Dynamic Behavior Analysis of Saturated Induction Motor

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Abstract

The paper demonstrates that selection of the flux linkages as state variables provides a simpler and easier way to investigate the dynamic behavior of an Induction Motor represented by the state equations model when the saturation phenomenon is considered.

1. Space Vector Model of an Induction Motor

The mathematical model of an Induction Motor can be written by means of three differential non linear equations, two complex and one real, using the space phasor approach [1].

The space phasors voltage equations of the Induction Motor are:

$$\overline{V}_1 = \overline{I}_1 r_1 + j\omega\overline{\lambda}_1 + p\overline{\lambda}_1 \tag{1}$$

$$\overline{V}_2 = \overline{I}_2 r_2 + j(\omega - \omega_2)\overline{\lambda}_2 + p\overline{\lambda}_2 \,. \tag{2}$$

The rotor quantities in Eq. 2 are referred to the stator. The speed ω is an electrical angular velocity of an arbitrarily chosen coordinate system [2] and ω_2 the electrical angular velocity of the rotor.

The equation of motion:

$$Jp\omega_r = T - T_L \,, \tag{3}$$

where ω_r is the mechanical angular velocity of the rotor, T_L the load torque and T the electromagnetic torque expressed as the product of the stator flux linkages and stator current space phasors (3).

$$T = -\frac{3}{2} P_p I_m \left(\overline{\lambda}_1 \cdot \overline{I}_1\right) \tag{4}$$

The model described by equations (1), (2) and (3) comprises three variables, voltage, current and flux linkages.

The flux linkages space phasor is related to the current space phasors by the inductance matrix [4].

In general

$$\overline{\lambda} = [L] \cdot \overline{I} \,. \tag{5}$$

2. The Leakage and Mutual Flux Linkages

It is well-known [1] that the flux linkages can be divided into leakage flux ψ and mutual flux λ. Mathematically the space phasors will be written as follows

$$\overline{\lambda}_1 = \overline{\psi}_1 + \overline{\lambda}_{12} \tag{6}$$

$$\lambda_2 = \psi_2 + \overline{\lambda}_{12} \,. \tag{7}$$

Considering (5) and taking into account that the leakage fluxes caused by stator or rotor current only and the mutual flux by both (stator and rotor) currents we can write

$$\overline{\lambda}_1 = L_1\overline{I}_1 + L_{12}\left(\overline{I}_1 + \overline{I}_2\right) \tag{8}$$

$$\overline{\lambda}_2 = L_2\overline{I}_2 + L_{22}\left(\overline{I}_1 + \overline{I}_2\right) \,, \tag{9}$$

where L_1 is the stator leakage inductance, L_2 the rotor leakage inductance and L_{12} the mutual inductance.

3. Induction Motor Model with Current Space Phasors as State Variables

Replacing in the voltage equations, (1) and (2) the flux linkages space phasors $\overline{\lambda}_1, \overline{\lambda}_2$ by expressions (8),(9) the current model will result in:

$$\begin{aligned} \overline{V}_1 &= \overline{I}_1 r_1 + j\omega L_1 \overline{I}_1 + j\omega L_{12}\left(\overline{I}_1 + \overline{I}_2\right) \\ &\quad + p\left[L_{12}\left(\overline{I}_1 + \overline{I}_2\right)\right] + p\left(L_1 \overline{I}_1\right) \end{aligned} \tag{10}$$

$$\begin{aligned} \overline{V}_2 &= \overline{I}_2 r_2 + j(\omega - \omega_2) L_2 \overline{I}_2 + j(\omega - \omega_2) L_{12}\left(\overline{I}_1 + \overline{I}_2\right) \\ &\quad + p\left[L_{22}\left(\overline{I}_1 + \overline{I}_2\right)\right] + p\left(L_2 \overline{I}_2\right) \,. \end{aligned} \tag{11}$$

3.1. The Saturation phenomenon

If the coordinate systems angular velocity ω is chosen equal to the supply line angular velocity $\omega_1 = 2\pi f_1$ the space phasors become regular phasors (vectors) [1] by dividing their moduli by $\sqrt{2}$.

The equations (10) and (11) for the steady state conditions (derivative terms equal to zero) are

$$\overline{V}_1 = \overline{I}_1 r_1 + j\omega_1 L_1 \overline{I}_1 + j\omega_1 L_{12}\left(\overline{I}_1 + \overline{I}_2\right) \tag{12}$$

$$\overline{V}_2 = \overline{I}_2 r_2 + j(\omega_1 - \omega_2) L_2 \overline{I}_2 + j(\omega_1 - \omega_2) L_{12}\left(\overline{I}_1 + \overline{I}_2\right) . \tag{13}$$

