COMPARISON OF THE PERFORMANCES OF ADI-FDTD AND EXPONENTIAL COEFFICIENT OPTIMIZED SYMPLECTIC FDTD METHODS

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ABSTRACT

Numerical dispersion performances of the ADI-FDTD and Symplectic FDTD methods have been compared. It has been shown that for time steps below the stability limits of the Symplectic FDTD method it has much better dispersion performance compared with the ADI-FDTD method and that the Symplectic FDTD method can be usefully employed for space increments in the order of $\lambda/25$ to $\lambda/50$.

I. INTRODUCTION

The finite-difference time-domain (FDTD) method has widely been used for the solution of electromagnetic problems [1]. The stability condition for this method [1] imposes a limitation on the time step. When the method is applied to electrically small problems this limitation necessitates unnecessarily small time steps which increase the computational time considerably. Alternating-direction implicit finite-difference time-domain (ADI-FDTD) method [2] is unconditionally stable and theoretically there is no limitation on the time step size of this method. But as the size of the time steps is increased the numerical dispersion error becomes significant and the time step size for the ADI-FDTD method is limited in use by the level of the numerical dispersion error that can be tolerated.

On the other hand Symplectic FDTD method [3] is a scheme which uses fourth-order finite differencing for space and a symplectic scheme using exponential differential operators for time. The method reduces the numerical dispersion errors significantly. It has been shown [3] that the stability limit of this method is much higher than the Yee's FDTD method and that the stability limit depends linearly on the number of the exponential coefficients.

In this presentation the performances of the ADI-FDTD method, with second order and fourth order finite differencing in space, are compared with the performances of the Symplectic FDTD method.

II. NUMERICAL DISPERSION PERFORMANCE

When an electromagnetic problem is simulated in a discretized domain the phase velocity of the electromagnetic wave differs slightly from the phase velocity of the natural medium. The variation in the phase velocity is not constant but varies with the frequency, direction of propagation and the sizes of the time and spatial steps. There has been many publications dealing with the numerical dispersion of the ADI-FDTD method [4-7]. In this presentation the expression used by Weiming Fu et. al.[5] is used for calculating the three dimensional (3-D) dispersion error of the ADI-FDTD method, which is given by:

$$\sin^2(\omega\Delta t) = \frac{4s^2\left[\eta^2 + s^2\left(\eta_x^2\eta_y^2 + \eta_y^2\eta_z^2 + \eta_z^2\eta_x^2\right)\right]\left[1 + s^6\eta_x^2\eta_y^2\eta_z^2\right]}{\left[\left(1 + s^2\eta_x^2\right)\left(1 + s^2\eta_y^2\right)\left(1 + s^2\eta_z^2\right)\right]^2} \quad (1)$$

where $\eta = \sqrt{\eta_x^2 + \eta_y^2 + \eta_z^2}$, and for 2nd order and 4th order space discretezition:

$$\eta_\gamma = \sin\left(\tilde{k}\Delta_\gamma\right) \quad ; \quad \gamma \in \{x, y, z\} \quad (2)$$

$$\eta_\gamma = \frac{27}{24}\sin\left(\tilde{k}\Delta_\gamma\right) - \frac{1}{24}\sin\left(3\tilde{k}\Delta_\gamma\right) \quad (3)$$

respectively. $\tilde{k}$ represents the numerical wavenumber, Δ ($\Delta=\Delta x=\Delta y=\Delta z$) is the cell size, Δt is the time increment, and $s = c\Delta t/\Delta$ is the stability factor. The A_γ parameters in these equations are defined as $A_x = \frac{\Delta}{2}\cos\varphi\sin\theta$, $A_y = \frac{\Delta}{2}\sin\varphi\sin\theta$ and $A_z = \frac{\Delta}{2}\cos\theta$.

The dispersion error of the Exponential Coefficient Optimized Symplectic FDTD Method can be optimized for chosen parameters of the method and the numerical error relationship for the Symplectic FDTD is given by[3]:

$$\cos(\omega\Delta t) = 1 + \frac{1}{2}\sum_{p=1}^{5} g_p \left\{4s^2\left(\eta_x^2 + \eta_y^2 + \eta_z^2\right)\right\}^p \qquad (4)$$

where,

$$g_p = \sum_{1\le i1\le j1<i2\le j2<\ldots<ip\le jp\le m} c_{i1}d_{j1}c_{i2}d_{j2}\ldots c_{ip}d_{jp} + \sum_{1\le i1<j1\le i2<j2\le\ldots\le ip<jp\le m} d_{i1}c_{j1}d_{i2}c_{j2}\ldots d_{ip}c_{jp} \qquad (5)$$

The η_γ parameters are defined in equation (2) and (3). c_i and d_i are time step coefficients to be determined.

