Design of Third-Order Square-Root-Domain Filters Using State-Space Synthesis Method

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Abstract

In this study, square-root-domain electronically tunable, third order low-pass filter is proposed. First circuit is third-order low-pass Butterworth filter and second circuit is third-order low-pass Chebyshev filter. Additionally, the cut-off frequency of the proposed filters can be electronically tuned by changing external currents. Time and frequency domain simulations are performed using PSPICE program for third order filters to verify the theory and to show the performance of them. For this purpose, the filters are simulated by using TSMC 0.35 μm Level 3 CMOS process parameters.

1. Introduction

Square-root-domain circuits are a subclass of companding circuits providing low power under low-voltage, having large dynamic range, operating in high frequencies and electronically tunability using DC current sources. Due to these properties, companding circuits are compatible with CMOS very large scale integration technology. Besides these properties, circuits are implemented in this technology; design of companding circuits has received great attention. Log-domain and square-root-domain filter circuits are an application of companding method. These circuits can be said that the most widely used translinear circuits. Log-domain circuits are proposed by Adams [1] and then they have been studied by Frey [2, 3]. Basic translinear principle uses the exponential I-V characteristic of BJTs or MOSFETs in weak inversion region [4, 5]. MOS translinear principle (MTL) derived from bipolar translinear principle (BTL) [4] by Seevinck [6] uses quadratic relationship between voltage and current of MOS transistors in strong inversion and saturation region. Starting from state-space equations, as well as quadratic relationship between the voltage and current of the MOS transistors, filters performed by using analog processing circuit blocks such as square-root and squarer/divider circuit are called square-root-domain filters [7–23].

Square-root-domain first-order filter circuits [14, 16, 18, 21], second order voltage-mode [11, 12, 13] or current-mode [9, 14, 22, 23] filter circuits and transconductance and transresistance circuits [16, 17, 24] have been studied by various researchers. However, as a result of the literature survey, it has been seen that the studies on square-root-domain third-order filter circuits was found to be minimal. Third-order filter circuits obtained using OTA and OTRA are presented in [25-27]. A state space Class AB synthesis method for the design of square-root-domain filter based on the MOSFET square law is proposed in [28]. A third-order low-pass elliptic filter using a square-root-domain differentiator is presented in [29]. A third-order elliptic low-pass LC filter is proposed in [30]. A realization of third-order active switched-capacitor filter is proposed in [31].

In this study, square-root-domain third order low-pass Butterworth and Chebyshev filters have been designed using state-space-synthesis method with square-root and squarer/divider circuits, MOS current mirrors, DC current sources, DC supply voltage, and grounded capacitors. State-space synthesis method is a very useful and efficient approach for the design of companding circuits [3]. It provides a general solution for realizing circuit function. The key aspect of the use of state-space methods in this study is that exactly relates internally non-linear filters to equivalent linear systems. Cut-off frequency of the proposed filters can be adjusted electronically by changing the value of the DC current sources.

2. Square-Root-Domain Third-Order Filter Design

A general third-order circuit function is as shown in Equation (1).

$$N(s)=\frac{P(s)}{Q(s)}=\frac{P(s)}{\left(s^2+\frac{\omega_{01}}{Q}s+\omega_{01}^2\right)(s+\omega_{02})} \tag{1}$$

Equation (1) can be transformed to the following state-space equations.

$$\dot{x}_1=-\frac{\omega_{01}}{Q}x_1+\omega_{02}x_3 \tag{2}$$

$$\dot{x}_2=-\omega_{02}x_2+\omega_{02}u \tag{3}$$

$$\dot{x}_3=-\omega_{01}x_1+\omega_{01}x_2 \tag{4}$$

Here, x_1, x_2 and x_3 represent the state variables. They are gate-source voltages of MOS transistors. Hence, the following transformations can be applied to the quantities in the equations [18, 19].

$$I_i=\frac{\beta}{2}(V_i-V_{th})^2 \quad , \quad i=1,2,3 \tag{5}$$

In Equation (5), I_i represents drain current of MOS transistors in saturation region, $\beta=\mu_0 C_{ox}(W/L)$ stands for transconductance, V_i represents gate-source voltage and V_{th} represents the threshold voltage. V_i voltages can be obtained as shown in the following equation:

$$V_i=\sqrt{\frac{2I_i}{\beta}}+V_{th} \quad , \quad i=1,2,3 \tag{6}$$

The relation given above can be organized to yield the nodal equations below after they are applied to Equations (2), (3) and (4).

