# ECONOMIC AND MINIMUM EMISSION DISPATCH

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## ABSTRACT

**In this paper, Hopfield Neural Network (HNN) and Lagrange Multiplier (LM) solutions to economic dispatch (ED), $NO_x$ emission dispatch (EmD), and economic-emission dispatch (EED) of a sample system consisting of six thermal generators are presented. Transmission losses are included. The results of HNN are compared with the results of LM.**

**Keywords : Economic - Emission Dispatch, Hopfield Neural Network, Lagrange Multiplier, Runge Kutta-4.**

**NOMENCLATURE**

| | |
|---|---|
| $F_T$ | :Total production cost |
| $F_i$ | :Production cost of the *i*th plant |
| $E_T$ | :Total $NO_x$ emission |
| $E_i$ | :$NO_x$ emission of the *i*th plant |
| $P_i$ | :Real power output of the *i*th generator |
| N | :Total number of units on system |
| $a_i, b_i, c_i$ | :Cost coefficients of the *i*th generator |
| $d_i, e_i, f_i$ | :$NO_x$ emission coefficients of the *i*th generator |
| $\Phi_T$ | :Total objective function |
| $w_1, w_2$ | :Weight factors |
| h | :Rate coefficient |
| $P_{i,min}$ | :Minimum generation limit of the *i*th generator |
| $P_{i,max}$ | :Maximum generation limit of the *i*th generator |
| $P_D$ | :Total demand |
| $P_L$ | :Total losses |
| $B_{ij}$ | :Transmission loss coefficients |
| $L(p_i, \lambda)$ | :Lagrange function |
| $\lambda$ | :Lagrange multiplier |
| $g(p_i)$ | :Equality constraint |
| E | :Hopfield network's energy function |
| $E^{obj}$ | :Optimization objective function |
| x | :*n* dimension variable vector of objective function |
| $T^{obj}$ | :$n \times n$ symmetrical matrix of objective function coefficients |
| $i^{obj}$ | :*n* dimension vector of objective function |
| $A^{eq}$ | :Equality constraint matrix |
| $b^{eq}$ | :$m^{eq}$ dimension equality constraint vector |
| $m^{eq}$ | :Equality constraints |
| $A^{in}$ | :Inequality constraint matrix |
| $b^{in}$ | :$m^{in}$ dimension ınequality constraint vector |
| $m^{in}$ | :Inequality constraints |
| $T^{constr}$ | :$n+m^{in}$ dimension feasible subspace projection matrix |
| s | :$n+m^{in}$ dimension feasible subspace offset vector |
| $A^{new}$ | :Extended constraint matrix |
| $b^{new}$ | : $(m^{eq}+m^{in})$ dimension extended constraint vector |
| I | :Identity matrix |
| $X^{new}$ | :Extended variable vector included slack variables |
| $c_0$ | :Penalty factor |
| $y_p$ | :Slack variable of the *p*th inequality constraint |
| f(x) | :The activation function of the variables |
| $\rho$ | :Momentum term's coefficient |
| $T^{hop}$ | :$n+m^{in}$ dimension matrix of Hopfield differential equation |
| $I^{hop}$ | :$n+m^{in}$ dimension vector of Hopfield differential equation |
| $\eta_1$ | :Coefficient which belongs to variables |
| $\eta_2$ | :Coefficient which belongs to slack variables |
| $P_{net}$ | :Total power except for transmisson loss |

## I. INTRODUCTION

The economic dispatch (ED) problem is to determine the optimal combination of power outputs for all generating units which minimizes the total fuel cost while satisfying load demand and operational constraints. A number of studies have been presented to solve ED problems such as Park et al.[1], and Yalcinoz and Short[2].

Under the strict governmental regulations on environmental protection, the conventional operation at minimum fuel cost can no longer be the only basis for dispatching electric power. The contributions of the electric energy industry to environmental pollution raise questions concerning environmental protection and methods of reducing pollution from power plants either by design or by operational strategies. Especially, emissions contribution of fossil-fired electric power plants which use coal, oil, gas or combinations as the primary energy resource cannot be neglected. These emissions are CO, $CO_2$, $SO_2$, $NO_x$ , particulates, and thermal emission. Emissions may be reduced through these methods : i) switching to fuels with low emission potential, ii) installing post-combustion cleaning system, and iii) dispatching of generation to each generator unit with the objective of minimum emission dispatch [3-4]. Selecting the third method is adequate because it is easy to implement and requires minimal additional costs, so, in this study it is used.