The equations (1)- (5) have been used to obtain the three dimensional numerical dispersion performances of the two methods. As the Symplectic FDTD Method has a stability limit it was only possible to compare the performances of the two methods for time steps corresponding to this limit or below.
The graphical results of the numerical dispersion study carried out for the 10 Exponential Coefficient Symplectic FDTD and the ADI-FDTD method are given by Figure 1 to Figure 5. The Figure 1 and Figure 2 show the dispersion errors against the angle θ for space increments $\lambda/25$ and $\lambda/50$ at the stability limit (s=2.45) of the Symplectic FDTD. The results are given for the conventional 2nd order ADI-FDTD as well as for the 4th order ADI-FDTD. The results show that although there is not a significant difference in error performance of the two ADI-FDTD methods, the errors of the Symplectic FDTD method are much smaller. No results are presented against the φ angle as there is no significant variation with φ. Figure 3 shows the dispersion error against space increment (Δ) at the stability limit (s=2.45) of the Symplectic FDTD method. For both of the ADI-FDTD methods the errors become unacceptably high for Δ values larger than $\lambda/50$. The Figure 4 and Figure 5 show the dispersion errors against the stability factor s up to the stability limit of the Symplectic FDTD for space increments $\lambda/25$ and $\lambda/50$. As the s increases (in other words as Δt increases*)* the errors for both of the ADI-FDTD methods increases while the errors of the Symplectic FDTD method remains low.

The stability limit of the conventional Yee's FDTD method is 0.577 [1] in 3-D, so it can not be used for stability factors above this limit. As the performance of the ADI-FDTD method is not acceptable for the stability factors (s) in the region 1-2.5 for space increments in the region $\lambda/25$ - $\lambda/50$, the Symplectic FDTD method can be usefully employed in these regions.

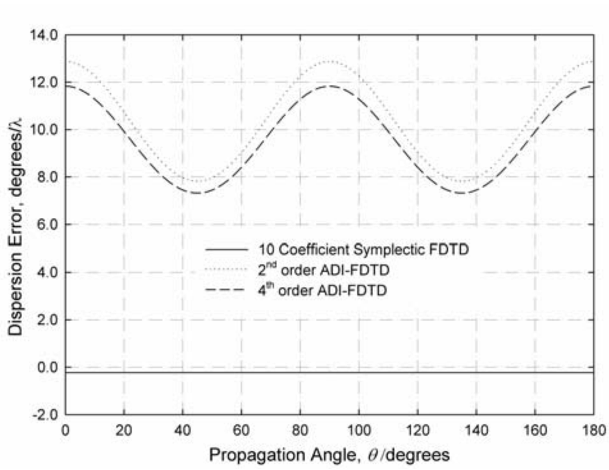

Figure 1. Dispersion error as a function of propagation angle θ, for φ=90^o, Δ=λ/25 and s=2.45.

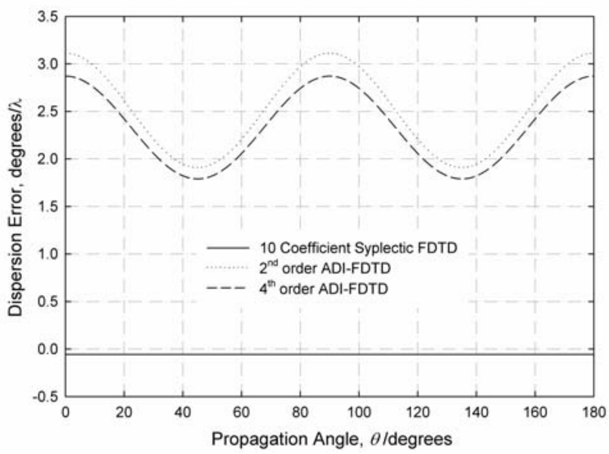

Figure 2. Dispersion error as a function of propagation angle θ, for φ=90^o, Δ=λ/50 and s=2.45.

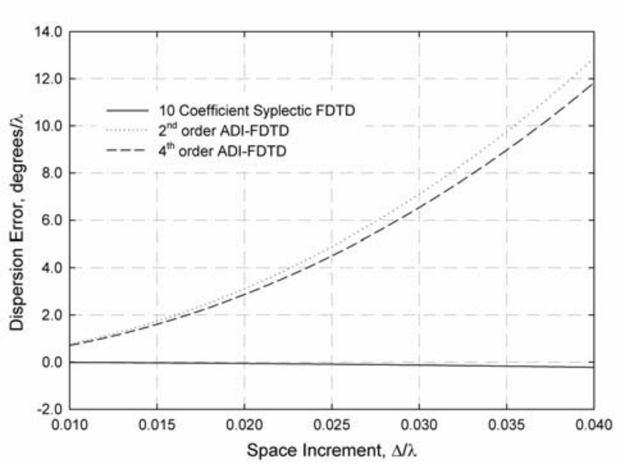

Figure 3. Dispersion error as a function of space increment per wavelength, for θ=0°, φ=90° and s=2.45.