Defining inductances $X_1 = \omega_1 L_1$; $X_2 = \omega_1 L_2$, $X_{12} = \omega_1 L_{12}$ and $s = \frac{\omega_1 - \omega_2}{\omega_1}$, the equations (12), (13) represent the classical steady state model of an induction motor Fig. 1 with neglected iron losses [4]. Since X_{12} represents the magnetization branch $X_{12} = X_m$. Consequently L_{12} is the magnetization inductance:

$$L_m = L_{12} = \frac{X_m}{\omega_1}$$

and the current flowing through it is the magnetization current $\overline{I}_m$

$$\overline{I}_m = \overline{I}_1 + \overline{I}_2 . \tag{14}$$

Also the mutual flux $\overline{\lambda}_{12}$ can be defined as the magnetization flux $\overline{\lambda}_m$. Thus,

$$\overline{\lambda}_m = \overline{\lambda}_{12} = L_{12}\left(\overline{I}_1 + \overline{I}_2\right) = L_m \overline{I}_m . \tag{15}$$

Due to the magnetic properties of iron the saturation curve $|\lambda_m| = f(|I_m|)$ is a nonlinear function (Fig. 2). The magnetizing inductance defined as

$$L_m = \frac{|\overline{\lambda}_m|}{|\overline{I}_m|} \tag{16}$$

is a nonlinear function of the magnetizing current.

It should be born in mind that the magnetizing current is defined for each working point by the input voltage, frequency and load [2]. Substituting the magnetizing current (14) into the voltage equations (10) and (11) the following equations are obtained:

$$\overline{V}_1 = \overline{I}_1 r_1 + j\omega_1 L_1 \overline{I}_1 + j\omega_1 L_m \overline{I}_m + p\left(L_m \overline{I}_m\right) + p\left(L_1 \overline{I}_1\right) \tag{17}$$

$$\overline{V}_2 = \overline{I}_2 r_2 + j(\omega_1 - \omega_2) L_2 \overline{I}_2 + j(\omega_1 - \omega_2) L_m \overline{I}_m + p\left(L_m \overline{I}_m\right) + p\left(L_2 \overline{I}_2\right) . \tag{18}$$

The leakage inductances L_1 and L_2 are considered constant [1]. But the magnetizing inductance L_m is a function of the magnetizing current which varies with the time, thus

$$p(L_m \overline{I}_m) = \overline{I}_m p L_m + L_m p \overline{I}_m . \tag{19}$$

It is very complicated [1] to calculate the term pL_m. A simpler way of taking into account the saturation phenomenon is given in the next section.

4. Induction Motor Model with Flux Linkages Space Phasors as State Variables

A different and much easier way to solve the voltage equations taking into account the saturation is to eliminate the currents from the voltage equations using the following relationship between the current space phasors and flux linkages space phasors

$$\overline{I}_1 = \alpha_{12}\overline{\lambda}_2 - \alpha_{11}\overline{\lambda}_1 \tag{20}$$

$$\overline{I}_2 = \alpha_{12}\overline{I}_1 - \alpha_{22}\overline{\lambda}_2 \, , \tag{21}$$

where the coefficients α_{ij} derived from (8) and (9) are defined in Appendix A.

The voltage equations of the Induction Machine represented by the flux linkages space phasors are:

$$p\overline{\lambda}_1 = \overline{V}_1 - r_1\alpha_{12}\overline{\lambda}_2 + r_1\alpha_{22}\overline{\lambda}_1 - j\omega\overline{\lambda}_1 \tag{22}$$

$$p\overline{\lambda}_2 = \overline{V}_2 - r_2\alpha_{12}\overline{\lambda}_1 +_2 \alpha_{11}\overline{\lambda}_2 - j(\omega - \omega_2)\lambda_2 \, . \tag{23}$$

Equation of motion:

$$p\omega_r = 1/J(T - T_L) \qquad \omega_r = \frac{\omega_2}{P_p} \tag{24}$$

$$T = -3/2\, P_p\alpha_{12}I_m\left(\overline{\lambda}_1 \cdot \overline{\lambda}_2^*\right) \, . \tag{25}$$

Solving numerically Eqs. (22),(23) and (24) (as five real equations - Appendix B) for given $\overline{V}_1, \omega$ and T_L the state of the Induction Motor can be computed.

For each state the flux linkages are computed. Then the currents and electromagnetic torque are calculated. Having the stator and rotor currents, the instant magnetization current can be obtained. For a given magnetizing current, the magnetizing flux linkage will be found [2] using the known saturation curve of the machine $|\overline{\lambda}_m| = f(|\overline{I}_m|)$. This allows to find the instantenous value of the saturated value magnetization inductance L_m. Of course changes of L_m will affect the α_{ij} coefficients of the model. (See Appendix A).

5. Digital Simulation of the Motor

It was shown that using the flux linkages as state variables is an easy way to include the saturation phenomenon in the motor model.

The motor is modeled by five real differential equations with flux linkages as state variables (Appendix B). A program was written using MATLAB-SIMULINK [5] in order to study the dynamic behaviour of the induction motor taking into account the saturation. The dynamic behaviour of the motor during direct on line is investigated.

By integrating the differential equations, the program computes the flux linkages and currents. At each step of computation $|I_m|$ is calculated from the values of the currents in the previous step. Then the magnetizing inductance L_m may by adjusted in accordance with a lookup table obtained from the experimental magnetization curve $|\lambda_m| = f(|\overline{I}_m|)$.