$$C\dot{V}_1 = -\frac{\sqrt{2I_1}}{Q}\frac{C\omega_{01}}{\sqrt{\beta}} + \sqrt{2I_3}\frac{C\omega_{01}}{\sqrt{\beta}} + \left(1-\frac{1}{Q}\right)C\omega_{01}V_{th} \quad (7)$$

$$C\dot{V}_2 = -\sqrt{2I_2}\frac{C\omega_{02}}{\sqrt{\beta}} + \sqrt{2I_u}\frac{C\omega_{02}}{\sqrt{\beta}} \quad (8)$$

$$C\dot{V}_3 = -\sqrt{2I_1}\frac{C\omega_{01}}{\sqrt{\beta}} + \sqrt{2I_2}\frac{C\omega_{01}}{\sqrt{\beta}} \quad (9)$$

In these equations, C is a capacitor value resembling a multifunction factor. $C\dot{V}_1$, $C\dot{V}_2$ and $C\dot{V}_3$ in Equations (7), (8) and (9) can be accepted as time dependent currents that are grounded via three capacitors.

The following equations I_{01}, I_{02} and I_B can be defined for use in Equations (7), (8) and (9).

$$\sqrt{I_{01}} = \frac{C\omega_{01}}{\sqrt{\beta}} \quad (10)$$

$$\sqrt{I_{02}} = \frac{C\omega_{02}}{\sqrt{\beta}} \quad (11)$$

$$\sqrt{I_B} = \frac{1}{\sqrt{2}}\left(1-\frac{1}{Q}\right)V_{th}\sqrt{\beta} \quad (12)$$

Equations (7), (8) and (9) can be arranged as follows:

$$C\dot{V}_1 = -\frac{2}{Q}\sqrt{\frac{I_{o1}I_1}{2}} + 2\sqrt{\frac{I_{o1}I_3}{2}} + 2\sqrt{\frac{I_{o1}I_B}{2}} \quad (13)$$

$$C\dot{V}_2 = -2\sqrt{\frac{I_{o2}I_2}{2}} + 2\sqrt{\frac{I_{o2}I_u}{2}} \quad (14)$$

$$C\dot{V}_3 = -2\sqrt{\frac{I_{o1}I_1}{2}} + 2\sqrt{\frac{I_{o1}I_2}{2}} \quad (15)$$

If we accept the quality factor as Q=1 and divide both side of the equations by 2. The state equations in Equations (13), (14) and (15) can be written as below:

$$\overline{C}\dot{V}_1 = -\sqrt{\frac{I_{o1}I_1}{2}} + \sqrt{\frac{I_{o1}I_3}{2}} \quad (16)$$

$$\overline{C}\dot{V}_2 = -\sqrt{\frac{I_{o2}I_2}{2}} + \sqrt{\frac{I_{o2}I_u}{2}} \quad (17)$$

$$\overline{C}\dot{V}_3 = -\sqrt{\frac{I_{o1}I_1}{2}} + \sqrt{\frac{I_{o1}I_2}{2}} \quad (18)$$

The following definition applies to Equations (16), (17) and (18).

$$\overline{C} = C/2 \quad (19)$$

Square-root-domain third order voltage-mode low-pass Butterworth filter circuit is shown in Figure 1 has been actualized using Equations (16), (17) and (18).

U, V_1, V_2 and V_3 represent the input and output voltage of the filter circuit, respectively. Additionally, using Equations (20) and (21), ω_{01} and ω_{02} pole frequency of the filter circuit can be determined depending on the I_{01}, I_{02}, β and C [9, 18].

$$\omega_{01} = \frac{\sqrt{\beta I_{01}}}{C} \quad (20)$$

$$\omega_{02} = \frac{\sqrt{\beta I_{02}}}{C} \quad (21)$$

Using Equations (2), (3) and (4), output variables of the square-root-domain third-order low-pass filter circuit can be determined depending on ω_{01}, ω_{02} and Q.

$$V_1 = \frac{-\omega_{01}^2\omega_{02}}{\left(s^2+\frac{\omega_{01}}{Q}s+\omega_{01}^2\right)(s+\omega_{02})}U \quad (22)$$

$$V_2 = \frac{s\left(s+\frac{\omega_{01}}{Q}\right)\omega_{02}-\omega_{01}^2\omega_{02}}{\left(s^2+\frac{\omega_{01}}{Q}s+\omega_{01}^2\right)(s+\omega_{02})}U \quad (23)$$

$$V_3 = \frac{-\omega_{01}\omega_{02}\left(s+\frac{\omega_{01}}{Q}\right)}{\left(s^2+\frac{\omega_{01}}{Q}s+\omega_{01}^2\right)(s+\omega_{02})}U \quad (24)$$

