

# Performance Investigation of Extended Kalman Filter Based Observer for PMSM Using in Washing Machine Applications

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## Abstract

**In this paper, sensorless speed control of permanent magnet synchronous motor (PMSM) with an Extended Kalman Filter (EKF) based observer is performed. The general model of the PMSM and EKF algorithm are presented. Simulation studies are carried out using MATLAB/Simulink, in order to explore effectiveness and usability of EKF in sensorless PMSM drive for washing machine applications. In order to investigate performance of proposed system in real conditions, real torque profile of a washing machine is used. Simulation results illustrate that the proposed system is quite effective for washing machine applications and robust against variations of motor parameters.**

## 1. Introduction

Recent years, the traditional drive systems are replaced with modern, electronically controlled, brushless drives in home appliances, especially washing machines. In former washing machine applications two different drive systems were used. The older designs utilize electromechanically controlled, two-speed, single-phase induction motors. Most washers have TRIAC controlled universal motors. However, with the evolution of power switches and the permanent magnet motor technology, these drives are becoming obsolete. A new concept for home appliances is to use permanent magnet synchronous motors (PMSM) for efficiency and comfort.

PMSM are widely used in automotive, space technologies, computer sciences, medical electronics, military applications, robotics and small household applications due to providing smooth and constant torque, high torque/current and torque/inertia ratio, high efficiency, durability and reliability [1,2]. However, due to cruel variations of the load torque of the feeding frequency can be source of dislocation phenomena. To overcome this problem, stator current must be commutated in synchronism with rotor position [3]. Thus, vector control of PMSM needs exact rotor position. The most primitive method for determining rotor position is employing of a position sensor such as high-resolution encoder or resolver. However, such sensors keep some disadvantages, especially in terms of increased cost and also larger size of machine, lower reliability, durability and increasing in noise etc. In view of these limitations of the mechanical sensors, different sensorless control and rotor position estimation methods have been considered. In these methods, rotor position and speed are estimated and used as feedback signals for speed and torque control [1-3].

Kalman filter is an observer that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and provides estimates of unknown variables with high precisely based on least-square techniques. Extended Kalman Filter (EKF) is extensively used in non-linear problems such as sensorless control of PMSM. If the speed and the position of the rotor considered in the dynamical model of PMSM, the EKF can relinearize the non-linear state problem for any estimation state. In literature, The EKF method has been applied to several sensorless controlled drives with an accomplished manner [4-6].

One of the earliest references for state estimation of synchronous motor with EKF is the study of Dhaouadi et al. [7]. They designed and implemented EKF for online estimation of rotor speed and position by only using measured motor voltage and current. Another inspiring and remarkable study belongs to Bolognani et al. [8]. Their estimation algorithm does not require information about mechanical parameters and initial conditions. There are also several studies of speed estimation of PMSM with EKF based observers. An open question of EKF application is determining the covariance matrices. Here, trial and error method is used mostly; nevertheless some authors are studying filter tuning techniques. One of these studies is done by Bolognani et al. [9]. They proposed a straightforward method for matrix choice which based on the complete normalization of EKF algorithm for PMSM.

In washing machine application; high performance motor control is defined as smooth rotation over the full speed range of the motor, full torque control at zero speed, and fast accelerations and decelerations [10]. In order to obtain this performance vector control techniques are used which are usually known as field-oriented control (FOC). The principle concept of the FOC is to separate a stator current into flux and torque producing components. Both components can be controlled separately after separation. This separation allows controlling PMSM similarly a separately excited DC motor. FOC of PMSM used in washing machine drum drive has many advantages. Increase in system efficiency at wash / spin, improvement in sound quality, decrease energy consumption can be provided by means of FOC [11,12].

In this study, the mathematical model of PMSM designed for washing machine application is given. EKF is applied in order to estimate position and speed. Speed control loop uses the estimated speed by EKF. Estimation errors for both speed and position were negligible. The proposed system is very simple and design procedure is straight forward. All system is modelled in MATLAB/Simulink.

The paper is organized as follows. The next section presents the mathematical model of PMSM for washers. In section III,

estimation method is presented and investigated in a detailed manner. In section IV, a summary for FOC is given. The simulation results are given in section V.

