Solution to Security Constrained Pumped-Storage Hydraulic Unit Scheduling Problem by Using Modified Subgradient Algorithm Based on Feasible Values

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Abstract

A *lossy* electric power system that contains thermal units and a pumped-storage (p-s) hydraulic unit is considered in this paper. The total fuel cost of the thermal units in an operation cycle is minimized under some possible electric and hydraulic constraints by means of a power dispatch method proposed by us and based on modified subgradient method operating on feasible values.

The proposed dispatch technique considers minimum and maximum reservoir storage limits of the p-s unit, upper and lower generation limits of the thermal units, upper and lower pumping/generation power limits of the p-s unit, maximum transmission capacities of the transmission lines, upper and lower limits of the bus voltage magnitudes and off-nominal tap ratios in a considered power system.

A nonlinear programming model is set up for the problem solution. Power system transmission loss is inserted into this model as equality constraints via the load flow equations. Since all constraints in the nonlinear programming model are functions of complex bus voltages and off-nominal tap ratios (once there are off-nominal tap changing transformers in the power system), they are taken as independent variables.

The proposed dispatch technique was tested on an example power system that has 12 buses with five thermal units and a p-s hydraulic unit. Optimal total cost value for the power system without any p-s unit is calculated first. Later on, the same optimal total cost value for the power system with a p-s unit is recalculated and the obtained saving in the optimal total cost value, due to the employment of the p-s unit, is presented. The numeric example, which is considered in this paper, was also solved by means of other dispatch technique that uses pseudo spot price electricity algorithm. Results obtained from the proposed method and from the other method are compared.

1. Introduction

The main function of p-s hydraulic units in electric power systems is to store inexpensive surplus electric energy that is available during off-peak load levels as hydraulic potential energy. This is done by pumping water from the lower reservoir of a p-s unit into its upper reservoir. The stored hydraulic potential energy is then used to generate electric energy during peak load levels (peak shaving hydraulic units). P-s units are generally operated over daily or weekly periods. Operation of a p-s unit over a period can reduce the total fuel cost in a power system.

Lee and Chen [1] solved a short term hydrothermal generation coordination problem including p-s and battery storage energy systems by using multi-pass dynamic programming. In this paper they did not consider transmission losses. Some previous papers that do not consider transmission losses can also be found in [2]. In our previous work, p-s hydraulic unit scheduling problem in a *lossy* electric power system is solved by using the pseudo spot price of electricity algorithm (PSPA) in [3]. In this paper some security constraints, such as bus voltage magnitude and transmission line maximum transmission capability constraints are not considered. In reference [4], combined optimization of wind farm and p-s unit in a market environment is investigated. The problem is modeled as two-stage stochastic programming problem that considers two random parameters: market price and wind generation. The researchers modeled two different joint operations of the units and compare it with the uncoordinated operation of them.

In the F-MSG algorithm [5], the upper bound for the cost function value is specified in advance and the algorithm tries to find a solution where the cost function is *less than or equal to* the upper bound and all constraint are satisfied. If it finds it (*feasible total cost*), the upper bound is *decreased* a certain amount, otherwise (*infeasible total cost*) the upper bound is *increased* a certain amount. The amount of decrease or increase on the upper bound for the next iteration depends on if any feasible or infeasible total cost value was obtained in the previous iterations. This process continues until absolute value of the change in the upper bound is less than a predefined tolerance value.

2. Statement of the problem

A nonlinear programming model for the economic power dispatch problem considered in this paper is given in the following:

$$Minimize\ F_T = \sum_{j=1}^{j_{max}} \sum_{i \in \boldsymbol{N}_s} F_i(P_{Gi,j})\, t_j, \quad (R) \qquad (1)$$

$$\begin{aligned} &P_{Gi,j} - P_{Load\ i,j} - \sum_{k \in \boldsymbol{N}_{Bi}} p_{ik,j} = 0 \\ &Q_{Gi,j} - Q_{Load\ i,j} - \sum_{k \in \boldsymbol{N}_{Bi}} q_{ik,j} = 0,\ i=1,2,..,N,\ j=1,2,..,j_{max} \end{aligned} \qquad (2)$$

$$\begin{aligned} &P_{PH,j} = P_{ps,j}, \quad j \in \boldsymbol{J}_{pump} \quad if \quad P_{ps,j} < 0 \\ &P_{GH,j} = P_{ps,j}, \quad j \in \boldsymbol{J}_{gen} \quad if \quad P_{ps,j} > 0, \quad i = ps \end{aligned} \qquad (3)$$