In accordance with Equation (22), the output of the circuit as shown in Figure 1 provides a third order inverting low-pass filter transfer function.

$$V_{AG} = V_1 \quad (25)$$

Consequently, using the output of the circuit shown in Figure 1 for Q=1 and $\omega_{01}=\omega_{02}=\omega_0$, third-order Butterworth low-pass filter voltage transfer functions accomplished as defined in Equation (26).

$$V_1 = \frac{-\omega_0^3}{\left(s^2+\omega_0 s+\omega_0^2\right)(s+\omega_0)}U \quad (26)$$

If we accept the quality factor as Q≠1 in Equations (13), (14) and (15), square-root-domain third order low-pass Chebyshev filter circuit can be performed as shown in Figure 2. In accordance with Equation (22), the output of the circuit as shown in Figure 2 provides a third order inverting low-pass Chebyshev filter transfer function.

In case of Q≠1 in Equation (22), the equation between I_{01} and I_{02} given by Equation (27) must be satisfied.

$$I_{02} = I_{01}/Q^2 \quad (27)$$

With 1 dB passband ripple and ω_0=1rad/s cut-off frequency, third order normalized low-pass Chebyshev filter's ω_{01}, ω_{02} and Q values given in Equation (22) are as shown below [32].

$$\omega_{01} = 0{,}997\ ,\ \omega_{02} = 0{,}494\ ,\ Q = 2{,}018$$

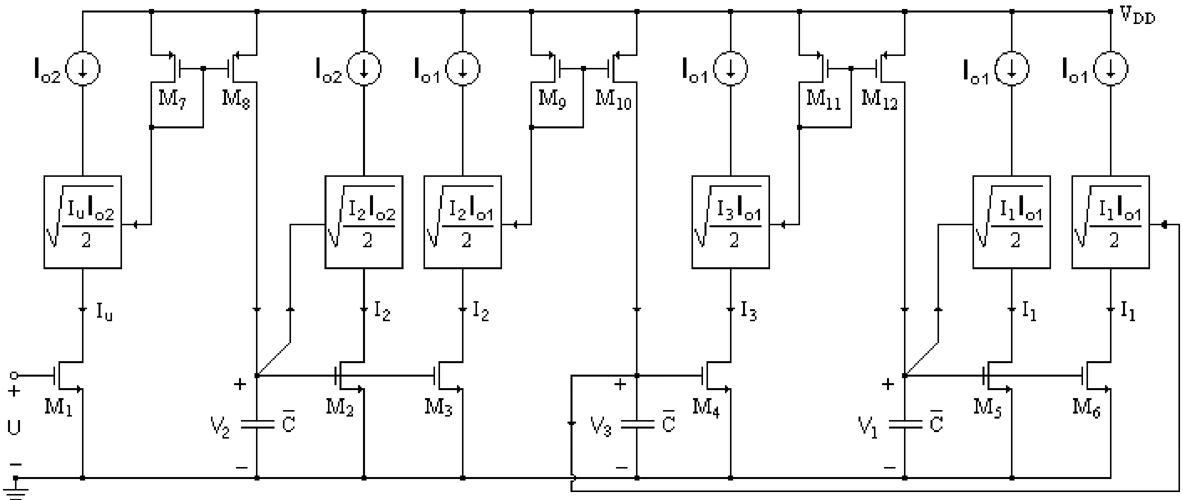

Fig. 1. Square-root-domain third order low-pass Butterworth filter circuit

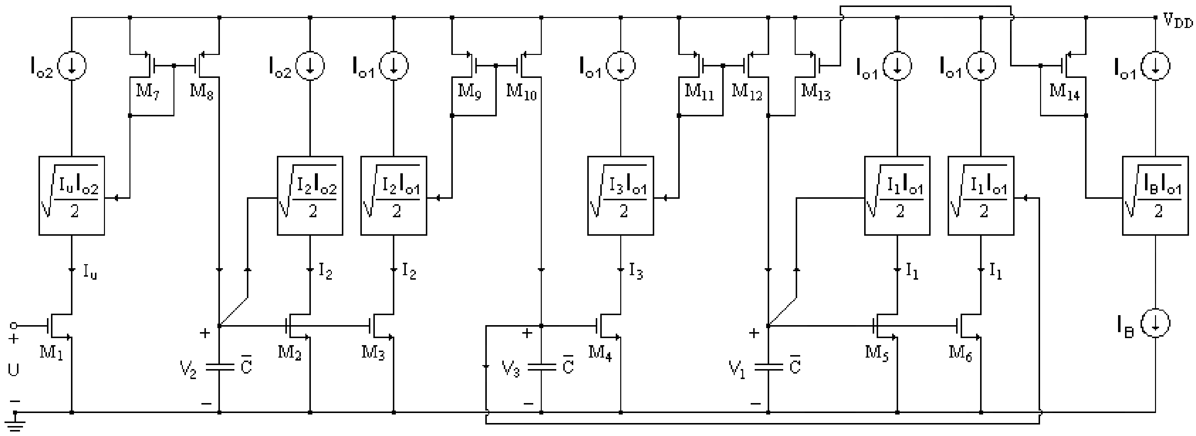

Fig. 2. Square-root-domain third order low-pass Chebyshev filter circuit

3. Simulation Results