## 2. Mathematical Model of PMSM

PMSM is commonly modelled and studied in two different reference frames (dq and  $\alpha\beta$ ) to reduce the complexity of differential motor equations and apply more efficient and to use more simple control strategies. In Fig. 1, dq reference frame, synchronously rotating with the motor rotor, and stationary  $\alpha\beta$  reference frame are depicted for modelling PMSM.

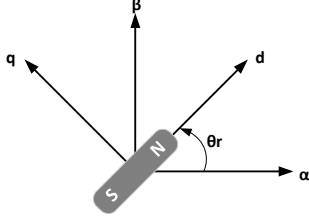


Fig. 1. Space vector diagram of PMSM

The voltage and electromagnetic torque equations of PMSM in dq reference frame can be written as [11]

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_s + pL_d & -\omega_r L_q \\ \omega_r L_d & R_s + pL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_r \lambda_m \end{bmatrix} \quad (1)$$

$$T_e = \frac{3}{2} \frac{P}{2} (\lambda_m i_q + (L_d - L_q) i_d i_q) \quad (2)$$

where  $v_d$  and  $v_q$  are dq axis components of stator voltages,  $i_d$  and  $i_q$  are dq axis components of stator currents,  $L_d$  and  $L_q$  are dq axis stator inductances,  $R_s$  is stator winding resistance,  $\lambda_m$  is permanent magnet flux linkage,  $\omega_r$  electrical rotor speed,  $T_e$  is electromagnetic moment,  $P$  is number of poles and  $p$  is the derivative operator.

The stationary reference frame motor voltage equations obtained by means of Clarke Transform can be given in matrix form as

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} L_0 + \Delta L \cos(2\theta_r) & \Delta L \sin(2\theta_r) \\ \Delta L \sin(2\theta_r) & L_0 - \Delta L \cos(2\theta_r) \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega_r \left( 2\Delta L \begin{bmatrix} -\sin(2\theta_r) & \cos(2\theta_r) \\ \cos(2\theta_r) & \sin(2\theta_r) \end{bmatrix} + \lambda_m \begin{bmatrix} -\sin(\theta_r) \\ \cos(\theta_r) \end{bmatrix} \right) \quad (3)$$

$$\dot{\theta}_r = \omega_r \quad (4)$$

where  $v_\alpha$  and  $v_\beta$  are  $\alpha\beta$  axis components of stator voltages,  $i_\alpha$  and  $i_\beta$  are  $\alpha\beta$  axis components of stator currents  $L_0$  is the average inductance and  $\Delta L$  is the zero-to-peak differential inductance is a direct measure of the spatial modulation of the inductance

$$L_0 = \frac{L_d + L_q}{2} \quad (5)$$

$$\Delta L = \frac{L_d - L_q}{2} \quad (6)$$

When  $L_d$  and  $L_q$  are almost equal,  $\Delta L$  is negligible, and motor voltage equations become

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} L_0 & 0 \\ 0 & L_0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \omega_r \lambda_m \begin{bmatrix} -\sin(\theta_r) \\ \cos(\theta_r) \end{bmatrix} \quad (7)$$

Considering the mechanical load, the dynamic mechanical equation of motor system is obtained as

$$T_e - T_L = J \frac{d\omega}{dt} + B \omega \quad (8)$$

where  $T_L$  is load torque,  $\omega$  is mechanical rotor speed,  $\theta$  is mechanical rotor position,  $J$  is moment of inertia and  $B$  is viscous damping.

## 3. Extended Kalman Filter Algorithm for PMSM

EKF is an optimal estimator that estimates the states of nonlinear dynamic systems based on least square sense [5]. A discrete time nonlinear system can be expressed in the following form

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}) + w_{k-1} \\ z_k &= h(x_k) + v_k \end{aligned} \quad (9)$$

where  $x_k$  and  $z_k$  are state variables and output of the system respectively. System noise  $w_k$  represents the uncertainties in the model, and  $v_k$  denotes the measurement noise. They are zero mean Gaussian noises with covariance  $Q_k$  and  $R_k$  respectively. They are uncorrelated with each other and the initial state  $x_0$ . The initial state  $x_0$  is a Gaussian random vector with mean  $\mu_0$  and error covariance  $P_0$ .