$$\begin{aligned} &P_i^{min} \le P_{Gi,j} \le P_i^{max}, \quad i \in \boldsymbol{N}_s \\ &Q_{Gr}^{min} \le Q_{Gr,j} \le Q_{Gr}^{max}, \quad r \in \boldsymbol{N}_Q, \quad j = 1,\cdots,j_{max} \end{aligned} \qquad (4)$$

$$P_{PH}^{max} \le P_{ps,j} \le P_{GH}^{max}, \quad j \in \left\{\boldsymbol{J}_{pump}, \boldsymbol{J}_{gen}\right\} \qquad (5)$$

$$p_{l,j} \le p_l^{max}, \qquad l \in \boldsymbol{L},\ j = 1,2,\cdots,j_{max} \qquad (6)$$

$$U_i^{min} \le U_{i,j} \le U_i^{max},\ i=1,2,\cdots,N,\ i \ne ref, vc,\ j=1,\cdots,j_{max} \qquad (7)$$

$$a_i^{min} \le a_{i,j} \le a_i^{max}, \; i \in N_{tap}, \;\; j = 1, \cdots, j_{max}. \tag{8}$$

$$V^{min} \le V_j \le V^{max}, \;\; j = 1, \cdots, j_{max} \tag{9}$$

$$\begin{aligned} V_j &= V_{j-1} + q_{PH}(|P_{PH,j}|)t_j \quad if \quad j \in \boldsymbol{J}_{pump} \\ V_j &= V_{j-1} - q_{GH}(P_{GH,j})t_j \quad if \quad j \in \boldsymbol{J}_{gen} \end{aligned} \tag{10}$$

$$V_0 = V_{j_{max}} = V^{start} = V^{end} \tag{11}$$

Since the beginning and final water volume values of the upper reservoir of the p-s unit are taken as the same in the considered problem, the total net water amount used by the p-s unit must be equal to zero.

$$q_{spentTOT} - q_{pumpTOT} = q_{net\ spent} = 0 \tag{12}$$

$$q_{spent\,TOT} = \sum_{j \in \boldsymbol{J}_{gen}} q_{GH}(P_{GH,j})t_j \tag{13}$$

$$q_{pump\,TOT} = \sum_{j \in \boldsymbol{J}_{pump}} q_{PH}(|P_{PH,j}|)t_j \tag{14}$$

2.1. Determination of Line Flows and Power Generations

To express the total system fuel cost function in terms of independent variables of our optimization model, line flows should be written in terms of complex bus voltages and off-nominal tap ratios (see equations (2)).

$$p_{ik,j} = U_{i,j}^2\left(\frac{g_{ik}}{a_{i,j}^2} + g_{sh\,i}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ik}\cos(\delta_{i,j}-\delta_{k,j}) + b_{ik}\sin(\delta_{i,j}-\delta_{k,j})\right] \tag{15}$$

$$p_{ki,j} = U_{k,j}^2\left(g_{ik} + g_{sh\,k}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ik}\cos(\delta_{k,j}-\delta_{i,j}) + b_{ik}\sin(\delta_{k,j}-\delta_{i,j})\right] \tag{16}$$

$$q_{ik,j} = -U_{i,j}^2\left(\frac{b_{ik}}{a_{i,j}^2} + b_{sh\,i}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ik}\sin(\delta_{i,j}-\delta_{k,j}) - b_{ik}\cos(\delta_{i,j}-\delta_{k,j})\right] \tag{17}$$

$$q_{ki,j} = -U_{k,j}^2\left(b_{ik} + b_{sh\,i}\right) - \frac{U_{i,j}U_{k,j}}{a_{i,j}} \times \left[g_{ij}\sin(\delta_{k,j}-\delta_{i,j}) - b_{ij}\sin(\delta_{k,j}-\delta_{i,j})\right] \tag{18}$$

In the above equations, $U_{i,j}$ and $\delta_{i,j}$ are voltage magnitude and phase angle of bus i in the j^{th} subinterval, respectively, $r_{ik} + jx_{ik}$ is the series impedance of the line between buses i and k, $g_{ik} + jb_{ik}$ is the *series* admittance of the line between buses i and k where $g_{ik} + jb_{ik} = 1/(r_{ik} + jx_{ik})$, $g_{shi} + jb_{shi}$ is the sum of the half line charging admittance and external shunt admittance if any at bus i, and $a_{i,j}$ is the off-nominal tap setting in the j^{th} subinterval with tap setting facility at bus i, $p_{ik,j}$ and $q_{ik,j}$ are the active and reactive power flows going from bus i to bus k at bus i border in the j^{th} subinterval, respectively. $-p_{ki,j}$ and $-q_{ki,j}$ are the active and reactive power flows going from bus i to bus k at bus k border in the j^{th} subinterval, respectively.