Several researchers have considered emissions either in the objective function or treated emissions as additional constraints. Kulkarni et al. [3], Song et al. [4], Dhillon et al. [5], and King et al. [6] presented EED dispatch.

In this paper, HNN and LM are used to solve economic-emission dispatch problem. As an illustration, only $NO_x$ emission reduction is considered. The equality constraint of power balance and inequality generator capacity constraints are taken into consideration. Also, transmission loss is considered. These methods have been demonstrated through a sample system consisting of six thermal generators. In simulation section, the results of HNN are compared with LM as the classical method.

## II. FORMULATION OF ECONOMIC-EMISSION DISPATCH

In this paper a system consisting of $N$ thermal generating units connected to a transmission network serving a received electrical load $P_D$ [MW] will be studied. The total cost rate of this system is, the sum of the cost rate of the individual units. The fuel cost curve is assumed to be approximated by a quadratic function of $P_i$ [MW] [1-7] :

$$F_T = \sum_{i=1}^{N} (a_i P_i^2 + b_i P_i + c_i) \text{ ..........[Rs/h]} \quad (1)$$

In Eq.(1), $a_i$ [Rs/MW$^2$-h], $b_i$ [Rs/MWh], and $c_i$ [Rs/h] are cost coefficients. For emission dispatch problem, the amount of $NO_x$ emission is expressed as a quadratic function like the cost function [3-6] :

$$E_T = \sum_{i=1}^{N} (d_i P_i^2 + e_i P_i + f_i) \text{ .......[kg/h]} \quad (2)$$

In Eq.(2), $d_i$ [kg/MW$^2$-h], $e_i$ [kg/MWh], and $f_i$ [kg/h] are $NO_x$ emission coefficients. Emission function as an objective is added to Eq.(1) as follows to obtain the objective function of the economic-emission dispatch problem [3-6] :

$$\Phi_T = w_1 \sum_{i=1}^{N} F_i + h.w_2 \sum_{i=1}^{N} E_i \text{ .....[Rs/h]} \quad (3)$$

$$\Phi_T = w_1 \sum_{i=1}^{N} (a_i P_i^2 + b_i P_i + c_i) + h.w_2 \sum_{i=1}^{N} (d_i P_i^2 + e_i P_i + f_i) \quad (4)$$

subject to

$$\text{i) } P_{i,\min} \le P_i \le P_{i,\max} \quad \text{(i=1...N)} \quad (5)$$

$$\text{ii) } \sum_{i=1}^{N} P_i = P_D + P_L \quad (6)$$

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j \quad \text{[MW]} \quad (7)$$

Now, the problem is to find the rate coefficient, "$h$". A practical way of determining $h$ is discussed by Kulkarni et al. [3]. It is necessary to obtain the rate coefficients of each generator at its maximum output:

$$\frac{F_i(P_{i,\max})/P_{i,\max}}{E_i(P_{i,\max})/P_{i,\max}} = h_i \quad \text{(i=1.....N) [Rs/kg]} \quad (8)$$

$h_i$ (i=1,...,N) is then arranged in ascending order; the maximum capacity of each unit, $P_{i,max}$, one at a time, starting from the smallest $h_i$ unit, until $\sum P_i \ge P_D$ .At this stage, $h_i$ associated with the last unit in the process in the rate coefficient $h$ [Rs/kg] for the given load.

## III. LAGRANGE MULTIPLIER

It is well known as the Lagrange function and is shown in Eq.(9) [7].

$$L(p_i, \lambda) = \Phi(p_i) + \lambda.g(p_i) \quad \text{(i=1,...,N)} \quad (9)$$

where

$$g(p_i) = P_D + P_L - \sum_{i=1}^{N} P_i = 0 \quad (10)$$

## IV. HOPFIELD NEURAL NETWORK

Neural networks are highly simplified models of the human nervous systems, exhibiting abilities such as learning, generalization, and abstraction. It is well known that the HNN converges very slowly and normally takes several thousand iterations. In this study momentum term is used to speed up convergence for the HNN. Also, Improved Euler Method and Runge Kutta-4 (RK-4) Method are used to solve differential equations in HNN.