By using the filter, for a sampling time  $T_s$ , the optimal state estimate  $\hat{x}_k$  with the covariance matrix  $P_k$  are obtained at the time  $k$ .

EKF algorithm has two steps: prediction and innovation with an initialization. In prediction step, the previous optimal estimate  $\hat{x}_{k-1}$  with covariance  $P_{k-1}$  and is used to make a prediction as given in (10)

$$\begin{aligned} \bar{x}_k &= f(\hat{x}_{k-1}) \\ \bar{P}_k &= J_f(\hat{x}_{k-1}) P_{k-1} J_f^T(\hat{x}_{k-1}) + Q_{k-1} \end{aligned} \quad (10)$$

In the innovation step, the predicted state vector is corrected through the measurements made at time  $k$  as given in (11)

$$\begin{aligned}\hat{x}_k &= \bar{x}_k + K_k (z_k - h(\bar{x}_k)) \\ P_k &= [I - K_k J_h(\bar{x}_k)] \bar{P}_k\end{aligned}\quad (11)$$

where the Kalman gain is defined as

$$K_k = \bar{P}_k J_h^T(\bar{x}_k) [J_h(\bar{x}_k) \bar{P}_k J_h^T(\bar{x}_k) + R_k]^{-1}. \quad (12)$$

Here  $J_f(\cdot)$  and  $J_h(\cdot)$  are the Jacobian matrices of nonlinear state equations  $f(\cdot)$  and  $h(\cdot)$  respectively.

In order to build an EKF based estimator for PMSM, at first, the state space equations of the motor needs to be discretized and linearized. To get the system equations in the most suitable form, the equations in stationary reference frame which are given in (7) are used [8]. Since dq-axis inductances of the motor used in this study are quite equal, it will not cause any essential errors.

By assuming the two stator currents, the electrical speed and position are chosen as system state variables, and two stator voltages are chosen as inputs

$$x = [i_\alpha \quad i_\beta \quad \omega_r \quad \theta_r]^T, \quad u = [v_\alpha \quad v_\beta]^T \quad (13)$$

and also assuming change of rotor speed is negligible compared with the electrical parameters, the following nonlinear dynamic equations are obtained:

$$\begin{aligned}\dot{i}_\alpha &= -\frac{R_s}{L_0} i_\alpha + \frac{\omega_r \lambda_m}{L_0} \sin \theta_r + \frac{v_\alpha}{L_0} \\ \dot{i}_\beta &= -\frac{R_s}{L_0} i_\beta - \frac{\omega_r \lambda_m}{L_0} \cos \theta_r + \frac{v_\beta}{L_0} \\ \dot{\omega}_r &= 0 \\ \dot{\theta}_r &= \omega_r\end{aligned}\quad (14)$$

Note that, by assuming the derivative of rotor speed is zero, all mechanical load parameters disappear from the equations, this will result in poor speed estimation at transient time.

After discretization with constant sampling time,  $T_s$ , by using forward rectangular rule, (14) becomes

$$\begin{aligned}i_{\alpha k} &= i_{\alpha_{k-1}} - T_s \frac{R_s}{L_0} i_{\alpha_{k-1}} + T_s \frac{\omega_{r_{k-1}} \lambda_m}{L_0} \sin(\theta_{r_{k-1}}) + T_s \frac{v_{\alpha_{k-1}}}{L_0} \\ i_{\beta k} &= i_{\beta_{k-1}} - T_s \frac{R_s}{L_0} i_{\beta_{k-1}} - T_s \frac{\omega_{r_{k-1}} \lambda_m}{L_0} \cos(\theta_{r_{k-1}}) + T_s \frac{v_{\beta_{k-1}}}{L_0} \\ \omega_k &= \omega_{r_{k-1}} \\ \theta_k &= \theta_{r_{k-1}} + T_s \omega_{r_{k-1}}\end{aligned}\quad (15)$$

