Dynamic Behavior of 1-D Array of the Memristively-coupled Chua's Circuits

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Abstract

Synchronization phenomena is a crucial instrument for neural information processing in biological systems. The behavior of biological synapse which connects neurons can be imitated by memristor. This the why the role of memristor in collective dynamics of coupled Chua's circuit has been explored in this paper. Complete synchronization phenomena has been investigated for memristively and resistively coupled Chua's circuit.

1. Introduction

Since memristor was introduced by R. S. Williams and his colleagues [7] using a thin film of titanium dioxide, it has received significant amount of attention. Memristor was formulated and named by Leon Chua [2] in 1971. Memristor has potential to reproduce the behavior of a biological synapse. A synapse is a structure that permits a neuron to pass an electrical to another cell. Therefore it is very important element of the network and its dynamical behavior. Synchronization phenomena is a well-known collective dynamics of coupled systems. In biological system, synchronization is an important mechanism for neural information processing. It has been already reported in [3] that the memristor plays a crucial role on the synchronization mechanisms of the bio-inspired network.

In order to understand the role of memristor as a coupling element, we focus on a network composed of Chua's circuit [1] coupled via memristor. Chua's circuit is a third order nonlinear circuit which is able to generate variety of dynamical behavior. Furthermore we investigate the same network using resistive coupling and compare with memristively coupled network. Synchronization of the memristively coupled network is investigated also.

This paper is organized as follows. Section II introduces Chua's Circuit with Resistor. Section III presents Locally Coupled Chua's Circuit with Memristors and Section IV concludes the work.

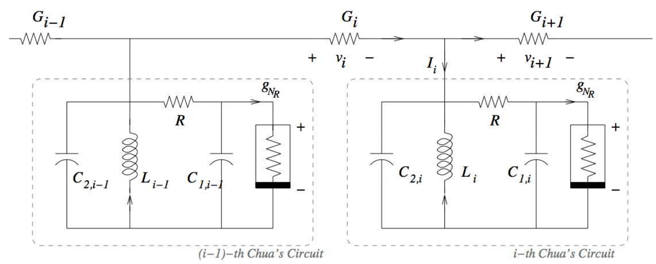


Figure 1. One dimensional Cellular Neural/Nonlinear Network circuit consist of Chua's circuits. Each Chua's circuit is coupled with resistor.

2. Locally Coupled Chua's Circuit with Resistor

Locally coupled Chua's circuits with linear resistors (Figure 1) are described by

$$\begin{cases} C_1 \dfrac{dv_{C1,i}}{dt} = G(v_{C2,i} - v_{C2,i}) - g_{N_R}(v_{C1,i}) \\ C_2 \dfrac{dv_{C2,i}}{dt} = G(v_{C2,i} - v_{C2,i}) + i_L + I_i \\ L \dfrac{di_{L,i}}{dt} = v_{C2,i}, i = 1, 2, 3, ..., N, \end{cases} \quad (1)$$

where i denotes the cell number and N is the number of cells (or Chua's circuits), $V_{C1,i}$, $V_{C2,i}$ are the voltage across capacitors C_1 and C_2 of i^{th} Chua's circuit, respectively, and $i_{L,i}$ is current flowing upward through the inductor of i^{th} Chua's circuit. The nonlinear function $g_{N_R}(v)$ is described by the piecewise-linear function

$$g_{N_R}(v_{C1}) = G_b v_{C1} + \frac{(G_a - G_b)}{2}(|v_{C1}+1| - |v_{C1}-1|) \quad (2)$$

[1]. I_i is a synaptic law of the i^{th} Chua's circuit. The synaptic law defines the coupling between the considered i^{th} Chua's circuit and all the cell within the prescribed neighborhood.

From the circuit given in Figure 1, I_i is obtained by

$$I_i = G_i v_i - G_{i+1} v_{i+1}. \quad (3)$$

Using $v_i = v_{C1,i-1} - v_{C1,i}$ equation from the Kirchhoff's voltage law, I_i is given by

$$I_i = G_i v_{C2,i-1} + G_{i+1} v_{C2,i+1} - (G_i + G_{i+1}) v_{C2,i}. \quad (4)$$

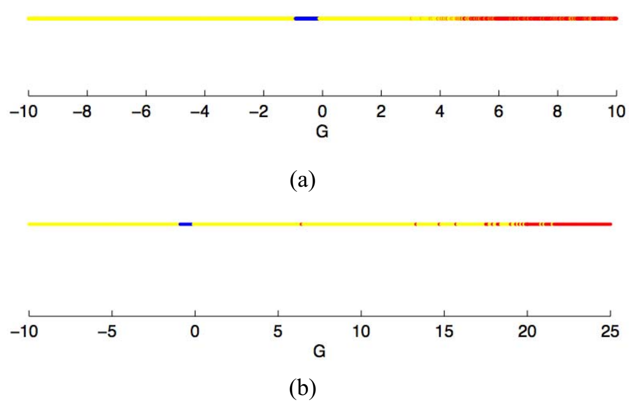


Figure 2. The network's dynamic behavior due to the coupling straight. Yellow, red and blue regions denote the regions of synchronous, non-synchronous and unstable behavior, respectively. (a) $L = 10$ (number of circuits) and (b) $L = 20$.