The data and parameters of the studied motor are given in Appendix C. Two different working conditions are considered:

No load; $T_L = 0.0$ Nm

Nominal load; $T_L = 26.0$ Nm

Both cases were investigated under three different values of L_m:

(a) A fixed value of magnetization inductance corresponding to a point in the saturated region: $L_m = 0.1541$ H.

(b) A fixed value of the magnetizing inductance corresponding to the linear part of the saturation curve: $L_m = 0.197$ H.

(c) A variable value of L_m, according to the point on saturation curve depending on the instantaneous value of the magnetizing current $|I_m|$.

The three options are shown in Fig. 3.

6. Simulation Results

The rotor angular velocity $\omega_2 = f(t)$ and electromagnetic torque $T = f(t)$ during direct on-line starting of the motor without load and full load are shown in Fig. 4 and Fig. 5 respectively. The starting process in the phase plane $T_L = f(\omega_2)$ for the above mentioned conditions is presented in Fig. 6 and Fig. 7. A closer look at the figures reveals that the number of oscillations of the electromagnetic torque, for "b" and "c" are similar, while in "a" case the number of oscillations is smaller, however the maximum instant value of the torque in case "a" is higher than in "b" and "c".

Case "a" is optimistic, which gives a more stable start and a shorter time to reach the steady angular velocity, but the results are not realistic.

The magnetizing current $I_m = f(t)$ and the magnetizing inductance $L_m = f(t)$ during starting are shown in Fig. 8 (no load) and Fig. 9 (nominal load).

Comparing the results (remembering that case "c" is closer to reality) it can be noted that during starting period the mean value of L_m approximates the one chosen for the "b" case. This means that the motor is working in the linear part of the saturation curve. Due to this fact the shaft load has no influence on the value of L_m as can be seen in Figures 8 and 9.

In the steady state the value of L_m is closer to that chosen for the "a" case since the motor working point moves to the saturation curve's knee. Now, the shaft load influences the value of L_m. The above mentioned facts can be explained by the high motor currents during a direct on-line starting. The high starting currents (approx. $6 \times I_n$) and the leakage inductance of the stator L_1 reduce the voltage applied to the magnetizing branch, moving the working point to the linear part of the saturation curve.

Thus, if the model to be implemented allows only a fixed value for L_m, the "b" option (air gap value) gives better results for transient (starting) conditions where high currents are involved. But the final value of the magnetizing current will be lower because of high value of the chosen L_m.

7. Conclusion

In the presented work a general mathematical model of a three phase induction motor using the flux linkages as state variables is given. Selection of the flux linkages as state variables provides a simple way to investigate the dynamic behavior of the motor when the saturation is taken into account. It was shown that the value of the chosen magnetizing inductance L_m affects the dynamics of the plant (motor and load), the oscillations in the developed electromagnetic torque, the rotor angular velocity and the starting period.

References

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[2] I. Boldea, S.A. Nasar, "Electric Machine Dynamics", McMillan Publishing Company, 1986.

[3] P.C. Krause, "Analysis of Electric Machinery", McGraw-Hill, 1986.

[4] A. Fitzgerald, Ch. Kingsley Jr., "Electric Machinery", McGraw Hill 2nd Ed., 1961.

[5] MATLAB-SIMULINK for Microsoft Windows. The Math. Work Inc. 1993.

[6] A. Alexandrovitz, "Transfer Function Approach to Dynamic Behaviour Investigation of Frequency Controlled Induction Motor", *ELMEKSEM'97*, Bursa, Turkey, 17–21 December, 1997.

APPENDIX A: α_{ii} Coefficients

$$\alpha_{11} = \frac{L_{11}}{\Delta} \quad ; \alpha_{22} = \frac{L_{22}}{\Delta} \quad ; \alpha_{12} = \frac{L_{12}}{\Delta} \quad ;$$

$$\Delta = L_{12}^2 - L_{11} \cdot L_{22}$$

$$L_{11} = L_1 + L_{12} \qquad L_{22} = L_2 + L_{12}$$

$$I_1 = I_{1d} + I_{1q} \quad ; I_2 = I_{2d} + I_{2q} \, .$$

APPENDIX B: Real State Space Equations of the Induction Motor [3],[4],[6]

$$p\lambda_{1d} = \quad V_{1d} - r_1\alpha_{12}\lambda_{2d} + r_1\alpha_{22}\lambda_{1d} + \omega\lambda_{1q} \qquad \text{(Volts)}$$

$$p\lambda_{1q} = \quad V_{1q} - r_1\alpha_{12}\lambda_{2q} + r_1\alpha_{22}\lambda_{1q} + \omega\lambda_{1d} \qquad \text{(Volts)}$$

$$p\lambda_{2d} = \quad V_{2d} - r_2\alpha_{12}\lambda_{1d} + r_2\alpha_{11}\lambda_{2d} + (\omega - \omega_2)\lambda_{2q} \quad \text{(Volts)}$$

