ISAR IMAGE FORMATION USING A NEW QUADRATIC INTERPOLATION FUNCTION

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ABSTRACT

Many synthetic aperture radar (SAR) image formation algorithms require the computation of multidimensional Fourier transform of irregularly sampled data samples. In this study a recently developed (2nd degree) quadratic interpolation algorithm is applied to ISAR image formation algorithm and compared its accuracy and complexity to nearest neighbor, linear and cubic interpolation algorithms.

I. INTRODUCTION

In this study Dodgson quadratic interpolation filter is applied to ISAR image formation. Nonetheless, the concepts we present here can be extended to work under various SAR imaging modalities. Especially for spotlight SAR, all image processing steps are the same.

In ISAR systems the radar data is taken from different angles and frequencies. According to the projection-slice theorem, Fourier space can be filled by taking FFT of this measured data. After this step data is collected on a polar grid in the Fourier domain. Digital polar format ISAR processors typically consist of three stages [1],

i) Polar to rectangular interpolation
ii) Inverse FFT
iii) Amplitude correction

Many different interpolation kernels have been proposed and analyzed. In most cases the interpolation kernel is a major factor in the quality of the final image. Typically 1D interpolation along range followed by 1D azimuth interpolation. The speed of processing is also frequently an issue, therefore, trade-off between time and quality needs to be studied.

Piecewise local polynomials are used extensively for reconstruction in image interpolation applications because they are simple, quick to evaluate and easy to implement. Examples are nearest neighbor, linear and Keys cubic interpolation. The first two are very simple to evaluate but give poor image quality. Third one takes longer but results in better quality image.

Detailed investigation of degree 0,1 and 3 piecewise local polynomials has been carried out, however, the degree 2 (quadratic) polynomials have been largely disregarded because of the phase distortion effect. In this study 2nd degree (quadratic) Dodgson interpolation function is applied and compared to the other interpolation kernels[4].

II. ISAR IMAGE MODEL

$\sigma(x,y)$ is complex reflectivity function of the scene being imaged. x is azimuth or along-track coordinate and y is slant-range coordinate. Radar uses linear FM chirp waveform. BxB is an area in the (*x,y*) plane covered by the radar footprint. Returned radar echos need to be demodulated and sampled. ISAR data s approximates the Fourier transform of σ ,

$$s(\theta_k, r_l) = C \int_B \int_B \sigma(x,y) \exp\{i2\pi(x \cdot r_l \sin\theta_k + y \cdot r_l \cos\theta_k)\} dxdy \qquad (1)$$

data sampled on a polar grid at locations given by:

$$\theta_k = \theta_0 + k\Delta\theta = \theta_0 + k \cdot PRF \qquad k = 1,\ldots\ldots,K$$
$$r_l = r_0 + k\Delta r = \frac{2f_0}{c} + l \cdot \kappa_r \cdot \frac{c}{2SW\gamma} \qquad l = 1,\ldots\ldots,L \qquad (2)$$

Where,
θ_0 : initial radar look angle (degrees)
f_0 : chirp starting frequency (Hz)
c : propagation constant (m/s)
SW: range swath width (m)
γ : chirp rate (Hz/s)
κ_r : range over sampling factor
PRF: pulse repetition frequency (Hz)

Each pulses containing L samples, total K pulses are collected. In equation (1) C, a complex-valued term which includes quadratic phase terms, will be ignored in this study. Sample rate is chosen slightly higher than Nyguist to avoid aliasing effects[3]. The finite radar beam width implies that the absorbed reflectivity function $\sigma(x,y)$ in equation (1) goes to zero outside of the beam footprint BxB. Consequently, the deramped phase history data $s(\theta_k, r_l)$ is band limited and conventional interpolation techniques can be used to resample s onto a rectangular grid. Let $s(\varepsilon_{k'}, n_{l'})$ denote the resampled version of $s(\theta_k, r_l)$. ISAR steps are given below.

ISAR Image Formation Steps:

Step 1. Resample onto rectangular grid

Resampling procedure defined from KxL polar grid onto a $K' \times L'$ rectangular grid. This step is necessary because IFFT is defined in rectangular grid.

Step 2. Zero Pad and Inverse FFT

FFT is used to invert equation (1) and reconstruct a discretized version of $\sigma(x,y)$. It is common to zero pad the interpolated phase history data by embedding it into a larger array size MxN (Where $M > K', N > L'$). Zero padding the phase history data yields an upsampled image. One typical upsampling factor is $M = 3K'/2$, $N = 3L'/2$ which gives a sampling rate of 1,5 pixels per resolution cell.

Step 3. Amplitude Correction and Image Trim

Amplitude correction is typically applied after the inverse FFT to compensate for non-uniform response in the passband of the interpolation filter h in the image domain. After amplitude correction the image is trimmed to remove image pixels at the edge of the scene that fall within transition band of the interpolator. A total of $M' \times N'$ pixels are extracted.

III. CONVENTIONAL INTERPOLATION ALGORITHMS

Interpolation is lowpass filtering operation. An ideal reconstruction filter has unity gain in the passband and zero gain in the stopband in order to transmit and suppers the signal's spectrum in these respective frequency ranges.