TSMC 0.35 μm Level 3 CMOS transistor parameters [33] have been used in PSPICE simulations of the designed square-root-domain third order voltage-mode low-pass Butterworth and Chebyshev filter. Transistor dimensions are chosen as W/L=10 μm/10 μm for M_1~M_6, W/L=220 μm/2 μm for M_7~M_{14}.

The circuit supply voltage is selected to be V_{DD}=3 V. The values of three capacitances of the circuit are chosen to be C=20 pF. The simulations are performed to tune the cut-off frequency by varying the values of the current sources. Varying the values of the current sources from 12 μA to 402 μA, the cut-off frequency of the filter is tuned from 157 kHz to 857 kHz. As a result, the cut-off frequency of the filter can be adjusted in the 700 kHz frequency range. The gain response obtained for the different values of the DC current sources of the third order low-pass Butterworth filter circuit have been given in Figure 3.

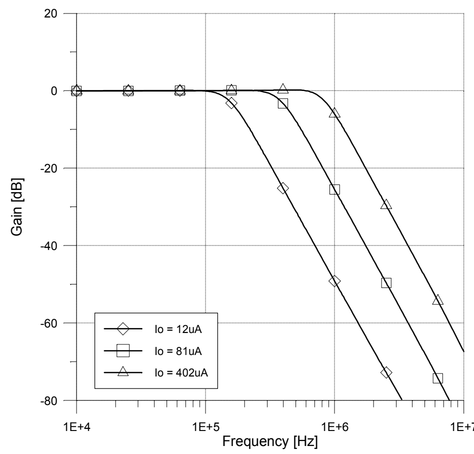

Fig. 3. Electronically tunable properties of Butterworth filter

The phase response obtained for the different values of the DC current sources of the third order low-pass Butterworth filter circuit have been given in Figure 4.

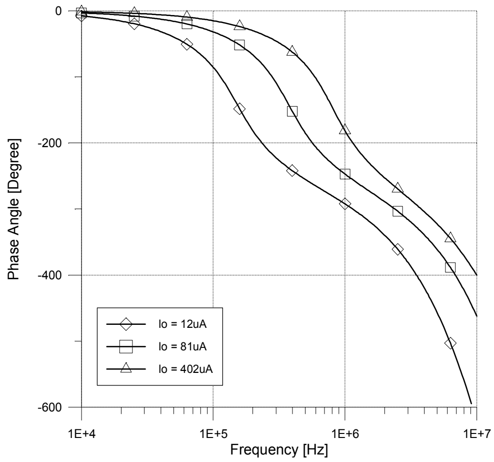

Fig.4. Electronically tunable phase response of Butterworth filter

The time-domain response of the Butterworth filter is shown in Figure 5. 0.25 V sine-wave input at a frequency of 850 kHz was applied to the filter. The total harmonic distortion was measured as 1.6%.