$$p\lambda_{2q} = \quad V_{2q} - r_2\alpha_{12}\lambda_{1q} + r_2\alpha_{11}\lambda_{2q} + (\omega - \omega_2)\lambda_{2d} \quad \text{(Volts)}$$

$$V_{1d} = V_{1\,\max} \quad ; \quad V_{1q} = 0 \quad V_{2q} = V_{2d} = 0$$

$$T = \frac{3}{2} P_p\alpha_{12} \left(\lambda_{2q} \cdot \lambda_{1d} - \lambda_{2d}\lambda_{1q}\right) \qquad \text{(Nm)}$$

$$Jp\omega_r = T - T_L \qquad \text{(Nm)} \qquad \omega_r = \frac{\omega_2}{P_p}$$

$$I_{md} = I_{1d} + I_{2d} \quad I_m = (I_{md}^2 + I_{mq}^2)^{1/2}$$

$$I_{mq} = I_{1q} + I_{2q}$$

APPENDIX C: Induction Motor Data

$V = 380V$; $I_n = 8.8A$ $f_n = 50H_z$ $nm = 1435rpm$

$P_n = 4KW$ $\cos\varphi_n = 0.81$

$r_1 = 1.31\Omega$; $r_2 = 1.19\Omega$

$L_1 = 0.0077H$; $L_2 = 0.0077H$

$J = 0.011Nmsec^2$; $P_p = 2$ (pole pairs).

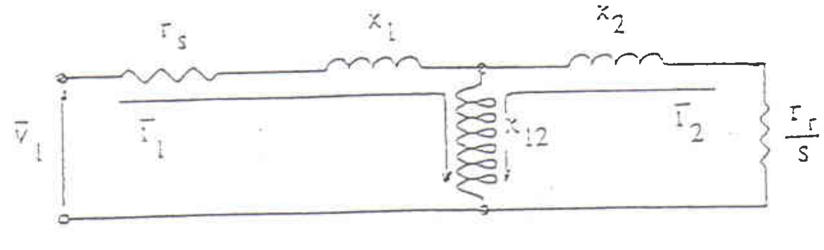

Fig. 1: Space phaser equivalent circuit of an induction motor.

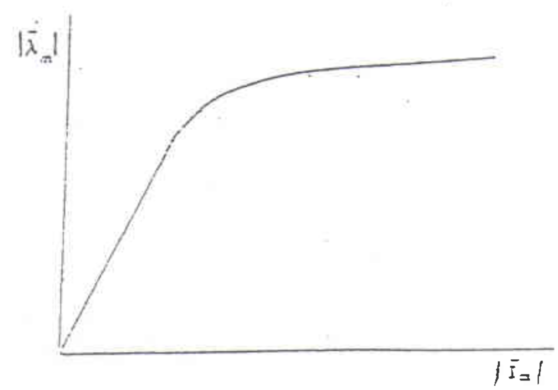

Fig. 2: Saturation curve $|\bar{\lambda}_m| = f|(\bar{I}_m)|$.

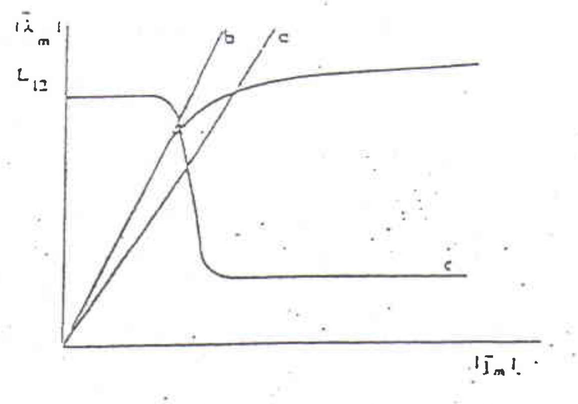

Fig. 3: a) air gap magnetization line

b) saturated magnetization line

c) magnetization inductance $L_{12} = f(|\bar{I}_m|)$.