The HNN method uses a mapping technique, which has been described in reference [2], to solve quadratic programming problems. For the mapping of quadratic programming problems, inequality constraints have been combined with a slack variable technique.

### A. Mapping Technique

Mapping technique refers to planning technique of energy function which is used to adapt economic-environmental dispatch problem to HNN form.

The differential equations of Hopfield's continuous model [2,6,8] are defined as follows :

$$\frac{dX^{new}}{dt} = \begin{bmatrix} -1/\eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -1/\eta_2 \end{bmatrix} X^{new} + T^{hop} F(x^{new}) + I^{hop} \quad (11)$$

This model is based on continuous variables and responses. Hopfield's energy function [2,6,8,9] is defined as follows :

$$E(x^{new}) = -\frac{1}{2} X^{newT} T^{hop} X^{new} - X^{newT} I^{hop} \quad (12)$$

The energy function is a quadratic function that is associated with the cost function and the emission function to minimize the optimization problem. The quadratic problem can be written as

$$\text{Min } E^{obj}(x) = -\frac{1}{2} X^T T^{obj} X - X^T i^{obj} \quad (13)$$

Under the equality and the inequality constraints :

$$A^{eq}X = b^{eq} \tag{14}$$

$$A^{in}X \le b^{in} \quad \text{or} \quad A^{in}X \ge b^{in} \tag{15}$$

and the side constraints may be given as

$$X_{i,\min} \le X_i \le X_{i,\max} \tag{16}$$

where $X=[x_1,\dots,x_n]$ is the vector of variables and $X_{i,min}$ are $X_{i,max}$ are lower and upper bounds, respectively.

The feasible solution can be described as :

$$X^{new} = T^{constr}X^{new} + s \tag{17}$$

where

$$T^{constr} = I - A^{newT}(A^{new}A^{newT})^{-1}A^{new} \tag{18}$$

$$s = A^{newT}(A^{new}A^{newT})^{-1}b^{new} \tag{19}$$

Energy function can be written according to penalty factor ($c_0$) as

$$E = E^{obj} + \frac{-1}{2}c_0\left\|X^{new} - (T^{constr}X^{new} + s)\right\|^2 \tag{20}$$

The network's weights ($T^{hop}$) and input biases ($I^{hop}$) are set as follows to satisfy the energy function Eq.(20) :

$$T^{hop} = T^{obj} + c_0\left[T^{constr} - I\right] \tag{21}$$

$$I^{hop} = i^{obj} + c_0\left[I - T^{constr}\right]^T s \tag{22}$$

In these equations, because of converted inequality constraints to equality constraints by introducing slack variables, variables $X$ are set as $X^{newT}=[x^T\ y^T]$. *"y"* is the vector of slack variables $[y_1, y_2, \dots, y_{m^{in}}]$. *F(x)* function is explained in next section.

### B. Mapping of Economic-Emission Dispatch

First, we have to set weights and input biases for the EED problems. We use $n$ neurons for generators and $m^{in}$ neurons for inequality constraints. The objective function of the constrained EED problem given in Eq.(4) is considered as the energy function of the HNN. Therefore weights $T^{obj}$ and input biases $i^{obj}$ of the objective function are set as follows :

$$\begin{aligned} T_{ii}^{obj} &= -2(w_1 a_i + h.w_2 d_i) \\ T_{ij}^{obj} &= 0 \\ i_i^{obj} &= -(w_1 b_i + h.w_2 e_i) \end{aligned} \tag{23}$$

The constraints of the EED problems can be handled by adding corresponding terms to the energy function. We can convert inequality constraints to equality constraints, then $A^{new}$ and $b^{new}$ can be written as

$$A^{new} = \left[A^{eq}; A^{in}\right] \text{ and } b^{new} = \left[b^{eq}; b^{in}\right] \tag{24}$$

where

$$A^{eq} = \left[\underbrace{1\ \ 1\ \ \cdots\ \ 1}_{\text{n generators}}\ \underbrace{0\ \ 0\ \ \cdots\ \ 0}_{\text{m}^{\text{in}}\text{ slack variables}}\right] \tag{25}$$

$$b^{eq} = P_D + P_L \tag{26}$$

$A^{in}$ and $b^{in}$ are defined from inequality constraint equations given in Eq.(5). Generation limits are taken as inequality constraints. Inequality constraints can be converted to equality constraints by using slack variables. For example, the upper limit of the *i*th generator may be converted to :

