

# Control of Pattern Tracking Nonholonomic Mobile Robot with Feedback Linearization

Onur Akbati and Galip Cansever

Yıldız Technical University, Dept. of Control and Automation Engineering, 34220 Davutpaşa, İstanbul, Turkey  
oakbati@yildiz.edu.tr, cansever@yildiz.edu.tr

## Abstract

In this paper the motion control of a pattern tracking nonholonomic mobile robot is presented. In many applications localization of the mobile robots is subjected, different from the existing studies, here a similar closed loop system has been developed, but instead of estimating local coordinates, control problem is constructed as keeping close to a pattern under predefined constraints. Since the dynamic equations of the model are nonlinear, feedback linearization have been applied to control the mobile robot. To implement the control algorithm Quanser's QBot and a portable computer connected to its head camera have been used.

## 1. Introduction

Mobile robots are used in lots of areas from industrial applications to space projects. A general problem in those applications is the trajectory tracking.

There are numerous studies over visual based localization and tracking of the mobile robots. A fundamental study based on linearization can be found in [1]. Different methods for the tracking control exist, such as backstepping method is used in [2], optimal tracking control method developed in [3], [4] has used pole placement method while, a novel linear interpolation method is used in [5]. In [6,7] robust controllers are designed and in [8] neural dynamics are used. Another classification can be done according to the application; in some cases instead of visual data, other sensing abilities are used, such as thermal vision [9] or iGPS and Odometry [10] and also there are other cases where computational time has more importance [11] or tracking vehicle is obstructed with objects [3,12]. And also in some other applications, a line with bounded curvatures is tracked with visual feedback [13] or keeping the visual data source inside the viewing area is the control aim [14]. Moreover in some of the studies estimators are used [6,15,16].

Most of the studies use the velocity model of the mobile vehicles. But in [2,7] torque model of the mobile vehicles are used. A combined kinematic-torque control law is developed in [2]. In [7] a sliding mode controller for trajectory tracking is designed. Such as in [6] a robust control method is used for noises in the visual data and also a visual state estimation method is developed to determine the velocity of the target. While in [15,16] extended kalman filter is used.

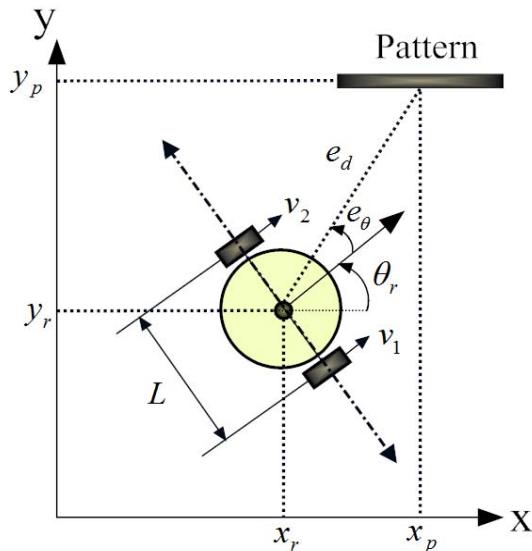
In this paper, feedback linearization is used to eliminate the nonlinear terms in the dynamic equations of the closed loop system and a simple damped system is obtained.

The rest of the paper is organized as follows: The kinematic equations of the mobile robot and the dynamic equations defining the error terms of the closed loop system are given in Section 2. Feedback linearization based control method is given

in Section 3. Experimental study achieved to prove the designed method and its results are given in Section 4. Conclusion and the future works are given in Section 5.

## 2. Mathematical Model

The mobile robot consists of two motor driven wheels and two free spinning passive wheels. The effects of the passive wheels are ignored, the robot is symmetrically shaped and the center of the mass is located at the geometric center of the body. Robot and the pattern are modeled under Cartesian coordinates. These Parameters and the vehicle are illustrated in Fig. 1.