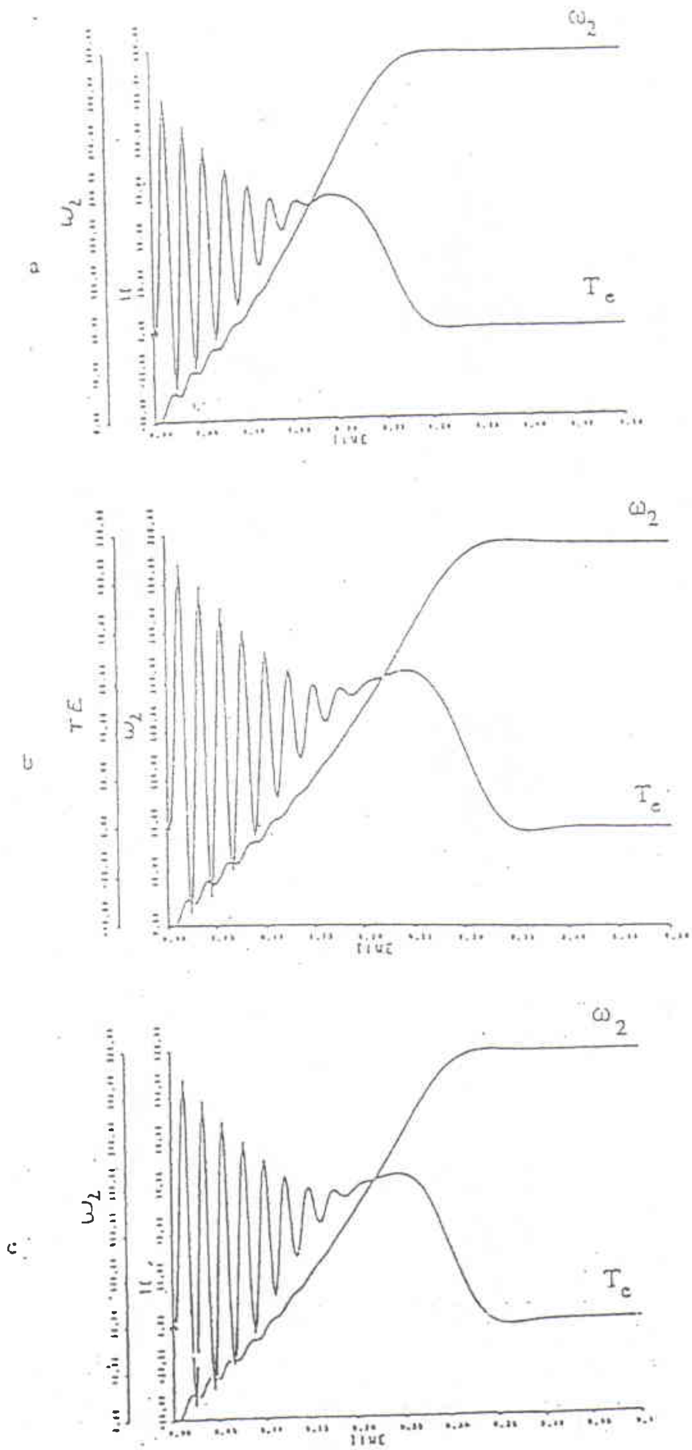

Fig. 4: Direct on-line starting ($T_l = 0$):

a) $L_{12} = 0.154\,H$

b) $L_{12} = 0.197\,H$

c) $L_{12} = f(|\bar{I}_m|)\,H$.

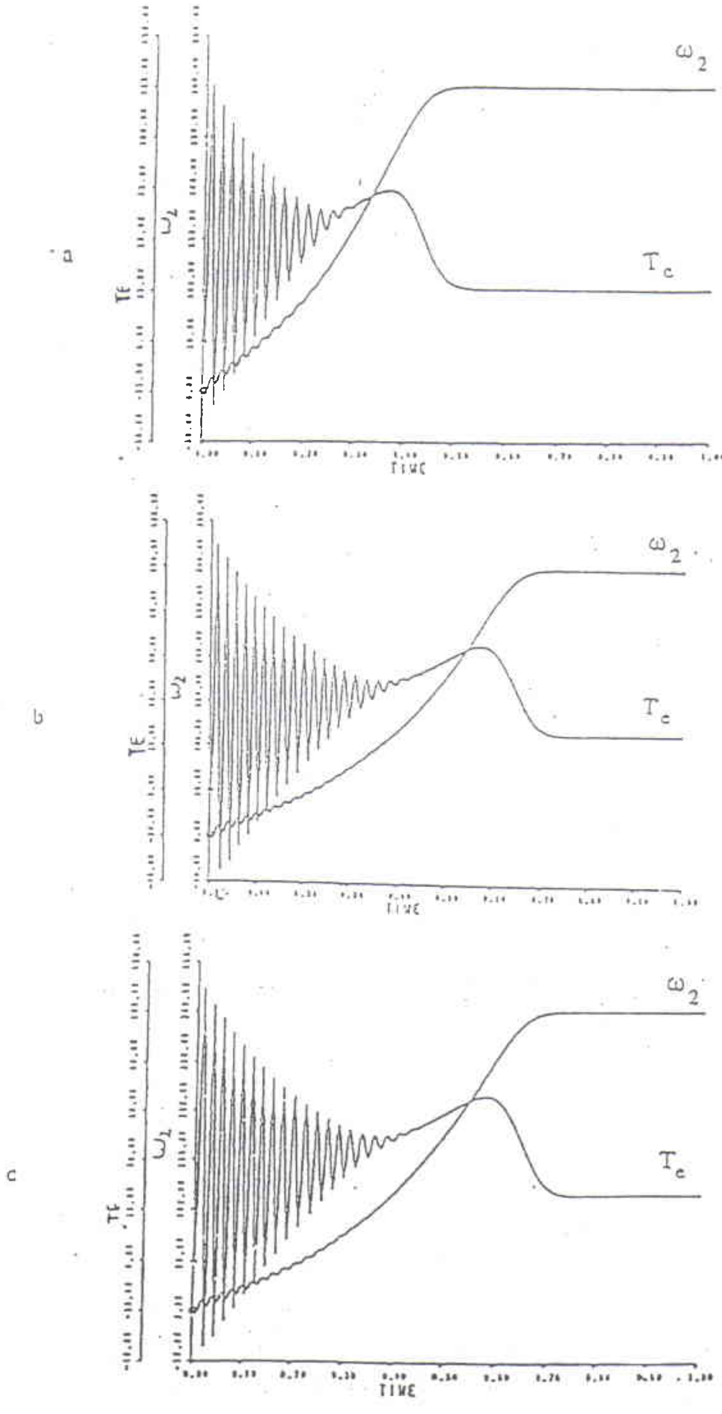

Fig. 5: Direct on-line starting ($T_l = 26Nm$):

