefficients matrix is 88 x 88 in the equation. This equation set is solved by using MATLAB. The potential values obtained from the solution of these simultaneous equations are shown in Figure 4.

The potential distribution on ground surface of the grounding electrode is important to calculate step and touch voltages. For the model used, step voltage is calculated as 10 V between r = 1 m and r = 2 m on the ground surface and touch voltage is calculated as 10.5 V. These voltages can be considered percent value of the electrode potential. For example, if potential of the grounding electrode is 1000 V, step and touch voltages become 100 V and 105 V, respectively.

100	79,484	63,8024	51,5883	41,5363	32,8698	25,1548	18,1463	11,699	5,7025	0
100	77,0727	62,0734	50,5111	40,8466	32,3964	24,8034	17,8671	11,4745	5,5559	0
73,3886	66,0622	56,5445	47,3227	38,8061	30,971	23,7239	16,986	10,7274	5,0122	0
59,1647	56,0837	50,0579	43,0177	35,8079	28,7585	21,9806	15,4957	9,3179	3,6609	0
49,6168	47,9161	43,7885	38,3196	32,2389	25,9614	19,6922	13,479	7,1653	0	0
42,2681	41,1102	38,0034	33,5615	28,3362	22,7276	16,9937	11,2615	5,5992	0	
36,1303	35,213	32,6413	28,8088	24,1555	19,0534	13,8106	8,5704	3,6824	0	
30,667	29,8436	27,4885	23,912	19,5445	14,7188	9,9174	4,8898	0	0	
25,525	24,6903	22,2596	18,5202	14,1398	9,1181	5,1705	0	0		
20,3547	19,4156	16,532	11,7602	7,3588	0	0	0			
14,2416	13,2172	9,5038	0	0						
0	0	0								

Figure 4. Potential values at the nodes in Volt

As a result, the three-dimensional potential distribution obtained with FDM on earth surface of the grounding rod is shown in Figure 5. This figure can be drawn at the 2D form due to the problem has an axisymmetric geometry.

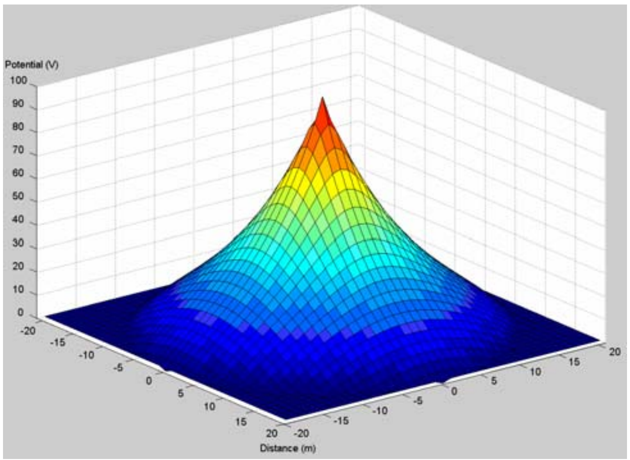

Figure 5. Three-dimensional plot of the earth surface potential distribution above the vertical grounding rod

The aim of this study is not only to determine step and touch voltages, but at the same time, potential distribution around the grounding electrode in soil is to obtain. This distribution is important for dangerous potential differences between grounding electrode and neighboring structures and for soil ionization. A division of the potential distribution around the grounding rod in the soil with 10% equipotential lines is shown in Figure 6.

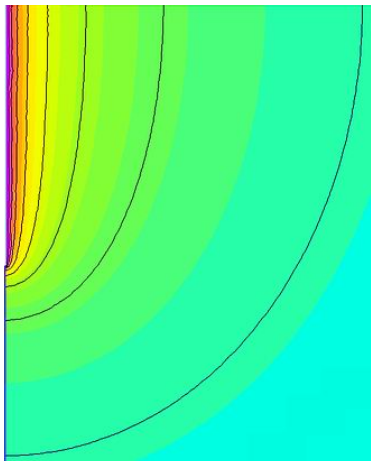

Figure 6. Potential distribution around the grounding rod

IV. CONCLUSION

In this study, an example for computation of the 3D potential distribution around a grounding rod using FDM is presented. The potential distribution is easily and truly calculated via developed software in this study. Touch and step voltages are calculated from the obtained potentials by FDM.

When the results of calculation are inspected, the potentials and potential difference per unit length are decreased by getting away from the rod electrode. The calculations can be made for different length of the rod electrode and buried depth of the rod electrode. If the calculations are made in that way (i.e. increasing length and buried depth of rod electrode), it is seen that grounding resistance and step voltages are decreased.

Consequently, in that way, computation of the potentials distribution and behaviour of grounding electrodes could be evaluated in different geometries, dimensions and buried depths. Thus, it might be designed of grounding system accurately.

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