# Retrieval of effective parameters of left handed materials by using different approaches

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## Abstract

**Two different expressions are used in this paper to extract bulk electromagnetic properties matching to effective permeability and permittivity, which characterize metamaterials. These parameters have been determined by calculation of S-parameters. We use global approaches with these different analytical methods of calculation. This choice of global approaches is justified by the fact that negative refraction index metamaterials belongs to particular category where the limits of validity of the effective medium theories are not defined .For designing and testing the studied structure, we use HFSS software as a computational approach based on finite element method** .

***Index Terms;*** *Metamaterial, Left Hunded Material, Effective parameters, Reflection method, Retrieval method, Analytical approach***.***

## 1. Introduction

The interest about metamaterials (MTMs) had been emerged in 1967 by the Russian Physics Veselago, where these artificial composite structures verify the negative refraction of electromagnetic waves by the exhibition of a simultaneous negative values of magnetic permeability and electric permittivity in the same frequency domain, so at results we have left handed system [1]. The subject of MTMs has restarted in 1999 by J. Pendry, who proposed a design of a composite materials with periodic structure of split ring resonators and thin wires, where the two types contains small resonators than the incident wavelength and allows a negative permeability and permittivity in the microwave range [2].

A year later, Smith et al finished the experimental verification of the first prototype of left-handed material(LHM). Smith's research group showed that the wave propagation in single negative materials could not restore as the case of double negative mcaterials [3]. Otherwise Schelby and Smith proposed a new manner to realize a double negative medium for microwave frequencies, consisting of two dimensional arrays of split ring resonators and continuous copper wire strips even the geometry is isotropic [4].

After the year 2000, there was a great expansion in the domain of MTMs and several applications come to light in different fields such as electronic, telecommunication, nanotechnology, electromagnetic cloaking which requires a material whose permittivity and permeability can be independently controlled.

In addition, it was a great development in many physics disciplines like optics, electrical and mechanical engineering, materials science …etc.

Metamaterial devices and design methods are constantly being imagined, simulated, designed and tested by using computational electromagnetic. This approach has appeared as a crucial enabling technology for microwave and wireless engineering [5]. The most popular methods are; Finite difference time domain method, transfer matrix method, method of moments and finite element method. CST microwave studio, ANSOFT HFSS, and COMSOL are well known as commercial softwares and also as electromagnetic computational tool that can report simulations in 2D and 3D. The design of MTM based on shape and geometry is the most interesting work among the others [6-7]. Especially, the design of split rings is very important to construct a new type of MTMs. Numerous types of different ring and ring-like structures such as circular, square, Ω-shaped, U-shaped, S-shaped and others are used to create new MTMs.

In the light of the known structures, we present a coupled "S" shaped structure. Using the commercial software HFSS, the S-parameters for a single unit cell are calculated with the mentioned boundaries along the wave propagation. In the analysis, S-parameters and retrieved effective material parameters are computed and visualized. From the simulation results, the real part of the refractive index is found to be negative at frequencies where both real parts of the permittivity and permeability are negatives.

## 2. Analysis and simulation

### 2.1 Analytical approaches

Reflection/transmission or S-parameters method is based on the reversal of the complex reflection and transmission coefficients determined for a slab of compound material. Initially proposed for normal incidence on the isotropic metamaterial [8].

Using the theory of homogenization to calculate the effective parameters (wave impedance, refractive index, permittivity, and permeability) of the homogenous slab from the transmission and reflection coefficients, where they are obtained by simulation under HFSS software.

To show the physical properties of the designed structures, S-parameters for a single unit cell are calculated with the mentioned boundaries along the wave propagation. The expression for the extracting normalized impedance (z ) is given by:

$$z=\pm\sqrt{\frac{\left(1+S_{11}{}^{2}\right)-S_{21}{}^{2}}{\left(1-S_{11}{}^{2}\right)-S_{21}{}^{2}}} \qquad (1)$$

The refractive index (n) of the under-study medium can be calculated as follows:

### 2.1.1 First approach

The S-parameter retrieval method introduced in [9]. It's the most common and successful approach [10]. It is being applied for different classes of metamaterials. The effective material parameters can be extracted from the S parameters as described in:

$$e^{jnk_0 d} = \frac{S_{21}}{S_{11}\left(\frac{z-1}{z+1}\right)} \tag{2}$$

$$n = \frac{1}{k_0.d}\cos^{-1}\left[\frac{1}{2S_{21}}.\left(1 - {S_{11}}^2 + {S_{21}}^2\right)\right] \tag{3}$$

Or

$$n = \frac{1}{jk_0.d}\,log\left[\frac{S_{21}}{\left(1-\left(S_{11}.\left(\frac{z-1}{z+1}\right)\right)\right)}\right] \tag{4}$$

Where $k_0$ and d are the wave vector of the incident wave and the thickness of the slab. n indicate the refractive index, respectively. Then, the electric permittivity **ε** and magnetic permeability **μ** can be computed from the equations :

$$\varepsilon = n/z \quad \text{and} \quad \mu = n.z \tag{5}$$

### 2.1.2 Second approach

To extract the effective medium parameters from the normal incidence scattering parameter data, we use the relations described in [11], which present a modified Nicolson-Ross-Weir approach [12-13].The NRW approach begins by introducing the composite terms:

$$V_1 = S_{11}+S_{21} \tag{6}$$

$$V_2 = S_{11}-S_{21} \tag{7}$$

The permeability, permittivity and index of refraction can then be obtained simply as:

$$\mu_r \sim \frac{2}{jk.d}.\frac{1-V_2}{1+V_2} \tag{8}$$

$$\varepsilon_r = \left(\frac{k}{k_0}\right)^2.\frac{1}{\mu_r} \tag{9}$$

$$n = \sqrt{\mu_r \varepsilon_r} = \frac{k}{k_0} \tag{10}$$

There are two others approximate expressions to extract permittivity:

$$\varepsilon_r \sim \frac{2}{jk.d}\,\frac{1-V_1}{1+V_1} \tag{11}$$

$$\varepsilon_r \sim \mu_r + j\,\frac{2S_{11}}{k_0.d} \tag{12}$$

There are three possibilities to trace effective permittivity allowing us to plot three different curves corresponding to refraction index where we can see ranges of frequencies in which, permittivity and permeability are negative simultaneously.

### 2.2. Design and Simulation

The proposed Left-Handed unit cell used in the simulations is shown in Fig.1. Its geometric parameters strongly affect the magnetic resonant frequency [14].

The structure has been dimensioned for resonance in the vicinity of 10 GHz. where two copper SRRs with S-shaped are printed on the opposite sides of dielectric substrate (FR4) who has a permittivity $\varepsilon_r$ = 4.4, and a thickness of 0.5 mm, the width of metallic ring is 0.4 mm .A unit cell measures 4 mm×2.5 mm×5.4 mm.