a) $L_{12} = 0.154\,H$

b) $L_{12} = 0.197\,H$

c) $L_{12} = f(|\bar{I}_m|)\,H$

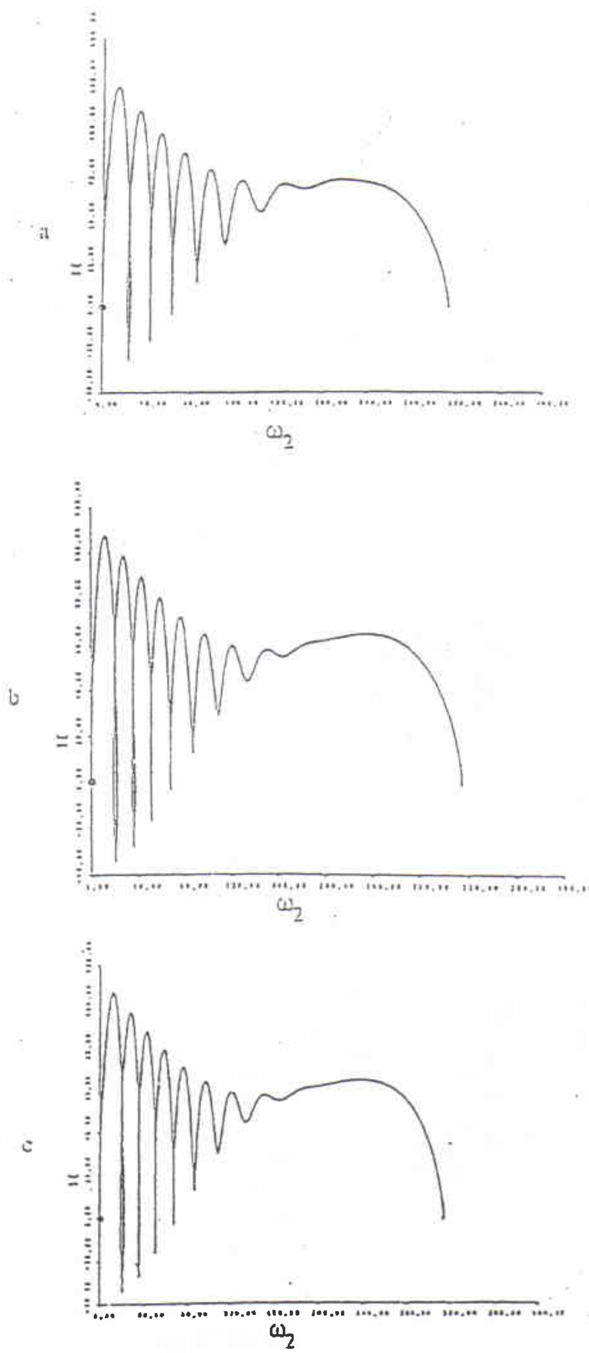

Fig. 6: Direct on-line starting in phase plane ($T_l = 0$):

a) $L_{12} = 0.154\,H$

b) $L_{12} = 0.197\,H$

c) $L_{12} = f(|\vec{I}_m|)\,H$.