**Fig. 1.** Model of the nonholonomic mobile robot

### 2.1. Kinematic Equations

The kinematic model of the robot depending on wheel velocities can be given as,

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} 0.5 \cos \theta_r & 0.5 \cos \theta_r \\ 0.5 \sin \theta_r & 0.5 \sin \theta_r \\ \frac{1}{L} & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1)$$

Instead of using velocities, the following input transformation can be regarded:

$$\begin{aligned} v_1 &= u_1 + u_2 \\ v_2 &= u_1 - u_2 \end{aligned} \quad (2)$$

Moreover to define the error terms of the system, some assumptions should be considered. The equations will hold only if the pattern is inside the front half plane of the robot. Since the pattern should be kept visible to the camera in application, the opposing situations are already discarded by this way.

System has two error terms symbolized as  $e_d$  and  $e_\theta$ , here  $e_\theta$  represents the orientation error to the pattern and  $e_d$  represents the distance error. It should be kept in mind that in the model, robot is considered as a point. But in application we should reduce the distance error with some constant to avoid collisions with the pattern. For this purpose, a safety circle is defined with radius  $r_{min}$ .

The error terms can be given as,

$$e_d = \sqrt{(x_p - x_r)^2 + (y_p - y_r)^2} - r_{min} \quad (3)$$

$$e_\theta = \text{atan} \left( \frac{y_p - y_r}{x_p - x_r} \right) - \theta_r \quad (4)$$

## 2.2. Dynamic Equations

Derivation of the error terms can be written as,

$$\dot{e}_d = \frac{\dot{x}_r(x_p - x_r) + \dot{y}_r(y_p - y_r)}{\sqrt{(x_p - x_r)^2 + (y_p - y_r)^2}} \quad (5)$$

$$\dot{e}_\theta = \frac{\dot{x}_r(y_p - y_r) - \dot{y}_r(x_p - x_r)}{(e_d + r_{min})^2} - \dot{\theta}_r. \quad (6)$$

Replacing the robot dynamics,

$$\dot{e}_d = -u_1 \frac{(x_p - x_r)\cos\theta_r + (y_p - y_r)\sin\theta_r}{e_d + r_{min}} \quad (7)$$

$$\dot{e}_\theta = u_1 \frac{(y_p - y_r)\cos\theta_r - (x_p - x_r)\sin\theta_r}{(e_d + r_{min})^2} - \frac{2u_2}{L} \quad (8)$$

obtained and redefining the trigonometric equations we get,

$$\dot{e}_d = -u_1 \text{cose}\theta \quad (9)$$

$$\dot{e}_\theta = \frac{u_1 \text{sine}\theta}{e_d + r_{min}} - \frac{2u_2}{L}. \quad (10)$$

As a result, terms depending on the local coordinates are eliminated. The remaining terms consist of trigonometric functions, errors and control signals.

## 3. Feedback Linearization

The ability to use feedback to convert a nonlinear state equation into a controllable linear state equation by cancelling nonlinearities requires the nonlinear state equation to have the suitable structure [18].

To achieve an exponentially stable system, the input signals  $u_1$  and  $u_2$  can be chosen as,

$$u_1 = \frac{\lambda_1 e_d}{\text{cose}\theta} \quad (11)$$

$$u_2 = \frac{\lambda_2 L e_\theta}{2} + \frac{\lambda_1 L e_d \tan(e_\theta)}{2(e_d + r_{min})} \quad (12)$$

and as a result, dynamic equations becomes:

$$\dot{e}_d = -\lambda_1 e_d \quad (13)$$

$$\dot{e}_\theta = -\lambda_2 e_\theta. \quad (14)$$

Here  $\lambda_1$  and  $\lambda_2$  are arbitrary constant parameters.

## 4. Experimental Studies

To apply the control algorithm, Quanser's QBot is used. This mobile vehicle is constructed over iRobot Create platform and consists of a Linux based target computer with wireless communication. The control algorithm constructed under MATLAB SIMULINK platform can be compiled and uploaded to this device to create a real-time application. The configuration of the application can be seen on Fig. 2.



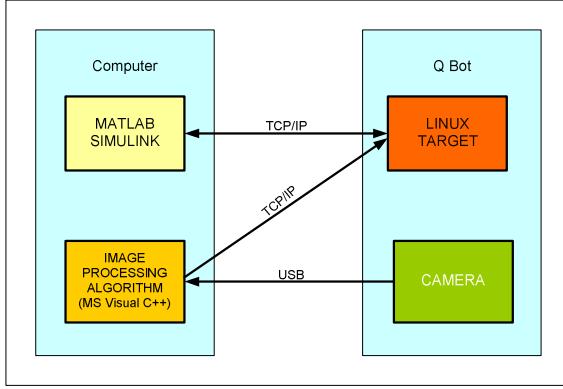
**Fig. 2.** System Configuration

For image processing operation an OpenCV based library, ARma is used [19]. This software tool directly gives the coordinates of the recognized pattern. After applying some transformations, it is possible to reach the error terms as defined in the theoretical part.