The total loss of the network in the j^{th} subinterval can be calculated by the following equations:

$$p_{loss\ ik,j} = p_{ik,j} + p_{ki,j} \tag{19}$$

$$P_{LOSS,j} = \sum_{i \in N}\sum_{k \in N, k \ne i} p_{ik,j}, \quad j = 1, 2, \cdots, j_{max}. \tag{20}$$

2.2. Converting Inequality Constraints into Equality Constraints

Since the F-MSG algorithm requires that all constraints need to be expressed in equality constraint form, the inequality constraints in the optimization model should be converted into the corresponding equality constraints. The following method is used for this purpose, since it does not add any extra independent variable into the optimization model in the conversion process [5]. The double sided inequality $x_i^- \le x_{i,j} \le x_i^+$ in the j^{th} subinterval can be written as the following two inequalities:

$$\begin{aligned} h_{i,j}^+(x_{i,j}) &= (x_{i,j} - x_i^+) \le 0 \\ h_{i,j}^-(x_{i,j}) &= (x_i^- - x_{i,j}) \le 0, \quad j = 1, 2, \cdots j_{max} \end{aligned} \tag{21}$$

Then, we can rewrite the above inequalities as continuous equality forms by the following:

$$\begin{aligned} h_{i,j}^{eq+}(x_{i,j}) &= \max\left\{0,\ (x_{i,j} - x_i^+)\right\} \\ h_{i,j}^{eq-}(x_{i,j}) &= \max\left\{0,\ (x_i^- - x_{i,j})\right\}, \quad j = 1, 2, \cdots j_{max} \end{aligned}. \tag{22}$$

If $x_i^- \le x_{i,j} \le x_i^+$, it is obvious that $(x_{i,j} - x_i^+) \le 0$, $(x_i^- - x_{i,j}) \le 0$ and, $\max\left\{0,\ (x_{i,j} - x_i^+)\right\} = 0$, $\max\left\{0,\ (x_i^- - x_{i,j})\right\} = 0$. So the inequality constraints in (21) can be represented by the corresponding equality constraints in (22). In this paper the inequality constraints, given in equations (4)-(9), are converted into the corresponding equality constraints in this way.

3. The Modified Subgradient Algorithm Based on Feasible Values

The nonlinear optimization problem for subinterval j can be represented in the standard form given below:

$$\text{Min } F_{T,j}(\boldsymbol{x}), \quad \textit{Subject to} \begin{cases} \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0} \\ \boldsymbol{x} \in K \end{cases} \tag{23}$$

where

$\boldsymbol{x} = [U_{1,j}, U_{2,j}, \cdots\cdots, U_{N,j}, \delta_{1,j}, \delta_{2,j}, \cdots\cdots \delta_{N,j}, a_{1,j}, a_{2,j}, \cdots, a_{N_{tap},j}]$ is the independent variable vector in subinterval *j*. $F_{T,j}(\boldsymbol{x})$ is the objective function which is given as

$$F_{T,j} = \sum_{i \in \{\boldsymbol{N}_s\}} F_i(P_{Gi,j})t_j + \psi \sum_{j \in \boldsymbol{J}_{gen}} q_{GH}(P_{GH,j})t_j - \psi \sum_{j \in \boldsymbol{J}_{pump}} q_{PH}(|P_{PH,j}|)t_j \tag{24}$$