The most simple interpolation algorithm is *nearest neighbor*. Interpolation kernel is given in equation (3).

$$h(x) = \begin{cases} 1 & |x| < 0.5 \\ 1/2 & |x| = 0.5 \\ 0 & otherwise \end{cases} \quad (3)$$

In image resampling this algorithm yields very fast operation but it produces an image with blocky artifacts. Figure 1 shows spectral characteristic of Nearest Neighbor interpolator. Side lobs lets high frequency components to leak into the image and low pass filter is not approximated in a close manner with this method.

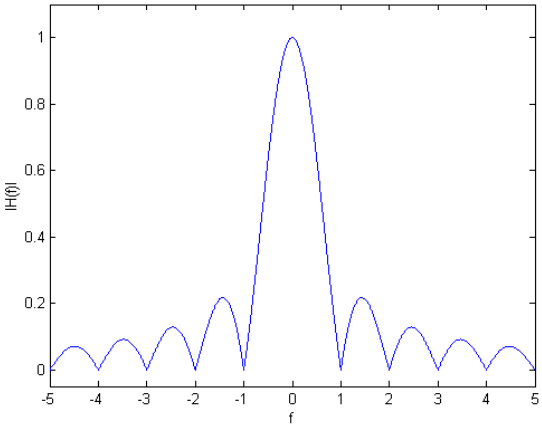

Figure 1 Spectral characteristic of nearest neighbor interpolator

More complicated example of interpolation function is *linear interpolation* kernel is given by,

$$h(x) = \begin{cases} 1 - |x| & 0 \le |x| < 1 \\ 0 & 1 < |x| \end{cases} \quad (4)$$

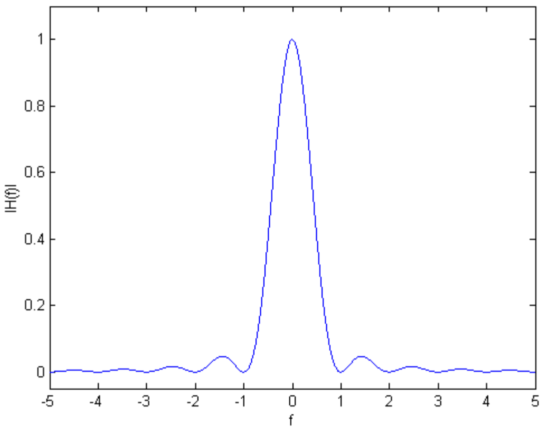

Figure 2 Spectral characteristic of linear interpolator

Linear interpolation gives better results then nearest neighbor for an extra computation cost. Unfortunately it also tends to blur the final image, because its Fourier spectrum is not close enough to ideal low pass filter. Figure 2 shows spectral characteristic of Linear interpolator.

Temporarily skipping over quadratics, (second degree) one of best *cubic interpolation* kernel Keys function is given by,

$$h(x)=\begin{cases}(a+2)|x|^3-(a+3)|x|^2+1 & 0\le |x|<1\\ a|x|^3-5a|x|^2+8a|x|-4a & 1\le |x|<2\\ 0 & \textit{otherwise}\end{cases} \quad (5)$$

Keys cubic interpolation function (a=-0.5) produces good image but computational cost is more than nearest neighbor linear and Dodgson.

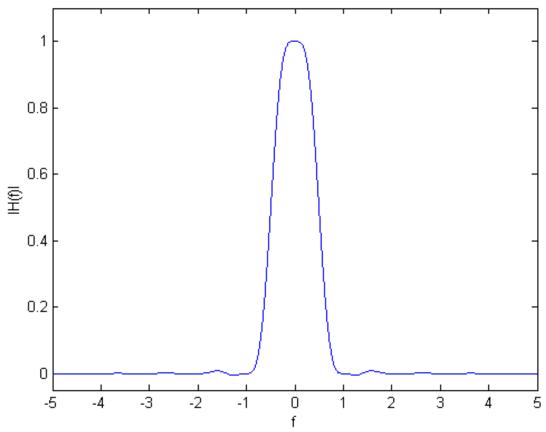

Figure 3 Spectral characteristic of Keys interpolation function

Figure 3 shows spectral characteristic of Keys interpolation function. If the frequency spectrums are analyzed, it can be easily seen that Keys cubic interpolation kernel is approximates better than Nearest Neighbor and Linear interpolation kernel.

IV. QUADRATIC INTERPOLATION

Quadratics have been largely disregarded in image resampling. Two distinct reasons for this are given in the literature.

First their filters are space-variant with phase distortion [2]. Schafer and Rabiner show that any quadratic will produce phase distortions if each quadratic piece starts and ends at the sample points[6]. Making an alternative assumption that each quadratic starts and ends halfway between sample points than quadratics with linear phase can be derived that are suitable for interpolation. All interpolation functions considered, as they are space invariant.