For the boundary conditions, we impose the conditions $v_{C1,0} = v_{C1,N}$ and $v_{C1,1} = v_{C1,N+1}$.

The chain of coupled Chua's circuit describes a one-dimensional Cellular Neural/Nonlinear Network (CNN). By making the following change of variables: $x = v_{C1,i}$, $y = v_{C2,i}$, $z = Ri_{L,i}$, $\alpha = \frac{C_2}{C_1}$, $\beta = \frac{C_2R^2}{L}$, $m_0 = RG_a$, $m_1 = RG_b$, $\hat{I}_i = RI_i$, $G = G_i = G_{i+1}$ and $\tau = \frac{t}{RC_2}$, the equation of the one-dimensional CNN in normalized dimensionless form are obtained

$$\begin{cases} \dot{x_i} = \alpha(y_i - x_i - f(x_i)) \\ \dot{y_i} = x_i - y_i + z_i + \hat{I}_i \\ \dot{z_i} = -\beta y_i, i = 1, 2, 3, ..., L, \end{cases} \tag{5}$$

where

$$f(x) = m_1 x + \frac{1}{2}(|x+1| - |x-1|) \tag{6}$$

and

$$I_i = G(x_{i-1} + x_{i+1} - 2x_i). \tag{7}$$

Furthermore, the coupling between the Chua's circuits which is defined above is known as diffusive coupling.

Dynamic behavior of the CNN in Eq. (5) has been investigated by Kapitaniak and L. O. Chua [5] and Suykens and Chua [8]. n-scroll hypercube attractors have been observed on the common state subspace of the cells [8]. In this work, the regions of synchronous, non-synchronous and unstable behavior of the CNN consisting of Chua's circuit cells which is defined in Eq. (5) experimentally investigate for the coupling straight G. There are many synchronization schemes in literature [6]. In this work synchronization meaning is complete synchronization which is characterized by the convergence of the all the trajectories, $[x_i(t)\ y_i(t)\ z_i(t)] = [x_j(t)\ y_j(t)\ z_j(t)]$. During the computer simulations, the parameters values are taken as $N = 10$, $\alpha = 9$, $\beta = 14.286$, $m_0 = -1/7$ and $m_1 = 2/7$. In Figure 2, yellow, red and blue regions denote the regions of synchronous, non-synchronous and unstable behavior, respectively.

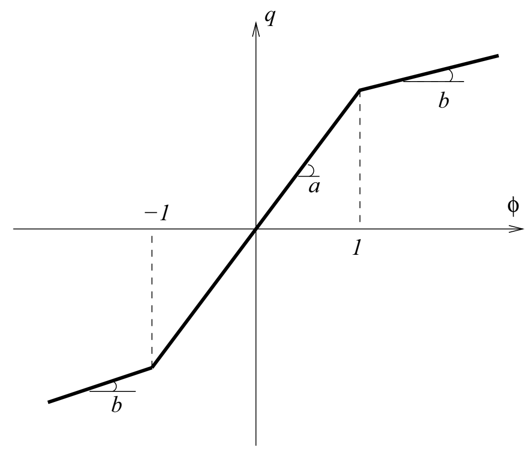


Figure 3. Piecewise linear characteristic of a flux-controlled memristor.

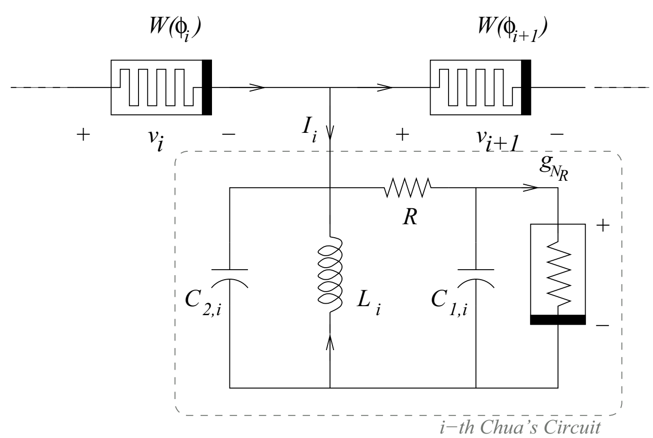


Figure 4. A cell of the CNN consist of Chua's circuit. Each cell is coupled to its neighbors with identical flux-controlled memristor.