In order to be able to apply the F-MSG algorithm to the problem described in section 2, the objective function in subinterval *j* is transformed into the form shown in equation (24). Necessary explanation about it is given in section 3.2. $\boldsymbol{h}(\boldsymbol{x}) = \left[h_1(\boldsymbol{x}), h_2(\boldsymbol{x}), \cdots\cdots, h_{N_{EQ}}(\boldsymbol{x})\right]$ in (23) is the equality constraint vector in subinterval *j*. It includes all the original equality constraints in subinterval *j*, which are shown in (2)-(3), and the equality constraints in subinterval *j*, which are obtained from converting all the inequality constraints shown in equations (4)-(9), into the corresponding equality constraints via the method given in Section 2.2. K is a sufficiently large compact set containing the potential values of $\boldsymbol{x}$. Region K is bounded by the upper and the lower limits of the voltage magnitudes of the buses in subinterval j and the upper and the lower limits of the tap settings of the transformers in subinterval j. Note that the voltage magnitude and phase angle of the reference bus, *are not* included into $\boldsymbol{x}$ since they are not independent variables and remain constant during the solution process. In solving the constrained optimization problem given by equation (23), the first step is to convert it into unconstrained one by constructing the dual problem. This can be done by using various LaGrange functions [6]. LaGrange function must guarantee that the optimal solution of the dual problem be equal to that of the primal constrained problem. Otherwise, there will be a difference between the optimal values of these problems; in other words, a duality gap will occur. The classical LaGrange function guarantees the zero duality gaps for the convex problems. However, if the objective function or some of the constraints are not convex, then the classical LaGrange function cannot guarantee this. Therefore, for the non-convex problems, suitably selected augmented LaGrange functions should be used. Considering the non-convex nature of our problem, we form the dual problem by using the following sharp augmented LaGrange function:

$$L(\boldsymbol{x},\boldsymbol{u},c) = F(\boldsymbol{x}) + c\|\boldsymbol{h}(\boldsymbol{x})\| - \langle \boldsymbol{u},\boldsymbol{h}(\boldsymbol{x})\rangle$$

$$= F(\boldsymbol{x}) + c\left(\left[h_1(\boldsymbol{x})\right]^2 + \left[h_2(\boldsymbol{x})\right]^2 + .. + \left[h_{N_{EQ}}(\boldsymbol{x})\right]^2\right)^{1/2} \quad (25)$$

$$-\left(u_1 h_1(\boldsymbol{x}) + u_2 h_2(\boldsymbol{x}) + \cdots + u_{N_{EQ}} h_{N_{EQ}}(\boldsymbol{x})\right)$$

where $u_1, u_2, \cdots, u_{N_{EQ}} \in \Re$ ($\Re$ stands for the set of real numbers) and $c \geq 0$ are LaGrange multipliers (dual variables) in subinterval j. The dual function associated with the constrained problem is defined as

$$H(\boldsymbol{u},c) = \underset{\boldsymbol{x} \in K}{\text{Min}} \; L(\boldsymbol{x},\boldsymbol{u},c). \quad (26)$$

Then, the dual problem is given by

$$\underset{(\boldsymbol{u},c) \in \Re^{N_{EQ}} \times \Re_+}{\text{Max}} \; H(\boldsymbol{u},c) \quad (27)$$

For the given dual problem, the conditions of guaranteeing zero duality gaps are proven in [7].

3.1. The F-MSG Algorithm Applied into the j^{th} Subinterval of the Dispatch Problem

Application of the F-MSG algorithm into the j^{th} subinterval of the dispatch problem is explained in the following.

Initialization Step: Select arbitrary initial active and reactive power generations for subinterval j. Then, perform an AC power flow calculation with selected active and reactive power generations to obtain the initial values for the voltage magnitudes and phase angles of the buses in subinterval j. Calculate the initial total cost $F_{T,j}$ for subinterval j by using the selected (or calculated) ψ (pseudo water price) value.

Step 1) Choose positive numbers ε_1, ε_2, Δ_1 and M (upper bound for m). Set $n=1$, $w=0$, $z=0$, and $H_n = F_{T,j}$.

Step 2) Choose $(\boldsymbol{u}_l^n, c_1^n) \in \Re^{N_{EQ}} \times \Re_+$ and $\ell(1) > 0$ and set $m=1$, $\boldsymbol{u}_m = \boldsymbol{u}_l^n$, $c_m = c_1^n$,

Step 3) Given $(\boldsymbol{u}_m, c_m)$, solve the following constraint satisfaction problem (CSP)

$$\begin{aligned}&\text{Find a solution } \boldsymbol{x}_m \in K \text{ such that}\\ &F_{T,j}(\boldsymbol{x}_m) + c_m\|\boldsymbol{h}(\boldsymbol{x}_m)\| - \langle \boldsymbol{u}_m, \boldsymbol{h}(\boldsymbol{x}_m)\rangle \leq H_n\end{aligned} \quad (28)$$

If a solution to (28) does not exist or $\ell(m) > M$, then go to Step 6; otherwise, if a solution $\boldsymbol{x}_m$ exists then check whether $\boldsymbol{h}(\boldsymbol{x}_m) = \boldsymbol{0}$. If $\boldsymbol{h}(\boldsymbol{x}_m) = \boldsymbol{0}$ (or if $\|\boldsymbol{h}(\boldsymbol{x}_m)\| \leq \varepsilon_1$) then go to step5*,* otherwise go to step 4.