$$P_i \le P_{i,\max} \Rightarrow P_i - P_{i,\max}y_p = 0$$

where $y_p \le 1$ ($y_p$ is a slack variable of the *p*-th inequality constraint ) and we can define $A_p^{in}$ and $b_p^{in}$ as :

$$\begin{array}{l} A_p^{in} = \begin{bmatrix} 0 & 0 & \dots & -1 & 0 & \dots & P_{i,\max} & 0 & \dots & 0 \end{bmatrix} \\ \qquad\qquad\qquad\quad\ \uparrow \qquad\qquad\quad \uparrow \\ \qquad\quad \text{i - th generator} \ (\text{n}+\text{p})\text{ - th column} \end{array} \tag{27}$$

and $b_p^{in}=0$ . Similarly the lower limits of generators can be fixed as in the above example. Then $A^{new}$ is created as

$$A^{new} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & \cdots & 0 & P_{1,\min} & 0 & \cdots & 0 & 0 \\ -1 & 0 & \cdots & 0 & 0 & P_{1,\max} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 0 & 0 & \cdots & P_{6,\min} & 0 \\ 0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 & P_{6,\max} \end{bmatrix}_{13\times18} \tag{28}$$

*F(x)* function in Eq.(11) is chosen as a symmetric ramp function which can be shown in Fig.1. The activation function of each neuron is modified to limit the output value between lower and upper bounds. It is described as

$$P_i = f(p_i) \rightarrow \begin{cases} P_{i,\min}, & P_{i,\min} > P_i \\ P_i, & P_{i,\min} \le P_i \le P_{i,\max} \\ P_{i,\max}, & P_i > P_{i,\max} \end{cases} \tag{29}$$

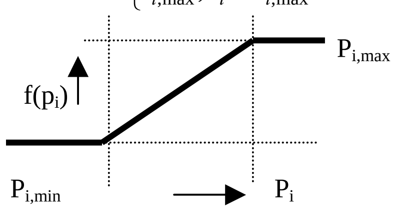


Figure 1. Input-output function of the variable i.

After finding $A^{new}$ and $b^{new}$ , $T^{constr}$ and $s$ can be determined using Eq.(18) and (19). Then we can set new weights and new input biases using Eq.(21) and (22). Finally, HNN is created for solving the constrained EED problem.

## V. TEST SYSTEM

The test system [3,5], which has six thermal generators, is chosen. The fuel cost and $NO_x$ emission equations are given in Table 1 and Table 2 [5].

**Table 1. Fuel cost [Rs/h] equations.**

$F_1=0.1525\ P_1^2+38.540\ P_1+756.8$
$F_2=0.1060\ P_2^2+46.160\ P_2+451.325$
$F_3=0.0280\ P_3^2+40.400\ P_3+1050$
$F_4=0.0355\ P_4^2+38.310\ P_4+1243.53$
$F_5=0.0211\ P_5^2+36.328\ P_5+1658.57$
$F_6=0.0180\ P_6^2+38.270\ P_6+1356.66$

**Table 2. $NO_x$ emission [kg/h] equations.**

$E_1=0.00420\ P_1^2+0.3300\ P_1+13.86$
$E_2=0.00420\ P_2^2+0.3300\ P_2+13.86$
$E_3=0.00683\ P_3^2-0.5455\ P_3+40.267$
$E_4=0.00683\ P_4^2-0.5455\ P_4+40.267$
$E_5=0.00460\ P_5^2-0.5112\ P_5+42.9$
$E_6=0.00460\ P_6^2-0.5112\ P_6+42.9$

**Table 3. Operating limits [MW].**

| Generator No. | Lower Limit [MW] | Upper Limit [MW] |
|---|---|---|
| 1 | 10 | 125 |
| 2 | 10 | 150 |
| 3 | 35 | 225 |
| 4 | 35 | 210 |
| 5 | 130 | 325 |
| 6 | 125 | 315 |

Transmission loss coefficients are taken from Ref.5. The operating limits of the generators are given in Table 3 [5].

In this paper, Improved Euler Method and RK-4 Method are used to solve differential equations in HNN. The time step (or step length) $\Delta t$ is given values from 0.0001 to 0.0003. Also, to speed up convergence to optimum point in HNN, momentum term is used as follows :

$$X_{n+1} = X_n + \Delta X_n + \rho.\Delta X_{n-1} \tag{30}$$

In Eq.(30), momentum term's coefficient is obtained from trials [6]. In this paper, $\rho$ is selected to 0.95 by trial-error.