Since the image processing software require higher computing power. It will not be possible to directly implement the entire algorithm to the Linux target located on the robot. Instead, the image processing algorithm is operated over a portable computer and the processed data is read through wireless communication by the robot. Meanwhile the process is started, stopped and tracked by MATLAB SIMULINK platform and all process data can be recorded.

At the first step, the image processing algorithm detects the pre-defined pattern and locates its coordinates in the camera coordinate system. The data that is sufficient to determine the error terms is served on a listening TCP connection. A data request from the Linux target over the robot starts the next step and it continues with calculation of the error terms in the target. The last step is calculating the control signals to be applied as

wheel speeds of the robot. An illustration of the data flow can be seen in Fig. 3.



**Fig. 3.** Data flowchart

The parameters of the application can be found in Table 1. All of the metric parameters are in millimeters and angles in radians.

**Table 1.** Parameters of the application

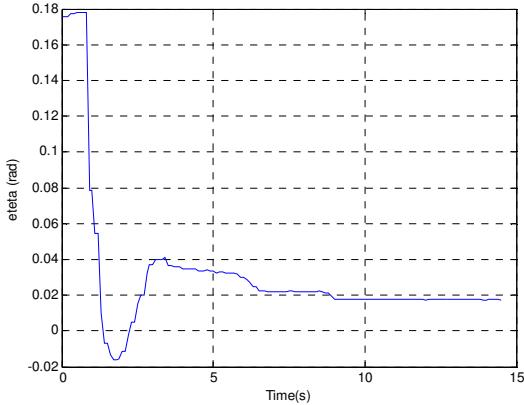
$\lambda_1$	$\lambda_2$	$L$	$r_{min}$	$e_d(0)$	$e_\theta(0)$
1	1.2	250mm	400mm	550mm	0.18

The application specific equations of the control signals are given below,

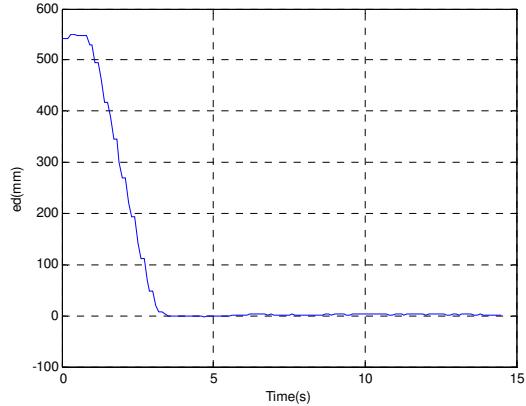
$$u_1 = \frac{e_d}{\cos e_\theta} \quad (15)$$

$$u_2 = 150e_\theta + \frac{125e_d \tan e_\theta}{e_d + 400} \quad (16)$$

The results of the application are given in Fig. 4 and Fig. 5.



**Fig. 4.** Orientation error due to time



**Fig. 5.** Distance error due to time

As it can be seen in the results, distance error converges to zero in a short time. On the other hand orientation error converges close to zero with an unexpected small overshoot. The reason for the overshoot is commonly about the time delay caused by the wireless communications. And the negligible steady state error is caused by the quantization error mostly caused by the type conversion transformations.

## 5. Conclusions

The motion control of a pattern tracking nonholonomic mobile robot has been presented in this paper. The feedback linearization based controller has been designed and applied over Quanser QBot platform successfully and the experimental results achieved are confirmative.

This application can be extended for wide range use by using improved cameras and other sensors. At the moment detection range is limited depending on the visual capabilities. Moreover studies can be extended by using other methods that can deal with uncertain time delays caused by wireless network.