3. Locally Coupled Chua's Circuit with Memristors

A memristor [2] is characterized by a relation of the type $g(\phi, q) = 0$. This relation can be express in the form of charged-controlled and flux-controlled memristor which are given by

$$\phi = g(q) \text{ and } q = g(\phi), \tag{8}$$

respectively.The terminal voltage of a charge-controlled and the terminal current of a flux-controlled memristors are given by

$$v = M(q)i \text{ and } i = W(\phi)v \tag{9}$$

where $M(q) = \frac{dg(q)}{dq}$ and $W(\phi) = \frac{dg(\phi)}{d\phi}$, called the memristance and the memductance, respectively. Like in [4], we assume that the memristor is characterized by the piecewise linear function shown in Figure 3.

Locally coupled Chua's circuits with memristor shown in

Figure 4 are described by the following state equations:

$$\begin{cases} C_1 \dfrac{dv_{C1,i}}{dt} = G(v_{C2,i} - v_{C2,i}) - g_{N_R}(v_{C1,i}) \\ C_2 \dfrac{dv_{C2,i}}{dt} = G(v_{C2,i} - v_{C2,i}) + i_L + I_i \\ L \dfrac{di_{L,i}}{dt} = v_{C2,i} \\ \dfrac{d\phi_i}{dt} = v_i \end{cases} \tag{10}$$

where

$$I_i = W(\phi_i)v_i - W(\phi_{i+1})v_{i+1}. \tag{11}$$

with the memductance is defined by

$$W(\phi) = \begin{cases} a & |w| < 1 \\ b & |w| > 1. \end{cases} \tag{12}$$

The terminal voltage of the memristor (v_i) is given by

$$v_i = v_{C2,i-1} - v_{C2,i} \tag{13}$$

hence the synaptic law is given by

$$\begin{aligned} I_i &= W(\phi_i)(v_{C2,i-1} - v_{C2,i}) \\ &\quad - M(\phi_{i+1})(v_{C2,i} - v_{C2,i+1}) \\ &= W(\phi_i)v_{C2,i-1} + W(\phi_{i+1})v_{C2,i+1} \\ &\quad -(W(\phi_i) + W(\phi_{i+1}))v_{C2,i} \end{aligned} \tag{14}$$

By making the change of variables like in the Section 2 with $\gamma = \frac{1}{RC_2}$, the state equations of the locally coupled Chua's circuits with memristors in normalized dimensionless form are given by

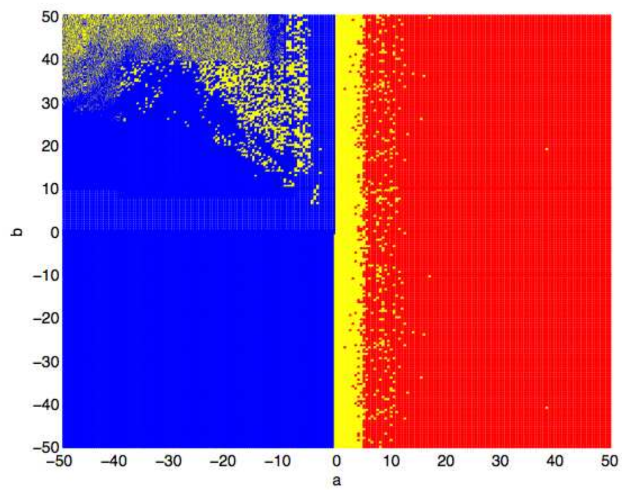
$$\begin{cases} \dot{x_i} &= \alpha(y_i - h(x_i)) \\ \dot{y_i} &= x_i - y_i + z_i + \hat{I}_i \\ \dot{z_i} &= -\beta y_i \\ \dot{\phi_i} &= \gamma(x_{i-1} - x_i), \;\; i = 1, 2, 3, ..., L, \end{cases} \tag{15}$$