## VI. SIMULATION RESULTS

In this section, simulation results of pure ED, pure EmD, and EED for the two conditions with transmission loss and without transmission loss are demonstrated. The results obtained from HNN Method are compared with the results of LM Method. The programs for these two optimization techniques were written in Matlab®. These programs are executed on a Pentium III 733 MHz PC with 64 MB RAM.

In this paper, these optimization techniques are applied to a test system which has six generating units, for 500, 600, and 700 MW loads and for four study modes as follows .

1. $w_1=1$ , $w_2=0$ Pure ED
2. $w_1=0.8$ , $w_2=0.2$ EED[1]
3. $w_1=0.5$ , $w_2=0.5$ EED[2]
4. $w_1=0$ , $w_2=1$ Pure EmD

with transmission loss and without transmission loss.

For the condition neglecting transmission loss,

1. HNN used Improved Euler Method
2. HNN used Runge Kutta-4 Method
3. HNN used momentum term and Open Euler M.
4. HNN used momentum term and RK-4 Method
5. Lagrange Multiplier

With the above techniques, simulation is implemented. With a system load of 600 MW, simulation is performed for the whole study modes and the results are demonstrated in Table 4 and Table 5.

**Table 4. Results of the condition neglecting transmission loss for $P_D$=600 MW.**

| METHODS | | STUDY MODES | | | |
|---|---|---|---|---|---|
| | | Pure ED | EED[1] | EED[2] | Pure EmD |
| HNN used Improved Euler Method | $P_T$[MW] | 599.98 | 599.97 | 599.97 | 599.97 |
| | $F_T$ [Rs/h] | 31912 | 31913 | 31914 | 31915 |
| | $E_T$ [kg/h] | 350.82 | 350.56 | 350.13 | 349.2 |
| HNN used Runge Kutta-4 Method | $P_T$[MW] | 599.97 | 599.96 | 599.95 | 599.97 |
| | $F_T$ [Rs/h] | 31912 | 31913 | 31914 | 31915 |
| | $E_T$[kg/h] | 350.78 | 350.42 | 349.66 | 349.22 |
| HNN used Momentum term + Open Euler M. | $P_T$[MW] | 599.97 | 599.96 | 599.95 | 599.95 |
| | $F_T$ [Rs/h] | 31912 | 31913 | 31914 | 31915 |
| | $E_T$ [kg/h] | 350.85 | 350.53 | 349.89 | 348.63 |
| HNN used Momentum term + Runge Kutta-4 | $P_T$[MW] | 599.98 | 599.97 | 599.97 | 599.97 |
| | $F_T$ [Rs/h] | 31911 | 31912 | 31913 | 31914 |
| | $E_T$ [kg/h] | 350.82 | 350.57 | 350.02 | 349.83 |
| Lagrange Multiplier | $P_T$[MW] | 600 | 600 | 600 | 600 |
| | $F_T$ [Rs/h] | 31447 | 31555 | 31813 | 32158 |
| | $E_T$ [kg/h] | 371.57 | 343.4 | 331.56 | 328.38 |

Results of Table 4 and Table 5 indicate how a reduction in $NO_x$ emission could be achieved by a change in generation dispatch schedules. This is obtained at the expense of fuel cost. The results of these methods do not violate the individual generator capacity limits, and the transmission losses are also nearly the same as LM method.

From the results of pure ED and pure EmD dispatches, it is observed that there is an increase in fuel cost of 3Rs/h and a reduction in $NO_x$ emission of 0.99 kg/h for HNN used momentum term and RK-4 Method together neglecting $P_L$. Thus, for a reduction of 1 kg of $NO_x$/h, there is an increase in cost of 3.03 Rs. For the case including $P_L$, this value is 13.64 Rs. According to LM Method, this value is 16.48 Rs for the condition neglecting $P_L$, and this value is 11.37 Rs for the condition including $P_L$.