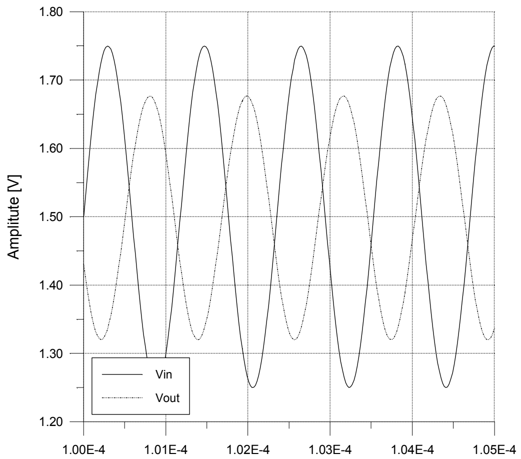

Fig.5. Time domain response of the proposed Butterworth filter

The gain response obtained for the different values of the DC current sources of the third order low-pass Chebyshev filter circuit have been given in Figure 6.

DC current source values are I_B=4.65 μA and I_{02}=2.79 μA, I_{02}=21.6 μA and I_{02}=98.7 μA for I_{01}=12 μA, I_{01}=88 μA and I_{01}=402 μA, respectively. Also, the values of DC input current of the block is connected to the M_5 transistor are I_{01}=4.11 μA, I_{01}=35.9 μA and I_{01}=195 μA, respectively.

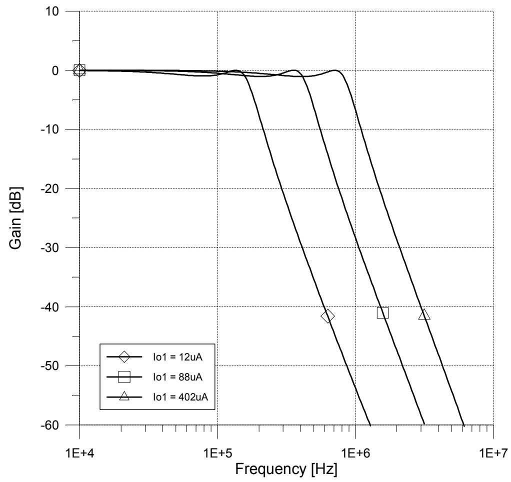

Fig.6. Electronically tunable properties of Chebyshev filter

The gain response of Chebyshev filter obtained for I_{01}=88 μA with 1 dB passband ripple is shown as larger in Figure 7.

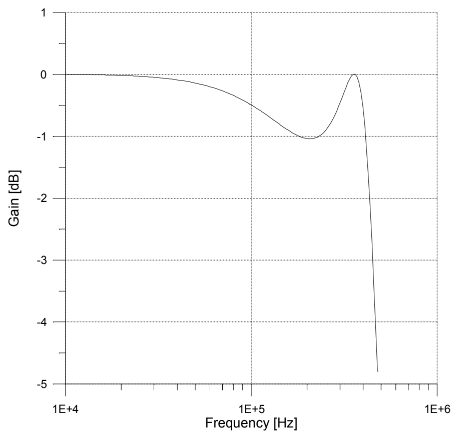

Fig.7. Change of Chebyshev filter's passband

The time-domain response of the Chebyshev filter is shown in Figure 8. 0.2 V sine-wave input at a frequency of 820 kHz was applied to the filter. The total harmonic distortion was measured as 1.5%.

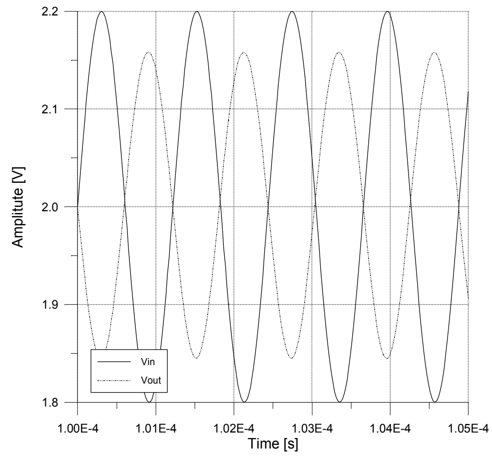

Fig. 8. Time domain response of the proposed Chebyshev filter

4. Conclusion

Square-root-domain third order voltage-mode low-pass Butterworth and Chebyshev filter circuits are proposed in this study. A systematic synthesis procedure to derive the filter circuits is also given. These circuits consist of only MOS transistors and grounded capacitors. Cut-off frequency of the filter circuits can be adjusted electronically by changing the value of the DC current sources. The most important feature of the circuits is electronic tunability, that is, the gain and phase response of the circuits can be controlled by DC current sources. PSPICE simulations are provided to confirm the theoretical analysis.

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