Step 4) Update dual variables as

$$\boldsymbol{u}_{m+1} = \boldsymbol{u}_m - \alpha\, s_m \boldsymbol{h}(\boldsymbol{x}_m) \quad (29)$$

$$c_{m+1} = c_m + (1+\alpha)\, s_m \|\boldsymbol{h}(\boldsymbol{x}_m)\| \quad (30)$$

where s_m is a positive step size parameter defined as

$$0 < s_m = \frac{\lambda\,\alpha\left(H_n - L(\boldsymbol{x}_m, \boldsymbol{u}_m, c_m)\right)}{\left[\alpha^2 + (1+\alpha)^2\right]\|\boldsymbol{h}(\boldsymbol{x}_m)\|^2} \quad (31)$$

where α and λ are constant parameters with $\alpha > 0$ and $0 < \lambda < 2$. Step size s_m corresponding to the dual variables $(\boldsymbol{u}_m, c_m)$ should also satisfy the following property:

$$\left(s_m \|\boldsymbol{h}(\boldsymbol{x}_m)\| + c_m - \|\boldsymbol{u}_m\|\right) > \ell(m). \quad (32)$$

Set $m = m+1$, update $\ell(m)$ in such a way that $\ell(m) \to +\infty$ as $m \to +\infty$, and go to step 3.

Step 5) If $w = 0$, it means that any infeasible total cost rate value has not been chosen yet, then set $\Delta_{n+1} = \Delta_n$, otherwise set $\Delta_{n+1} = (1/2)\Delta_n$. If $\Delta_{n+1} < \varepsilon_2$, then stop, $\boldsymbol{x}_m$ is an approximate optimal primal solution, and $(\boldsymbol{u}_m, c_m)$ is an approximate dual solution; otherwise set $H_{n+1} = \min\left\{F_{T,j}(\boldsymbol{x}_m),\ H_n - \Delta_{n+1}\right\}$, $z = z+1$, $n = n+1$, and go to step 2.

Step 6) If $z = 0$, it means that any feasible cost rate value has not been chosen yet, then set $\Delta_{n+1} = \Delta_n$; otherwise, set $\Delta_{n+1} = (1/2)\Delta_n$. If $\Delta_{n+1} < \varepsilon_2$ then stop, and in this case, the last calculated feasible $\boldsymbol{x}_m$ is an approximate optimal primal solution, and $(\boldsymbol{u}_m, c_m)$ is an approximate dual solution; otherwise, set $H_{n+1} = H_n + \Delta_{n+1}$, $w = w+1$, $n = n+1$ and go to step-2.

The following problem is solved by using GAMS® solver:

$$\begin{aligned}&\text{Minimize} \quad f = 0\\ &\text{Subject to} \begin{cases} L(\boldsymbol{x},\boldsymbol{u},c) - H_n \leq 0 \\ \boldsymbol{x} \in K \end{cases}\end{aligned} \quad (33)$$

where f is a 'fictitious' objective function which is identically zero, or can be taken as any constant value [5]. The way of updating the dual variables $(\boldsymbol{u}_m, c_m)$ in step 4 will force the solution in Step 3 to converge to the feasible solution (see Theorems in [8]).

Note that an AC load flow calculation is carried out only in the initialization step of the algorithm just to obtain the selected initial complex bus voltages. No more load flow calculation is carried out in the subsequent steps of the algorithm.

3.2 The Proposed Solution Technique for the Dispatch Problem

When the F-MSG algorithm is applied to the dispatch problems with specific ψ value for all subintervals of the considered problem, if the water constraint of the p-s unit is satisfied (see equation (11) or (12)), it means that the optimal solution is found. But if the water constraint is *not* satisfied, the following solution technique, where the F-MSG algorithm and common pseudo water price (ψ *(R/acre-ft)*) for the discharge and pumping rate functions of the p-s unit are used, is proposed to solve the dispatch problem described in the above. Two fictitious terms are added into the original fuel cost; water discharge cost (positive one) and water pumping cost (negative one) in equation (24). When ψ gets higher and higher values, the p-s unit starts to pump more and more water amounts in subintervals with low load values and discharge less and less water amounts in subintervals with high load values. Therefore, $q_{net\ spent}$ given in (12) gets more and more negative values. When ψ gets smaller and smaller values, $q_{net\ spent}$ takes more and more positive values. The matter in the following algorithm is to find a critical pseudo water price value so that the p-s unit water constraint given in (12) is satisfied within a selected tolerance value. From the optimal solution of the system without p-s unit, an average value of the cost rate for the thermal units for the operation period can be calculated. With the help of this average thermal cost rate, an average value of pseudo water price (ψ^{avg}) can be calculated by assuming that the p-s unit is operating in generation mode in subintervals with high load levels and pumping mode in subintervals with low load levels. In this calculation, the generation and pumping powers of the p-s unit can be taken as equal to midpoint of the generation and pumping power ranges, respectively. The proposed solution algorithm is given in the following.