The second reason is more subjective, which states that using three points for interpolation would result in two points on one side of the interpolated point and only one point on the other side [5]. Linear interpolation uses one point on each side, and cubic interpolation uses two points on each side. In case of quadratic three *nearest* sample points are used for interpolation. In the same way that linear interpolation is calculated from two, and cubic from the four nearest. Samples can be chosen from either side without considering their position.

Quadratic, Dodgson interpolation kernel is given by,

$$h(x)=\begin{cases}-2|x|^2+1 & |x|\le 1/2\\ |x|^2-(5/2)|x|+3/2 & 1/2<|x|\le 3/2\\ 0 & \textit{otherwise}\end{cases} \quad (6)$$

Figure 4 shows spectral characteristic of Dodgson interpolator. In this interpolation method approximation to the ideal low-pass filter is successfully done without phase distortion. High frequency components are less than Linear and Nearest Neighbor algorithms, and unit gain in the pass band is more stable.

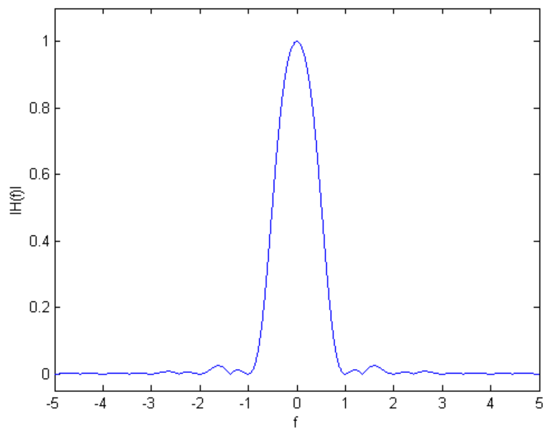

Figure 4 Spectral characteristic of Dodgson interpolator

V. APPLICATION RESULTS

Data set that is used in this study, is supplied from Electroscience Laboratory at Ohio-State University. Back scattered electromagnetic field from 6 perfect conductor targets measured for different looking angles and frequencies. Frequency varies between 2-18 GHz with 50 MHz steps, and looking angle is ±10° for 0.25° steps.

The error is defined as the order of approximation of targets' places,

$$E=\sum_{i=1}^{N}\sum_{j=1}^{M}\left[\frac{f(x_i,y_j)-g(x_i,y_i)}{g(x_i,y_i)}\right]^2/(N\times M) \quad (7)$$

In this study looking angle is taken ±2° and frequency 10.25-11.75 GHz as a part of the data set is used to generate ISAR images.

If the most simple interpolation method, nearest neighbor, is used to generate ISAR image as a result error rate is 1.231 %. Linear interpolation is 1st degree interpolation method and requires more processing time than nearest neighbor, but produces more accurate image. ISAR image reconstruction using linear interpolation brings an error rate of 1.224 %. Applying Keys cubic interpolation to ISAR data, resulted image has an error rate of 1.206 %.

Applying quadratic interpolation to ISAR data, resulted image has an error rate of 1.209 %. Figure 5 shows ISAR image which is formatted by using Dodgson quadratic interpolation algorithm.

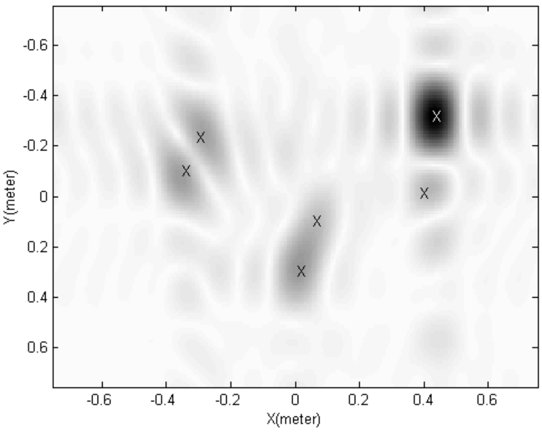

Figure 5: ISAR image using Dodgson Interpolation

Execution times and errors for different interpolation algorithms applied to ISAR data given in Table 1. Keys function brings best accuracy, but takes longer than the other methods. In case of large volume of data computational cost is more important. 2nd degree interpolation function can be used with only slight accuracy sacrifice.

Method	Execution time (s)	Error (%)
Nearest Neighbor	0.20	1.231
Linear	0.36	1.224
Dodgson (degree 2)	0.72	1.209
Keys (degree 3)	1.20	1.206

Table 1: Execution times and Errors for different interpolation algorithms

The best passband and stopband performance of the four functions is shown by Keys, followed by Dodgson's quadratic. According to the application results for two dimensional reconstruction quadratic interpolation took 60 % of the time of a cubic for slightly reduced quality, while a linear interpolation took 30 % of the time of a cubic for significantly degraded quality.

VI. CONCLUSION

In this study Dodgson's quadratic interpolation function applied to ISAR image formation and compared to conventional algorithms. In comparison its been found that 2nd degree interpolation filter can be used for ISAR image formation. Especially if large volume of data needs to be processed for reconstruction it takes long time with cubic interpolation. Thus 2nd degree interpolation algorithm can be preferred for ISAR image formation, in expense of small accuracy change.

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