Now, the Jacobian matrix, first order partial derivatives of system equations with respect to state variables,

$$J_f = \frac{df(x_{k-1}, u_{k-1})}{dx_{k-1}}, \text{ can be written as}$$

$$J_f = \begin{bmatrix} 1 - T_s \frac{R_s}{L_0} & 0 & T_s \frac{\lambda_m}{L_0} \sin(\theta_{r_{k-1}}) & T_s \frac{\omega_{r_{k-1}} \lambda_m}{L_0} \cos(\theta_{r_{k-1}}) \\ 0 & 1 - T_s \frac{R_s}{L_0} & -T_s \frac{\lambda_m}{L_0} \cos(\theta_{r_{k-1}}) & T_s \frac{\omega_{r_{k-1}} \lambda_m}{L_0} \sin(\theta_{r_{k-1}}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & T_s & 1 \end{bmatrix} \quad (16)$$

The outputs of the system are stator currents,  $i_\alpha$  and  $i_\beta$ , then the output matrix and its Jacobian matrix becomes

$$z = h(x_k) = [i_{\alpha k} \quad i_{\beta k}]^T \quad (17)$$

$$J_h = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (18)$$

In implementation of EKF, one critical point is the choice of the values of noise covariance matrices  $P_0$ ,  $Q$  and  $R$ .  $P_0$  is related to errors in knowledge of initial states, and it only changes the magnitude of transient response, not settling time and any steady state characteristics. Changing  $Q$  and  $R$  affects both transient and steady state performance of the filter [7]. It is common to use diagonal matrices for  $Q$  and  $R$  to eliminate the need of extra information to calculate off-diagonal terms [7, 9].

There are some studies in the literature to develop tuning rules for  $Q$  and  $R$  matrices. Convergence and stability of the filter is more sensitive to covariance matrix  $Q$  than  $R$ . Increasing values of  $Q$  corresponds to heavy system noise and parameter uncertainty. An increment of  $Q$  will increase the filter gain, resulting in faster but large magnitude transient response and ripples. Decreasing values of  $Q$  causes weaker correction and state estimation. Increasing values of  $R$  means that measurements are affected much by noise and should be weighted less by filter. This will result in smaller filter gain and slower transient characteristic [7, 9, 12-14].

In this study, by considering these facts and making some trial and error studies, the matrices  $Q$  and  $P_0$  are chosen as

$$P_0 = 10I(4), \quad Q = \text{diag}([1 \quad 1 \quad 60 \quad 0.5]) \quad (19)$$

where  $I(4)$  is  $4 \times 4$  identity matrix and  $\text{diag}$  is diagonal matrix. When determining matrix  $R$ , also the measurement noises that added to the simulation are considered and it is chosen as

$$R = 10^{-8} I(2) \quad (20)$$

where  $I(2)$  is  $2 \times 2$  identity matrix.

#### 4. Sensorless Field Oriented Control of PMSM

The basic idea of FOC is to imitate the dc motor's operation by separately controlling the torque producing and field-generating currents. Fig. 2 illustrates the basic concept of the FOC algorithm for the PMSM.

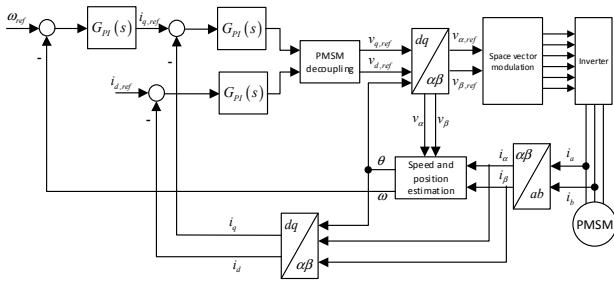


Fig. 2. Sensorless vector control diagram of PMSM

The torque produced in the synchronous machine is maximum when the stator and rotor magnetic fields are orthogonal. If the flux is oriented correctly all the time (maintaining the rotor and stator flux in quadrature), better dynamic response and less torque ripple are achieved. To do this, the stator current component in quadrature with the rotor flux is controlled to generate the commanded torque (current), and the direct component is set to zero. FOC allows decoupling and controlling the torque and the magnetizing flux components of stator current. So, there are two separate current control loop in FOC. Since FOC is performed in dq reference frame, the controllers are isolated from time varying current and voltages, and the system is not limited by bandwidth of controllers [15,16].