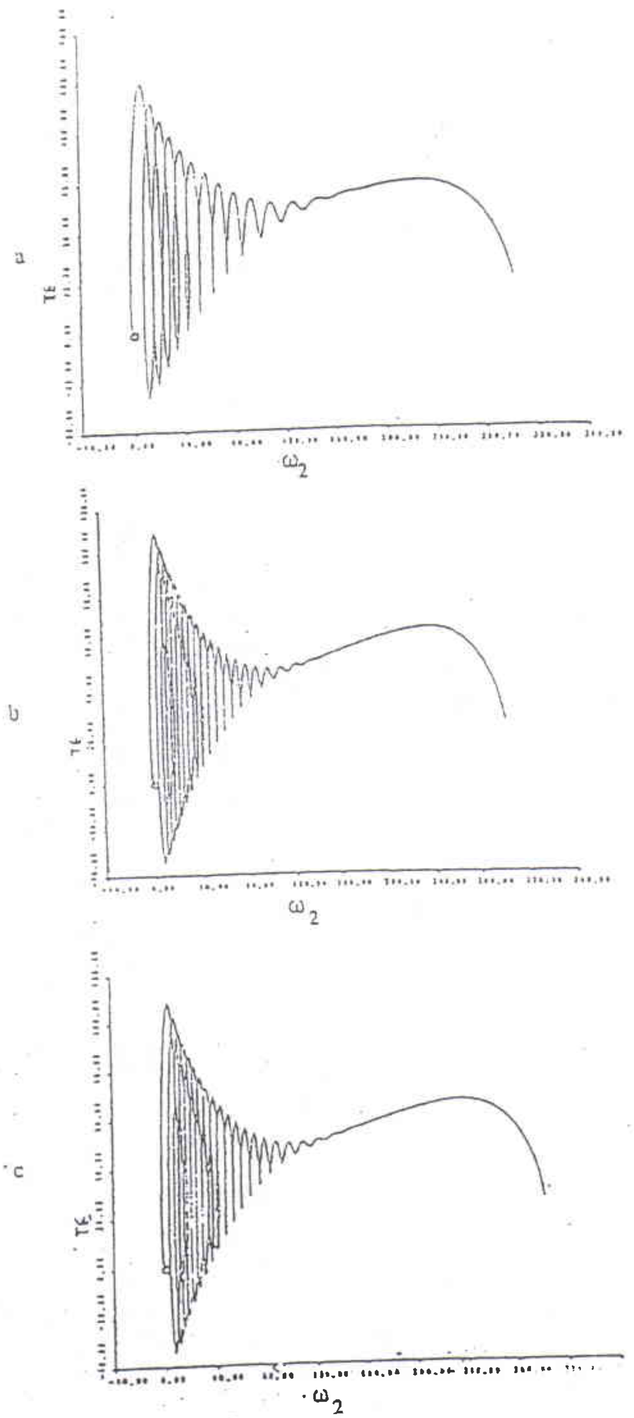

Fig. 7: Direct on-line starting in phase plane ($T_l = 26Nm$):

a) $L_{12} = 0.1541\,H$

b) $L_{12} = 0.197\,H$

c) $L_{12} = f(|\bar{I}_m|)\,H$.

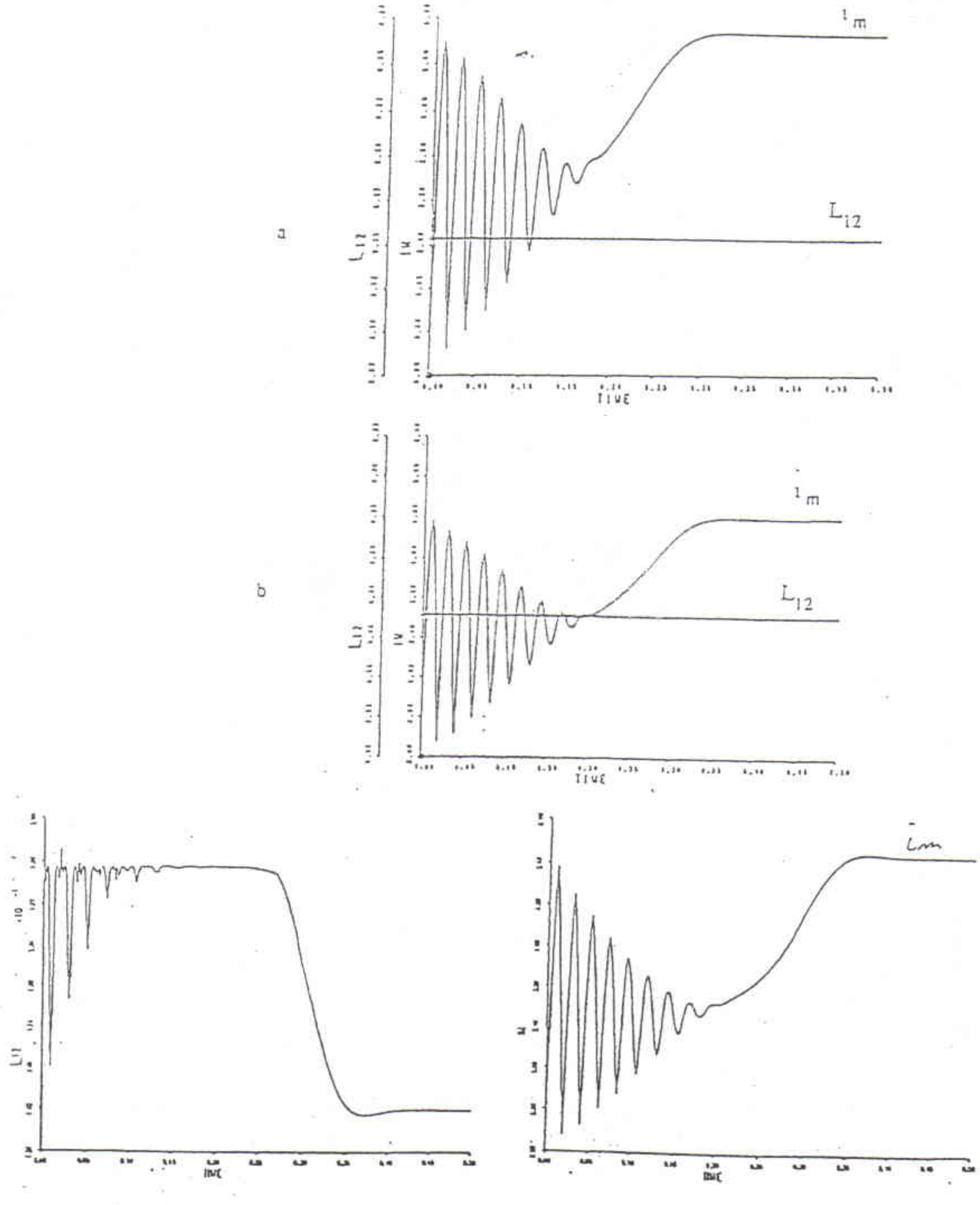

Fig. 8: Magnetizing current and magnetizing inductance during direct on-line starting ($T_l = 0$):