where

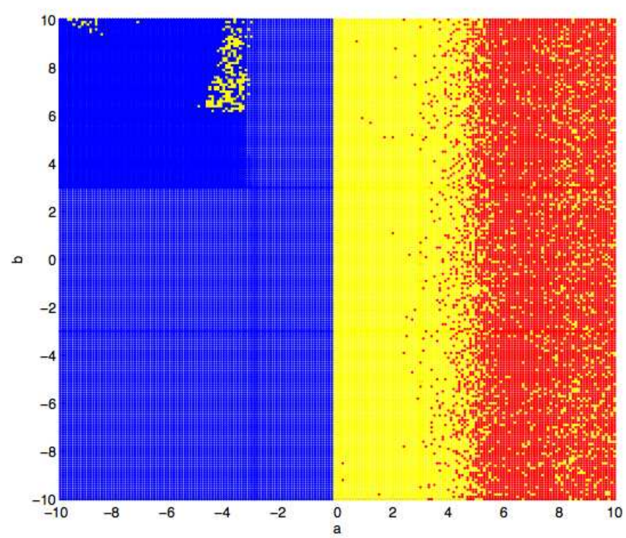
$$\begin{aligned} \hat{I}_i &= W(\phi_i)y_{i-1} + W(\phi_{i+1})y_{i+1} \\ &\quad -(W(\phi_i) + W(\phi_{i+1}))y_i. \end{aligned} \tag{16}$$

In order to understand the effect of the coupling with memristor, the same parameters values and the same number of Chua's circuit are taken with the Section 2 where Chua's circuits were coupled with resistor. Here we investigate the effect of the nonlinear characteristic of the memristor in order to classify the network dynamic such as synchronization (complete), non-synchronization and instability. Like the previous section, the network dynamic is pictured in the parameter space of the memristor (see Figure 5) where yellow, red and blue regions denote the regions of synchronous, non-synchronous and unstable behavior, respectively.

In Figure 6.a ((1,1) to (3,2)), the state spaces projected to $x_i - y_i$ sub-space of the ten Chua's circuits for $a = b = 20$ show that each circuit is in chaotic regime. Also in Figure 6.a ((3,3) to (5,4)), the subspaces of $x_i - x_{i+1}$ space which shows the synchronization between each neighborhood circuits are shown. Consequently the network is in synchronous regime. The same simulation has been repeated for twenty Chua's circuit ($L = 20$) and simulation result which shows the regimes in the parameter space is given in Figure 6.b.

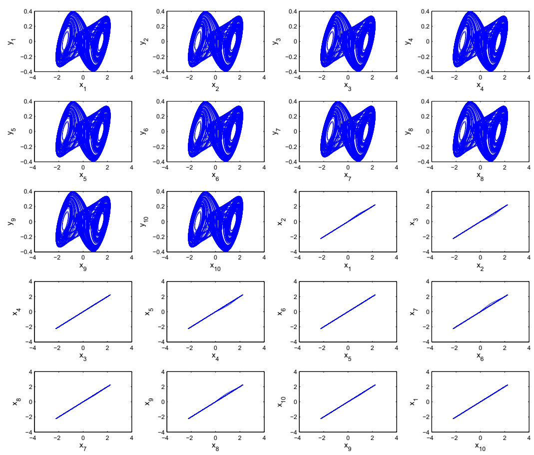


(a)

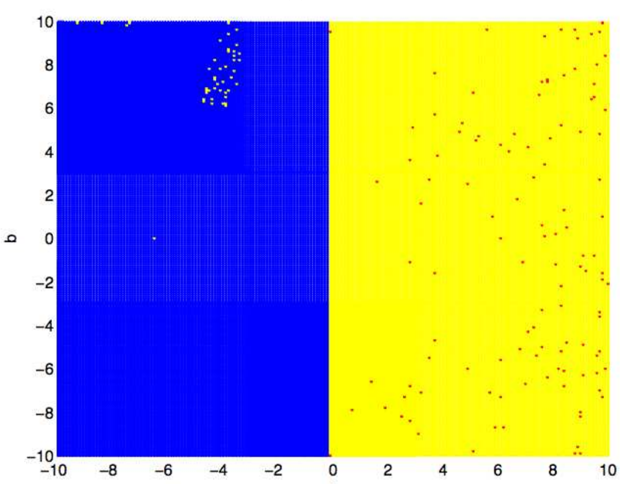


(b)

Figure 5. The network dynamic due to the parameter space of the memristor. Yellow, red and blue regions denote the regions of synchronous, non-synchronous and unstable behavior, respectively for (a) $|a| < 10$ and $|b| < 10$ and (b) $|a| < 50$ and $|b| < 50$.



(a)



(b)

Figure 6. (a) Dynamic behavior of the each cell for $a = 20$ and $b = 20$. (b) Same analysis with Figure 5 for $L = 20$.

4. Acknowledgment

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5. References

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