In order to perform FOC, following steps are performed.

- Sensing of the motor phase voltages and currents.
- Transforming them into the two-phase system ( $\alpha\beta$ ) by means of Clarke transformation.
- Calculation of the flux space vector magnitude and position angle of the rotor.
- Transforming of stator currents into the dq reference frame by using Park transformation.
- Separately controlling of torque ( $i_q$ ) and flux ( $i_d$ ) components of stator current.

- Calculation of reference stator voltage in dq reference frame by using decoupling block.
- Transformation of reference voltages into two-phase system ( $\alpha\beta$ ).
- Generation of three phase motor voltage by using space vector modulation.

The position of the magnetizing flux of PMSM must be known, in order to decompose currents into torque and flux components. Therefore, the rotor position and velocity must be known high precisely.

## 5. Simulation Studies

To investigate the proposed EKF based observer for sensorless speed control of PMSM, the simulation diagram in Fig. 3 is constructed in Matlab/Simulink.

The electrical and mechanical part of the PMSM is modelled as in (1), (2) and (8).

As mentioned in previous section to control the d and q axis currents are controlled separately by PI controllers. The outputs of these current loops are d and q axis voltages that should be supplied to the motor respectively. The voltage information in dq reference frame is transformed into fundamental frame and then they are converted to gate signals of inverter by help of space vector modulation.

The EKF algorithm is integrated to the simulation through an embedded Matlab function. The inputs of EKF block are current and voltages in stationary  $\alpha\beta$  reference frame since the  $\alpha\beta$  model of PMSM is used. In practice three phase motor currents are directly gathered from motor drive and there is no need for position information to transform these values to  $\alpha\beta$  reference frame. Therefore, in simulation, the real position information is used to get through the  $\alpha\beta$  reference frame from dq reference frame. As seen from the figure, the measurement noises are added to the currents before the EKF block.

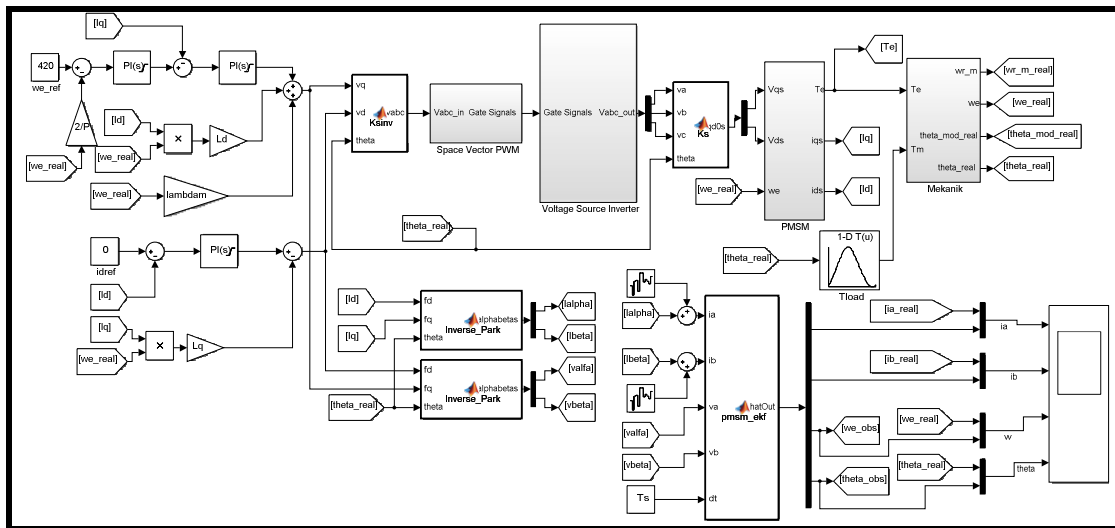


Fig. 3. Block diagram of sensorless speed control of PMSM with EKF based observer.