## 6. References

- [1] Y. Kanayama, Y. Kimura, F. Miyazaki, T. Noguchi, "A stable tracking control method for an autonomous mobile robot", *In Proc. IEEE International Conf. on Robotics and Automation*, 1990, pp. 384-389.
- [2] R. Fierro, F. L. Lewis, "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics", *In Proc. 34<sup>th</sup> IEEE Conf. on Decision and Control*, New Orleans, LA, USA, 1995, pp. 3805-3810.
- [3] K. H. Park, S. B. Cho, Y. W. Lee, "Optimal tracking control of a nonholonomic mobile robot", *In Proc. ISIE 2001 IEEE International Symp. on Industrial Electronics*, Pusan, Korea.
- [4] C. Y. Tsai, K. T. Song, "Visual tracking control of a mobile robot using a new method in image plane", *In Proc. 12<sup>th</sup> IEEE Conf. Advanced Robotics*, Tokyo, Japan, 2005, pp. 540-545.
- [5] G. Scaglia, A. Rosales, L. Quintero, V. Mut, R. Agarwal, "A linear-interpolation-based controller design for trajectory tracking of mobile robots", *Control Engineering Practise*, vol. 18, pp. 318-329, November, 2009.
- [6] C.Y. Tsai, K. T. Song, "Dynamic visual tracking control of a mobile robot with image noise and robustness",

*Image and Visual Computing*, vol. 27, pp. 1007-1022 August, 2008.

- [7] J. M. Jung, J. H. Kim, "Sliding mode control of trajectory tracking of nonholonomic wheeled mobile robots", *Robotics and Autonomous Systems*, vol. 15, no.3, pp. 575-587, June, 1999.
- [8] G. Yuan, S. X. Yang, G. S. Mittal, "Tracking control of a mobile robot using a neural dynamics based approach", *In Proc. IEEE International Conf. on Robotics and Automation*, Seoul, Korea, 2001, pp. 163-168.
- [9] A. Treptow, G. Cielniak, T. Duckett, "Real time people tracking for mobile robots using thermal vision", *Robotics and Autonomous Systems*, vol. 54, pp. 729-739, July, 2006.
- [10] Y. Hada, E. Hemeldan, K. Takase, H. Gakuhari, "Trajectory tracking control of a non-holonomic mobile robot using iGPS and Odometry", *In Proc. IEEE Conf. on Multisensor Fusion and Integration for Intelligent Systems*, Tokyo, Japan, 2003, pp. 51-57.
- [11] H Zhang, J. P. Ostrowski "Visual motion planning for mobile robots", *IEEE Transactions on Robotics and Automation*, vol. 18, no. 2, pp. 199-208, April, 2002.
- [12] L. Rui, D. Zhijiang, S. Lining, "Moving object tracking based on mobile robot vision", *In Proc. IEEE International Conf. on Mechatronics and Automation*, Changchun, China, 2009, pp. 3625-3630.
- [13] J. B. Coulaud, G. Campion, G. Bastin, M. De Wan, "Stability analysis of a vision based design for an autonomous mobile robot", *IEEE Transactions on Robotics*, vol. 22, no. 5, pp. 1062-1069, October, 2006.
- [14] L. Freda, G. Oriolo, "Vision-based interception of a moving target with a nonholonomic mobile robot", *Robotics and Autonomous Systems*, vol. 55, pp. 419-432, February, 2007.
- [15] J. Chen, W. E. Dixon, D. M. Dawson, M. McIntyre "Homography-based visual servo tracking control of a wheeled mobile robot", *IEEE Transactions on Robotics and Automation*, vol. 22, no. 2, pp. 407-415, April, 2006.
- [16] Y. Ma, J. Kosecka, S. Sastry, "Visual guided navigation for a nonholonomic mobile robot", *IEEE Transactions on Robotic and Automation*, vol. 15,no. 3, pp. 4521-535, June, 1999.
- [17] F. Kühne, J. M. G. da Silva Jr., W. F. Lages "Mobile robot trajectory tracking using model predictive control", *In Proc. IEEE Latin American Robotics Symposium*, Sao Luis, Brazil, 2005
- [18] H. K. Khalil, "Nonlinear systems", Prentice Hall, New Jersey, USA, 2002.
- [19] G. Evangelidis. (2011). ARma library: Pattern Tracking for Augmented Reality,. Available:  
<http://xanthippi.ceid.upatras.gr/people/evangelidis/arma/>