Step-1) Take $\psi = \psi^{avg}$

Step-2) Set $test\_up = 0,\quad test\_down = 0,\quad j = 1$

Step-3) Get the initial values of the independent variables for the current subinterval and solve the dispatch problem by using the F-MSG algorithm.

Step-4) Set $j = j + 1$. If $j > j_{max}$ then go to step-5; otherwise go to step-3.

Step-5) Calculate $q_{net\ spent} = q_{spentTOT} - q_{pumpTOT}$. If $\left| q_{net\ spent} \right| \le TOL$ then stop, the solution is obtained; otherwise go to step-6.

Step-6) If $test\_down = 0$ *and* $q_{net\ spent} > 0$ then set, $test\_down = 1$, $\psi^{low} = \psi$ and $\psi^{high} = 1.5 \times \psi^{low}$, $\psi = \psi^{high}$ and $j = 1$, go to step-3: otherwise go to step-7.

Step-7) If $test\_up = 0$ *and* $q_{net\ spent} < 0$ then set $test\_up = 1$, $\psi^{high} = \psi$, $\psi^{low} = 0.5 \times \psi^{high}$, $\psi = \psi^{low}$ and $j = 1$, go to step-3; otherwise go to step-8.

Step-8) If $test\_down = 1$ *and* $test\_up = 0$ *and* $q_{net\ spent} > 0$ then set, $\psi = 1.5 \times \psi^{high}$, $\psi^{high} = \psi$ and $j = 1$, go to step-3; otherwise go to step-9

Step-9) If $test\_up = 1$ *and* $test\_down = 0$ *and* $q_{net\ spent} < 0$ then set, $\psi = 0.5 \times \psi^{low}$, $\psi^{low} = \psi$ and $j = 1$, go to step-3; otherwise go to step-10

Step-10) If $test\_up = 1$ *and* $test\_down = 0$ *and* $q_{net\ spent} > 0$ then set $test\_down = 1$, $\psi^{low} = \psi$, $\psi = 0.5(\psi^{high} + \psi^{low})$ and $j = 1$, go to step-3; otherwise go to step-11

Step-11) If $test\_down = 1$ *and* $test\_up = 0$ *and* $q_{net\ spent} < 0$ then set $test\_up = 1$, $\psi^{high} = \psi$, $\psi = 0.5(\psi^{high} + \psi^{low})$ and $j = 1$, go to step-3; otherwise go to step-12

Step-12) If $test\_up = 1$ *and* $test\_down = 1$ *and* $q_{net\ spent} > 0$ then set, $\psi^{low} = \psi$, $\psi = 0.5(\psi^{high} + \psi^{low})$ and $j = 1$, go to step-3; otherwise go to step-13

Step-13) If $test\_up = 1$ *and* $test\_down = 1$ *and* $q_{net\ spent} < 0$ then set $\psi^{high} = \psi$, $\psi = 0.5(\psi^{high} + \psi^{low})$ and $j = 1$, go to step-3.

3. Numerical Example

The proposed dispatch technique was tested on an example power system that has 12 buses, five thermal units and a p-s hydraulic unit (connected to bus 6). Please refer to reference [3] for the necessary data related with the test system.

The initial parameters, explained in section 3.1, 3.2 are chosen as $\varepsilon_1 = 5 \times 10^{-5}$, $TOL = 5\ ccf$, $\varepsilon_2 = 0.05$, $\Delta_1 = 50\ R$, $M = 250$, $\boldsymbol{u}_1^1 = [0,0,...0,0]_{(1x52)}$, $c_1^1 = 2500$, $\ell(m) = m$ for all subintervals. The maximum active power transmission capacity limit for all transmission lines is taken as 150 *MW*. The upper and lower limits of the bus voltage magnitudes for all buses (except ref. bus) are taken as 0.9 *pu* and 1.1 *pu*, respectively. Reactive power generation limits for all units are taken as $Q_{Gr}^{max} = 2.0\ pu$, $Q_{Gr}^{min} = -2.0\ pu$.