For the load torque, real washing machine load profile that changing with motor position is used. In Fig. 4, real speed, position and torque curves of the motor are given with 10 kg

laundry load. The torque curve is multiplied with 100 to be seen better.

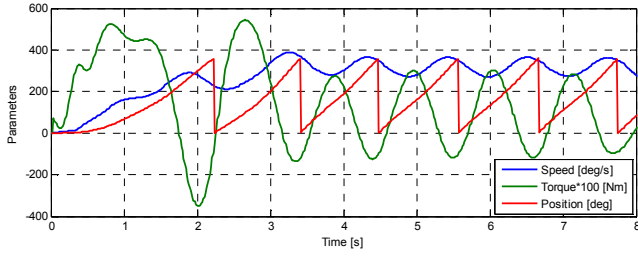


Fig. 4. Real load profile of washing machine.

As expected, this load profile is highly coherent with the form in (21).

$$T_L = A \sin(\theta + \phi) \quad (21)$$

Here  $A$  is a constant with related to the laundry quantity. The offset on the torque curve is originated from friction and other constant inertia.

To simulate this load profile, a look up table is used as seen in Fig. 3. The input of the load block is real position. Only one steady state period of load profile is used and repeated during the simulation.

Motor parameters are given in Table 1.

Table 1. Motor parameters

Parameter	Value
Rated power (P)	900 W
Rated torque ( $T_e$ )	2 Nm
Rated speed (n)	4000 $\text{min}^{-1}$
Number of poles (P)	8
Stator winding resistance ( $R_s$ )	2.5 $\Omega$
q axis inductance ( $L_q$ )	0.017 H
d axis inductance ( $L_d$ )	0.016 H
Permanent magnet flux linkage ( $\lambda_m$ )	0.1183 Vs $\text{rad}^{-1}$
Moment of inertia (J)	0.001 $\text{kg m}^2$

In simulations, 420 rad/s is used as reference speed. In Fig. 5 and 6, position and speed estimation results are given. In these simulations, rated motor parameters are used. As seen from figures, motor is settled in reference frame and the estimation error is small enough. Absolute position error is 0.4 rad, and absolute speed error is 3.5 rad/s. Therefore relative errors are  $0.4/2\pi$  and  $3.5/420$  for position and speed respectively.

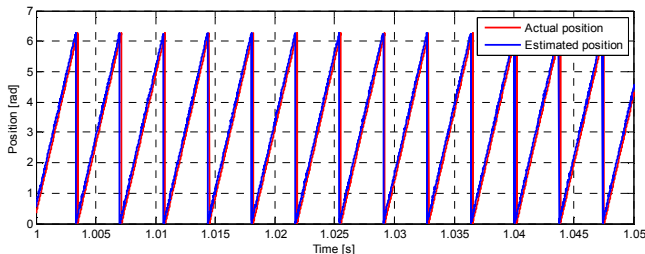


Fig. 5. For rated motor parameters actual and estimated position

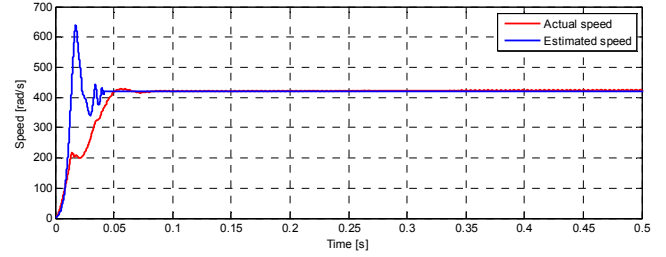


Fig. 6. For rated motor parameters actual and estimated speed

Once motor begins to turn, the temperature rises and stator winding resistance and inductances will change. To take these effects into consideration, the simulations are repeated for the cases where resistance increased by 50% and  $L_d$  inductance reduced by 30%. Simulation results are given in following figures. For the case in Fig. 7 and 8, relative errors are  $0.3/2\pi$  and  $4.5/420$  for position and speed respectively. Relative errors in Fig. 9 and 10 are  $0.25/2\pi$  and  $8/420$  for position and speed respectively.