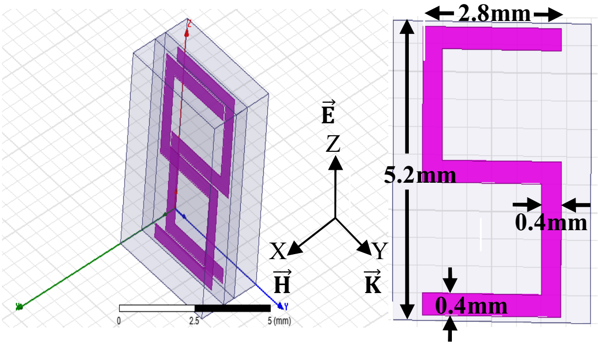


**Fig. 1.** Geometry of the double S structure.

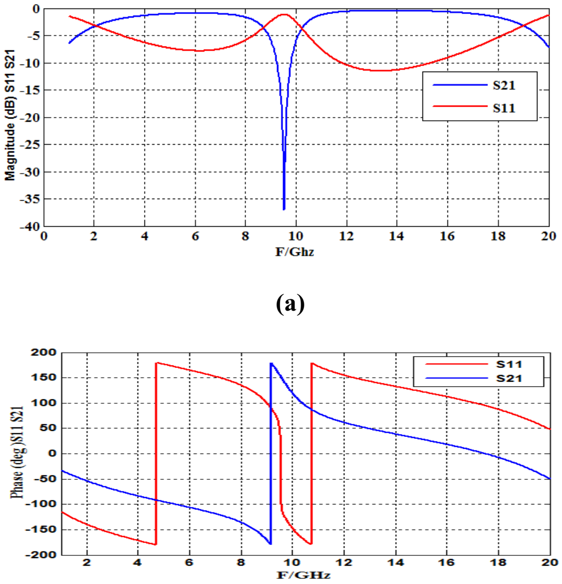
Metamaterial unit cell is designed and simulated using the commercial software package, ANSOFT's High Frequency structure Simulator (HFSS), based on finite-element method (FEM). For the simulation of the unit cell, boundary conditions of magnetic and electric wall are applied respectively according to axes y and x. The structure is excited by an electromagnetic wave with the propagation vector k in z direction. The H field of the electromagnetic wave has to be along (parallel) the axis of the cell (x) in order to induce the current through the rings which gives the desired effect.

The Simulation is made on a frequency band between 1GHz and 20 GHz with an incremental strep of 20MHz.

## 3. Results and discussion

In figure 2, computed results for transmission ($S_{21}$) and reflection ($S_{11}$) characteristic for the MTM unit cell are given. Magnitudes of $S_{11}$ and $S_{21}$ presented in figure 2(a) shows a resonant at 9.54 GHz with an order of 37.1dB.

It's clear from figure 2(b), the dip in the phase of $S_{21}$ which indicates the presence of a negative region observed at 9.15GHz.

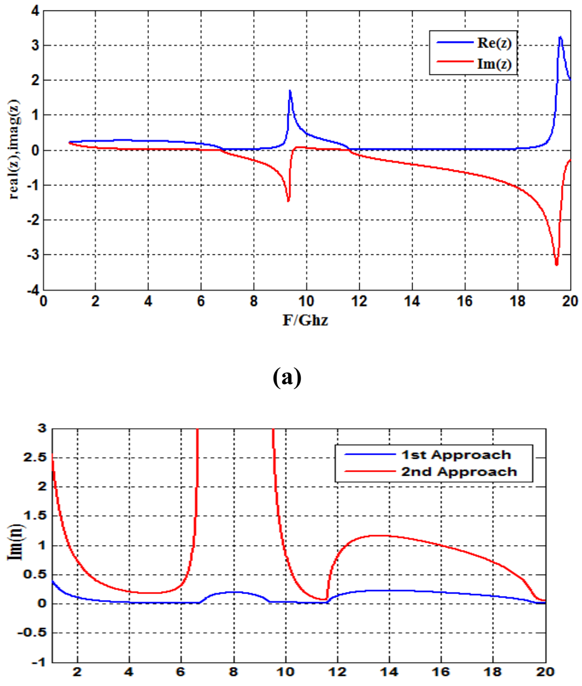


(a)

(b)

**Fig. 2.** Single MTM unit cell: (a) Magnitude spectra of $S_{11}$ and $S_{21}$ versus frequency. (b) Phase of $S_{11}$ and $S_{21}$ versus frequency.

In this structure, real part for wave impedance and imaginary part for refractive index are greater than zero, that's why they ensure the verification of passivity for the medium as it is appears in figure (3). We note that the trace of the imaginary part for refractive index is done by the two approaches.



(a)

(b)

**Fig. 3.** Single MTM unit cell : (a) real and imaginary parts of the wave impedance. (b) Imaginary part of the refractive index.

We analyze the reflection and transmission coefficients calculated from transfer matrix simulations to determine the effective material parameters $(\varepsilon_{eff}, \mu_{eff}, n_{eff})$ using a MATLAB code based on previous equations. Like we presented in the last curv, these parametrs and illustrated successively in figure 4,5 and 6 . Effective permittivity and permeability are also compared with previously published work [15], they were in good agreement.

Permittivity of MTM unit cell is resonant in real part and lies in the negative band from 6.8 to 19.56 GHz .this permittivity agree with Drude response behavior the frequency range. Whereas the real part of effective permeability resonates and occurs in the negative band from 9,17 to 11.65 GHz .this permeability agree with Lorentz behavior the frequency range. The two approaches show the same negative band with different peaks.