The thermal units connected to bus 9 and 11 are chosen as inefficient units with respect to the other thermal units. These thermal units' minimum generation limits are assumed to be as zero. If those units' active generations drop below their actual minimum active generation limits during the optimization procedure, they are taken as equal to zero and those units are operated as synchronous compensators. We solved the dispatch problem for two cases where the p-s unit is off-line in case-1 and on-line in case-2.

Case-1) We have applied the F-MSG solution algorithm given in Section 3.1 to the dispatch problems of all six intervals of the test system without the p-s unit by using the initial generations shown in Table 1. Table 2 shows the solution point generations, active loss and fuel cost values in all subintervals. It also contains the total fuel cost for the considered operation period; $F_T = 124987.1462\ R$. The same dispatch problem was also solved by means of the PSPA where the total fuel cost was found to be $F_T = 125268.540\ R$ [3]. *It is seen from the presented figures that the F-MSG algorithm gives a total fuel cost that is 281.4 R lower than the one supplied by the PSPA algorithm.*

Case-2) In this case we have applied the solution algorithm given in Section 3.2 to the dispatch problem of the system with the p-s unit. We have also used the same initial generations, given in Table 1, in this case. Note that the p-s unit is taken as in *idle operation mode* initially in all subintervals (see the column with $i = 6$ in Table 2) With the help of data in Table 2 and in reference [3], the initial pseudo water price is calculated as $\psi^{avg} = 5.0$ *R/acre-ft*. The solution point data in this case is presented in Table 3. Negative and positive active powers in the column with $i = 6$ in Table 3 represent pumping (load) and generation powers of the p-s unit, respectively. The total fuel cost in this case is found to be $F_T = 124305.989\ R$. From solution via the PSPA the total fuel cost value was found to be $F_T = 124604.982\ R$ [3]. *From the given figures, it is seen that the proposed method gives a total fuel cost that is 300 R lower than the one supplied by the PSPA algorithm.* The change of $q_{net\ spent}$

Table 1. Selected initial unit generations ($S_{base} = 100\ MVA$) .

j	Gen (pu)	i = 1	4	6	7	9	11
1	P	0.7720	0.45	0.00	0.40	0.20	0.20
	Q	0.0887	0.08	0.00	0.08	0.08	0.08
2	P	2.5810	1.50	0.00	1.40	0.45	0.35
	Q	1.5261	0.70	0.00	0.70	0.70	0.70
3	P	2.9778	1.70	0.00	1.70	0.55	0.45
	Q	1.3911	1.00	0.00	1.00	1.00	1.00
4	P	2.5810	1.50	0.00	1.40	0.45	0.35
	Q	1.5261	0.70	0.00	0.70	0.70	0.70
5	P	1.1043	0.70	0.00	0.60	0.35	0.30
	Q	0.5433	0.20	0.00	0.20	0.20	0.20
6	P	0.7720	0.45	0.00	0.40	0.20	0.20
	Q	0.0887	0.08	0.00	0.08	0.08	0.08

Table 2. Solution point generations of the thermal units, active loss and fuel cost values for case-1.

j	$P_{LOSS,j}$ (pu)	Gen. (pu)	i = 1	4	7	9	11	$F_{T,j}$ (R)
1	0.03498	P	1.120687	0.449999	0.464284	—	—	11933.0349
		Q	0.09647	0.107481	0.208685	0.109423	0.021402	
2	0.34459	P	2.801414	1.793206	1.749968	—	—	26057.4631
		Q	1.244917	0.737129	0.841460	0.770782	0.907631	
3	0.40333	P	3.109558	1.80000	1.749997	0.380683	0.363062	33716.4652
		Q	1.316160	1.030548	1.055186	1.036765	1.026195	
4	0.34443	P	2.799222	1.795185	1.749988	—	—	26056.9539
		Q	1.242847	0.747038	0.836033	0.767213	0.907430	
5	0.07285	P	1.273808	0.768846	1.030184	—	—	15290.0380
		Q	0.383335	0.109179	0.563497	0.249953	0.223700	
6	0.03427	P	1.092322	0.450004	0.491901	—	—	11933.1911
		Q	-0.004501	0.122717	0.232010	0.101068	0.021403	
							F_T (R)	**124987.1462**