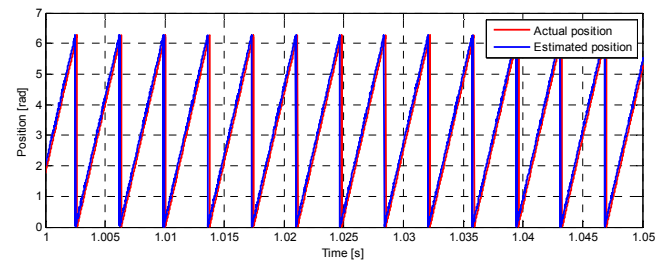


Fig. 7. For  $\hat{R}_s = 1.5 R_s$  actual and estimated position

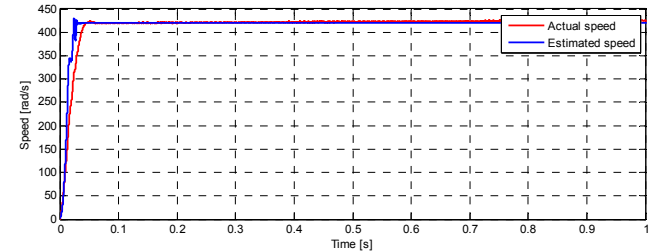


Fig. 8. For  $\hat{R}_s = 1.5 R_s$  actual and estimated speed

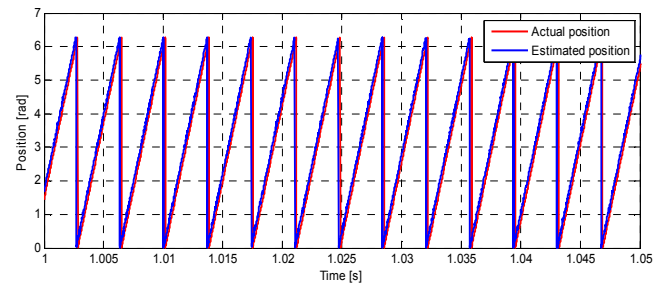
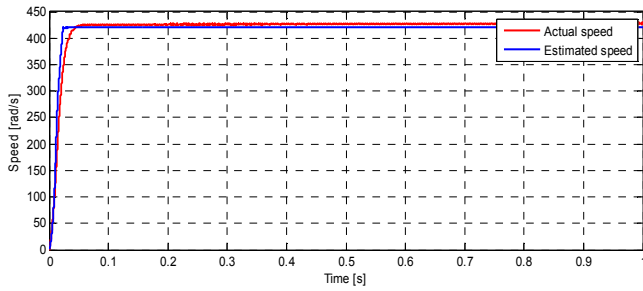


Fig. 9. For  $\hat{R}_s = 1.5 R_s$  and  $\hat{L}_d = 0.7 L_d$  actual and estimated position



**Fig. 10.** For  $\hat{R}_s = 1.5R_s$  and  $\hat{L}_d = 0.7L_d$  actual and estimated speed

As seen from figures, the filter can tolerate the parameter changes in the model and errors are acceptable. As said before since derivative of speed is neglected in EKF algorithm, there is not any variable to manipulate the transient response of the filter. When transient response of speed is slow, at steady state position errors reduce but speed error increase. The desired optimal response can be found by changing the covariance matrix  $Q$ .

## 6. Conclusions

In this study, sensorless FOC of PMSM is presented for washing machine applications. Extended Kalman Filter has been used in order to estimate position and speed of drum. The mathematical model of the PMSM and EKF algorithm has been given in the study and MATLAB/Simulink has been used for all simulations. Real torque profile of a washing machine is used in order to examine performance of whole system in real conditions. Estimation errors for both speed and position are acceptable. The proposed system is very simple and design procedure is easy. In addition that simulation results show that the system can tolerate parameter uncertainty of the motor.

## 7. Acknowledgment

This research is supported by The Scientific and Technological Research Council of Turkey (TUBITAK) in the framework of Development of Unique Field Oriented Control Method for Motors of Household Appliances with project number 5130069.

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