**Table 5. Results of the condition with transmission loss for $P_D$=600 MW.**

| METHODS | | STUDY MODES | | | |
|---|---|---|---|---|---|
| | | Pure ED | EED[1] | EED[2] | Pure EmD |
| HNN used Improved Euler Method | $P_{net}$[MW] | 599.97 | 599.97 | 599.97 | 599.97 |
| | $F_T$ [Rs/h] | 33434 | 33437 | 33442 | 33452 |
| | $E_T$ [kg/h] | 379.2 | 379 | 378.68 | 378.08 |
| | $P_L$ [MW] | 34.32 | 34.39 | 34.5 | 34.7 |
| HNN used Runge Kutta-4 Method | $P_{net}$[MW] | 599.98 | 599.98 | 599.99 | 599.99 |
| | $F_T$ [Rs/h] | 33435 | 33438 | 33443 | 33453 |
| | $E_T$[kg/h] | 379.22 | 379.02 | 378.69 | 378.08 |
| | $P_L$ [MW] | 34.34 | 34.4 | 34.51 | 34.72 |
| HNN used Momentum Term+ Open Euler Method | $P_{net}$[MW] | 599.97 | 599.97 | 599.97 | 599.97 |
| | $F_T$ [Rs/h] | 33434 | 33436 | 33441 | 33452 |
| | $E_T$ [kg/h] | 379.26 | 379.07 | 378.75 | 378.07 |
| | $P_L$ [MW] | 34.3 | 34.37 | 34.47 | 34.71 |
| HNN used Momentum Term+ Runge Kutta-4 Method. | $P_{net}$[MW] | 599.97 | 599.97 | 599.97 | 599.97 |
| | $F_T$ [Rs/h] | 33415 | 33421 | 33425 | 33432 |
| | $E_T$ [kg/h] | 380.86 | 380.35 | 379.99 | 379.54 |
| | $P_L$ [MW] | 33.88 | 34.02 | 34.12 | 34.26 |
| Lagrange Multiplier | $P_{net}$[MW] | 599.92 | 599.94 | 599.99 | 600 |
| | $F_T$ [Rs/h] | 33229 | 33247 | 33268 | 34254 |
| | $E_T$ [kg/h] | 452.07 | 399.56 | 380 | 361.92 |
| | $P_L$ [MW] | 33.63 | 21.1 | 28.05 | 28.7 |

To obtain the performance of HNN method, the error is calculated as the percentage difference between the values of HNN method and LM method. The error is formulated as

$$\text{Err} = \frac{\text{HNN's cost - LM's cost}}{\text{LM's cost}} \times 100\% \qquad (31)$$

The maximum error is 6.53% for the HNN method. The minimum error is (-16.12)% for the HNN method. The negative sign refers to the advantage of the HNN method. In addition , while the error of $F_T$ decreases from pure ED to pure EmD, the error of $E_T$ increases. The error values are almost the same for all differential equation techniques.

Also, the error is calculated as the percentage difference between HNN used Improved Euler Method and HNN used other techniques to find out which differential equation solution technique is the best. Although, in general the results of the HNN used momentum term and RK-4 method together are very good, the whole error values are very little and can be neglected.

The iterations of the HNN methods are (4-78) for 500 MW load, (156-713) for 600 MW load, and (246-578) for 700 MW load. The HNN used momentum term and RK-4 method together has the minimum iterations. The LM method has no iterations except some exceptions.

The execution (CPU) times are (0-0.06)s for 500 MW load, (0.50-1.72)s for 600 MW load, and (0.55-3.08)s for 700 MW load. Although the HNN method used momentum term and RK-4 method together has the maximum execution times, it can be neglected. The LM method takes almost no times.

The minimum memories are 2340 Bytes for the condition neglecting $P_L$ and 2850 Bytes for the condition including $P_L$ for LM method. The maximum memories are 7140 Bytes for the condition neglecting $P_L$ and 8790 Bytes for the condition including $P_L$ for HNN used momentum term and RK-4 method together.

## VII. CONCLUSIONS

In this study, HNN and LM solutions to the economic-emission dispatch problem have been presented. Although it is well known that the HNN converges very slowly and it takes several thousand iterations, this paper has presented an analysis of the performance of the HNN methods which have achieved efficient and accurate solutions for test system six generating units for 500, 600, and 700 MW loads. A comparison of HNN method with LM method has been presented. The errors of HNN method are negligible even HNN has an advantage of $NO_x$ emission. The HNN method has achieved very fast solutions according to a lot of studies in literature. The paper demonstrated that the HNN method can be applied easily to the economic-emission dispatch problems.

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