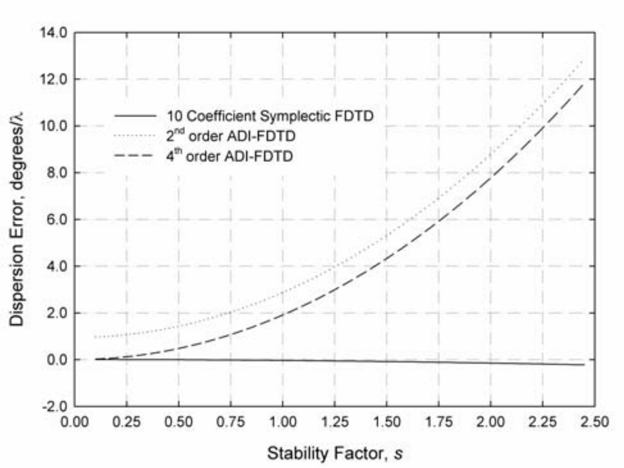

Figure 4. Dispersion error as a function of stability factor, for θ=0°, φ=90° and Δ=λ/25.

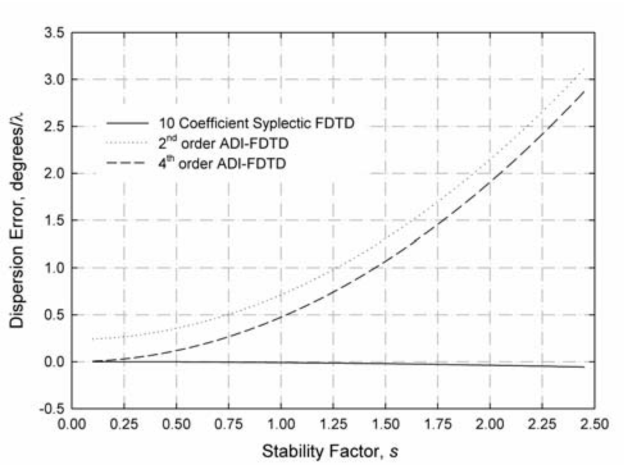

Figure 5. Dispersion error as a function of stability factor, for θ=0°, φ=90° and Δ=λ/50.

III. CONCLUSION

It has been shown that ADI-FDTD method has large dispersion errors when the space increments are in the order of $\lambda/25$ to $\lambda/50$ for stability factors of larger than 1. It has also been shown that for time steps below the stability limits of the Symplectic FDTD method it has much better dispersion performance compared with the ADI-FDTD method. Therefore the symplectic FDTD method can be usefully employed for space increments in the order of $\lambda/25$ to $\lambda/50$ for stability limits above 1 and below the stability limit of the Symplectic FDTD .

REFERENCES

1. K. S. Yee, “Numerical solution of initial boundary value problem involving Maxwell’s equation in isotropic media,” IEEE Transaction on Antennas Propagation, vol. AP-14, no. 5, pp. 302–307, May 1966.
2. F. Zheng and Z. Chen, Toward the development of a three-dimensional unconditionally stable finite-difference time-domain method, IEEE Transaction on Microwave Theory and Techniques, vol. 48, pp. 1050–1058, Septemper 2000.
3. M. Kusaf and A. Y. Oztoprak, Higher Stability Limits for the Symplectic FDTD Method by Making Use of Chebyshev Polynomials, IEEE Microwave and Component Letter, vol. 16, no. 11, pp. 579–581, Novenber 2006.
4. F. Zheng and Z. Chen, Numerical dispersion analysis of the unconditionally stable 3-D ADI-FDTD method, IEEE Transaction on Microwave Theory and Technique, vol. 49, no. 5, pp. 1006–1009, May 2001.
5. W. Fu and E. L. Tan, Stability and dispersion analysis for higher order 3-D ADI-FDTD method, IEEE Transaction on Antennas Propagation, vol. 53, pp. 3691–3696, Nov. 2005.
6. A. P. Zhao, Analysis of the numerical dispersion of the 2-D alternating-direction implicit FDTD method, IEEE Transaction on Microwave Theory and Technique, vol. 50, pp. 1156–1164, Apr. 2002.
7. G. Sun and C. W. Trueman, Analysis and numerical experiments on the numerical dispersion of two-dimensional ADI-FDTD, IEEE Antennas and Wireless Propagation Letter, vol. 2, pp. 78–81, 2003