Table 3. Solution point *pu* generations of the units, active loss and fuel cost values for case-1.

j	$P_{LOSS,j}$ (pu)	Gen. (pu)	i = 1	4	6	7	9	11	$F_{T,j}$ (R)
1	0.079371	P	1.741613	0.549161	-1.156998	0.945605	—	—	15781.4661
		Q	0.567267	-0.352467	0.055062	0.188376	0.176718	0.015946	
2	0.330094	P	2.524354	1.719509	0.336229	1.750000	—	—	24890.7296
		Q	1.145830	0.708990	0.341458	0.576144	0.769927	0.915621	
3	0.457635	P	2.609415	1.798319	1.300009	1.749816	—	—	25437.7056
		Q	1.010610	0.885057	-0.122814	1.750060	0.951334	1.140551	
4	0.330094	P	2.524354	1.719509	0.336229	1.750000	—	—	24890.7296
		Q	1.145830	0.708990	0.341458	0.576144	0.769927	0.915621	
5	0.125465	P	1.834451	0.792560	-0.644999	1.143451	—	—	17523.8929
		Q	1.069664	0.295330	-1.001902	0659725	0.262799	0.305414	
6	0.079371	P	1.741613	0.549161	-1.156998	0.945605	—	—	15781.4661
		Q	0.567267	-0.352467	0.055062	0.188376	0.176718	0.015946	
								F_T (R)	**124305.989**

versus ψ is given in Table 4. The effect of ψ on the convergence of $q_{net\ spent}$ to 0 *acre-ft* is clearly seen from the table. The change of the stored water amount in the upper reservoir of the p-s unit with respect to its pumping/generation power is given in Table 5. The total active load values in each subinterval are also shown in Table 5.

4. Discussion and Conclusion

In this paper, we propose an economic dispatch method based on F-MSG algorithm for a security constrained p-s unit scheduling problem. The dispatch technique is tested on 12-bus test system with 5 thermal units and a p-s unit. The inclusions of the p-s unit with 0.67 cycle efficiency into the power system decreases the total fuel cost from 124987.1462 R to 124305.989R for the given daily load schedule, saving 681.15 R daily. We are currently performing research on the application of the F-MSG algorithm to security constrained p-s unit scheduling problem with *non-convex total fuel cost curve.* To our knowledge, the proposed solution technique *has not been* applied to the problem considered in this paper.

Table 4. The change of $q_{net\ spent}$ versus pseudo water price ψ

$q_{net\ spent}$ (*acre-ft*)	253.8569	-220.4578	98.7536	-45.2369	-12.2563	7.8547	**-0.1559**
ψ (*R/acre-ft*)	5	7.5	6.25	6.875	6.5625	6.40625	**6.484375**

Table 5. The change of the stored water amount versus pumping/generation power of the p-s unit.

j	0	1	2	3	4	5	6
V_j (*acre-ft*)	**10000**	11417.0658	10348.0819	8508.0745	7439.0906	8583.0901	**10000.1559**
$P_{s,j}$ (*pu*)	—	-1.156998	0.336229	1.300009	0.336229	-0.644999	-1.156998
$P_{LOAD,j}$ (*pu*)	—	2.0	6.0	7.0	6.0	3.0	2.0

6. List of Symbols

R : a fictitious monetary unit.

N : number of buses in the network.

$\mathbf{N}_S$: set that contains all thermal units in the network.

$\mathbf{N}_{Bi}$: set that contains all buses *directly* connected to bus *i*.

$\mathbf{N}_{tap}$, $\mathbf{L}$: sets that contains all tap changing transformers and lines in the network, respectively.

$\mathbf{N}_Q$: set that contains all buses where a reactive power generator is connected.

t_j : length of time interval *j*.

$p_{l,j}$: active power flow on line *l* in the j^{th} subinterval.

$P_{Gi,j}$, $Q_{Gi,j}$: active and reactive power generations of the i^{th} unit in the j^{th} subinterval, respectively.

$P_{Load\ i,j}$, $Q_{Load\ i,j}$: active/reactive loads of the i^{th} bus in the j^{th} subinterval, respectively.

$P_{LOSS,j}$: total active loss in the j^{th} subinterval.

p_l^{max} : maximum active transmission capacity of line l .

N_{EQ} , N_{VAR} : number of equality constraints and independent variables, respectively.

Please refer to reference [3] for the meaning of the other symbols that are used in this paper.

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