a) $L_{12} = 0.1541\,H$

b) $L_{12} = 0.197\,H$

c) $L_{12} = f(|\bar{I}_m|)\,H$.

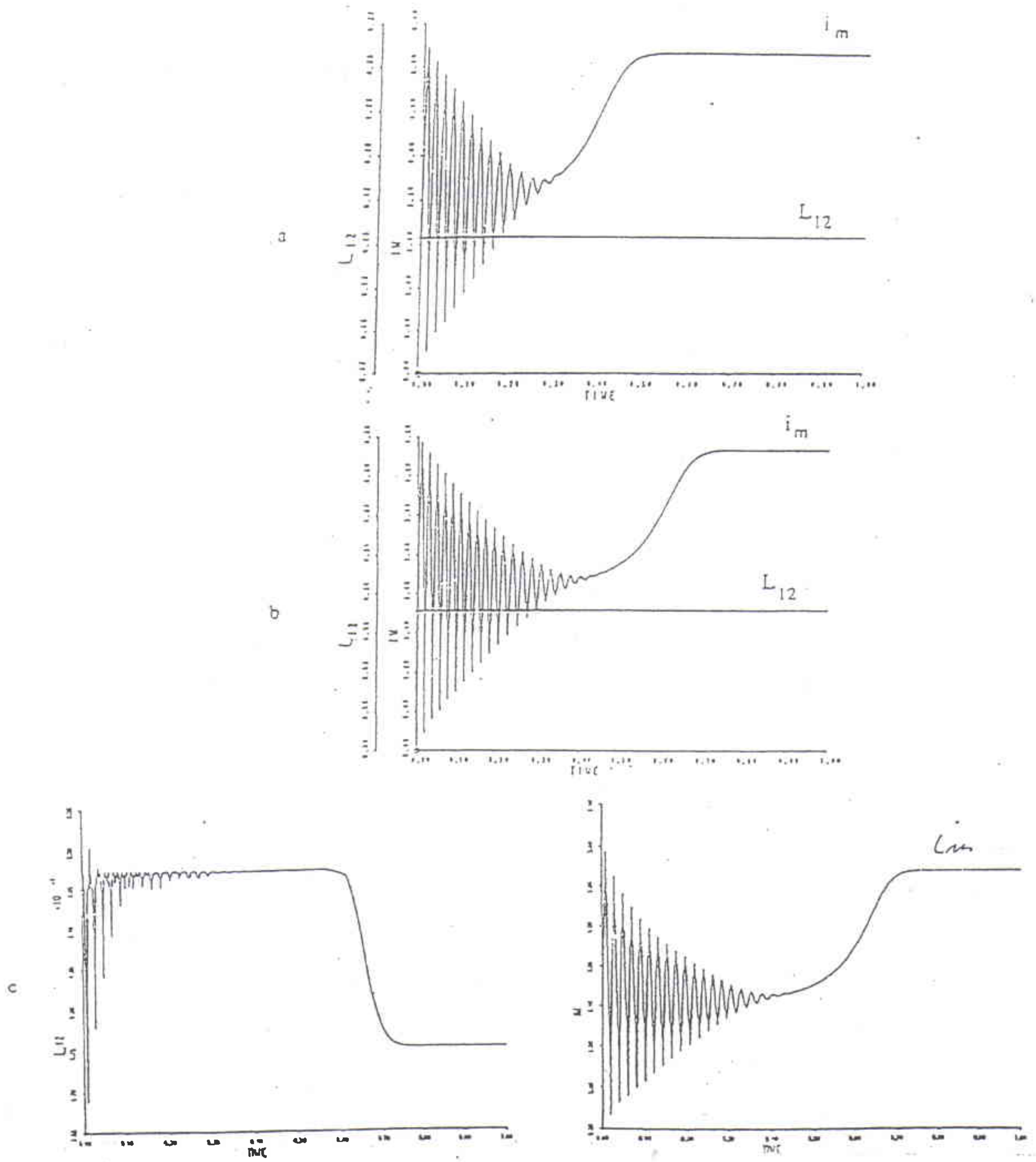

Fig. 9: Magnetizing current and magnetizing inductance during direct on-line starting ($T_l = 26Nm$):

a) $L_{12} = 0.1541\,H$

b) $L_{12} = 0.197\,H$

c) $L_{12} = f(|\bar{I}_m|)\,H$.