It can be said that the frequency region with negative permeability is very narrow however the frequency region with negative permittivity is relatively wider which can be explained relatively with " Eq12" in the second approach , thus the key issue of fabricating LHMs is the realization of the negative permeability. It is desirable to realize the negative permeability metamaterials by controlling the permittivity of the constituent materials. The negative band for the refractive index exists beteween 9.17 to 11.74 GHz, it's where the permittivity permeability are simultaneously negative .This result confirm that the S-shaped resonators exhibits left-handed properties without using an additional array of thin wires.

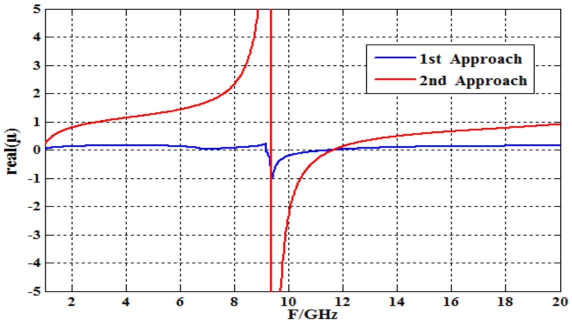


**Fig. 4.** Single MTM unit cell: Real part of the permeability by using two approaches.

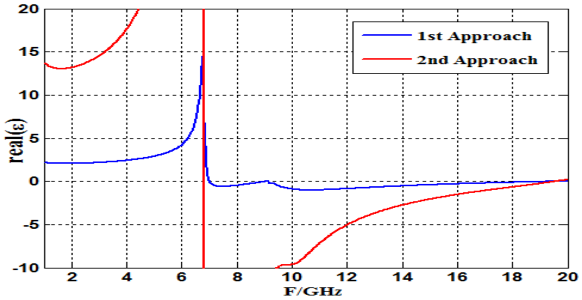


**Fig. 5.** Single MTM unit cell: Real part of the permittivity by using two approaches.

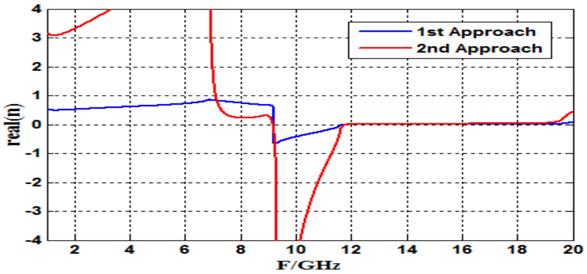


**Fig. 6.** Single MTM unit cell: Real part of the refractive index by using two approaches.

The difference between the two traces (approaches) in the last figure for refraction index is due to the disparity of the expressions (analytical functions) used in each approach. Extraction manner affects also on the curves ; we compute directly the refraction index in the first approach while in the second approach we calculate first the permittivity and permeability in their approximate form, then we deduce it.

Furthermore,these two approaches are well clarified by using approximative formulas . In this figure ,we use the expressions of permittivity mentioned in the previous equations.

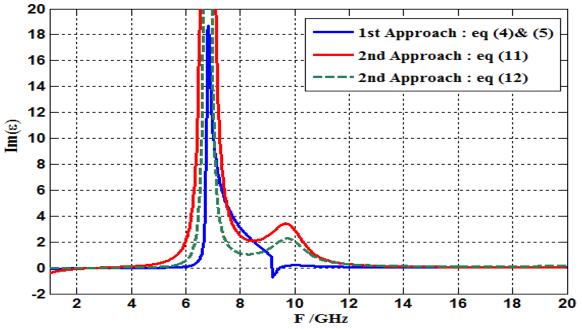


**Fig. 7.** Single MTM unit cell: Imaginary part of the permittivity by using two approaches.

## 4. Conclusion

We have studied a metamaterial structure from the unit cell of a coupled "S" shaped geometry. Then effective material parameters are retrieved by using two different approaches and computed using S parameter. Therefore, the essential role of global approaches relies on measuring and calculating observable magnitudes. The LHM property is shown by the simultaneous negative permittivity and permeability and negative refractive index without the need of additional thin wires. The negative permeability is realized by controlling the permittivity of the constituent materials. Modification of a certain number of parameters such as the structures dimensions and nature of substrate or split resonators affect directly the effective parameters that characterize MTM. Although these approaches don't give the same results in peaks but they always indicate the resonant frequency and essentially negative bands for each parameter.

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