HYBRID SR ARQ SCHEME USING RATE 2/3 4-STATE ASYMMETRIC TCM AND CODE COMBINING TECHNIQUE FOR AWGN CHANNEL

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Abstract-In this paper, we evaluate the weight spectra of the rate 2/3, 4-state asymmetric trellis coded modulation (ATCM) scheme. Using these weight spectra and the code combining technique, the throughput of hybrid selective repeat automatic repeat request (SR ARQ) scheme is obtained over additive white Gaussian noise (AWGN) channel. Numerical results show that the proposed scheme performs better than the scheme using the rate 2/3, 4-state symmetric TCM at the values of E_s/N_0 between 2-9 dB.

Key words: TCM, Hybrid ARQ, throughput, data transmission.

I.INTRODUCTION

In data transmission, increasing the reliability of the communication is very important without increasing the energy need for the system. One solution of increasing the reliability of data communication is the application of hybrid automatic repeat request (HARQ) schemes. There are three basic ARQ schemes which are called stop and wait ARQ (SW ARQ), go back N ARQ (GBN ARQ) and selective repeat ARQ (SR ARQ) [1]. The last one has the best throughput efficiency. In order to increase throughput efficiency, the packets combining techniques [2] are also applied to the HARQ schemes. In these kinds of schemes, received erroneous packets are not discarded, but are combined with their repeated copies in an optimum manner and then the decoder decodes the combined sequence for obtaining data in the receiver. Packet combining techniques achieve power efficiency and are grouped into two categories: Code combining [2] and diversity combining [3]. The diversity combining technique is applied to HARQ schemes using TCM for AWGN and fading channels [3-6]. The code combining technique is applied to HARQ schemes using TCM for AWGN channels [7,8].

Trellis coded modulation is a combined coding and modulation technique which increases reliability and so improves the performance of communications without increasing power need and bandwidth expansion for communication systems [9]. It is possible to say that the throughput of HARQ protocol using TCM is better than that of HARQ protocol which is not using TCM, because by using TCM the transmitted signals are less affected from the channel noise and received more correctly, so the number of retransmission is reduced. The more the squared free Euclidean distances between signals of TCM are increased, the more correctly they are received. Using asymmetric trellis coded modulation (ATCM) instead of TCM is a way of increasing the squared free Euclidean distances between signal points. There are some studies for HARQ scheme using TCM in the literature [3-8]. At the studies made for HARQ scheme using TCM in the literature, performance improvements have been obtained by changing the code rate, the number of states of trellis diagram etc. [3-8]. One of the criteria used for performance improvement of TCM is the squared free Euclidean distance (d_{free}^2) which depends on the code rate, the number of states of trellis diagram and the signal constellations used. Employing asymmetric signal constellation for TCM achieves performance gain over traditional TCM in which the symmetric signal constellation is used.

In this paper, we obtain hybrid SR ARQ scheme using a rate 2/3, 4-state asymmetric TCM and code combining technique over AWGN channel. In the following, we first mention asymmetric TCM and we compute the weight spectra of the rate 2/3, 4-state asymmetric trellis coded modulation (ATCM) scheme. Then we introduce the operation of the scheme. Using the weight spectra and code combining technique, the throughput of hybrid selective repeat automatic repeat request (SR ARQ) scheme is obtained for additive white Gaussian noise (AWGN) channel.

II. ASYMMETRIC TRELLIS CODED MODULATION

Trellis coded modulation is a combined coding and modulation scheme wherein a rate m/(m+1) convolutional code is combined through a suitable mapping function with an $M=2^{m+1}$ points signal constellation. This improves the reliability of a communication without bandwidth expansion relative to an uncoded 2^m point modulation [9]. In traditional TCM schemes symmetric signal constellation, in which the signals are equally spaced as shown in Fig. 1a, is used for mapping. Employing asymmetric signal constellation, shown in Fig 1b, instead of symmetric signal constellation for mapping increases the squared free distance of TCM and improves the throughput of the HARQ scheme.

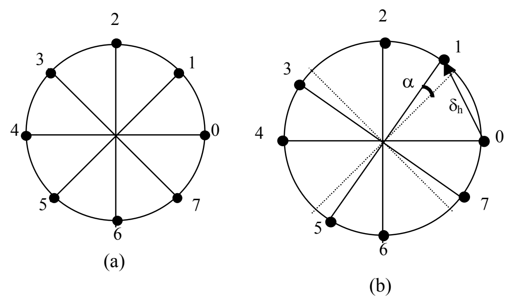

Figure -1. a) Symmetric 8PSK signal constellation.
b) Asymmetric 8PSK signal constellation.

The spectral coefficients of TCM are used for obtaining the throughputs of the HARQ schemes. So in this section the spectral coefficients of the rate 2/3, 4-state asymmetric TCM are calculated for AWGN channel.

The state transition diagram of 4-state trellis diagram [9] is shown in Fig.2.

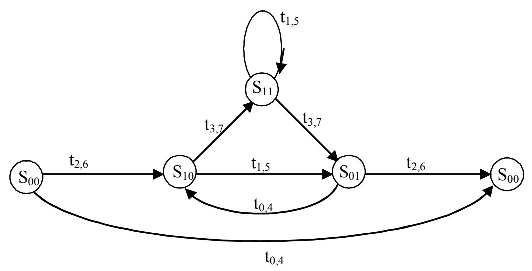

Figure- 2. The state transition diagram for 4-state TCM.

The transfer function of the state transition diagram shown in Fig. 2 is given as

$$T(D,I) = t_{0,4} + \frac{t_{2,6}^2\left(t_{1,5} - t_{1,5}^2 + t_{3,7}^2\right)}{\left(1-t_{1,5}\right)\left(1-t_{0,4}t_{1,5}\right) - t_{0,4}t_{3,7}^2} \quad (1)$$

The transitions between the states of the state transition diagram are obtained from asymmetric 8 PSK signal constellation of the rate 2/3, 4-state ATCM such that

$$t_{h,(h+4)}\Big|_{I=1} = D^{\delta_h^2} + D^{\delta_{h+4}^2} \quad (h = 0,1,2,3) \quad (2)$$

where the squared free Euclidean distances are such as

$$\begin{aligned} \delta_h^2 &= 2\left[1-\cos(h\times\frac{2\pi}{8})\right] \quad (h = 0,2,4,6) \\ \delta_h^2 &= 2\left[1-\cos(h\times\frac{2\pi}{8}+\alpha)\right] \quad (h = 1,3,5,7) \end{aligned} \quad (3)$$

The squared Euclidean distances obtained in Eq.(3) are applied to Eq.(2) and the state transitions are obtained. By using state transition in Eq.(1) and solving Eq.(1), the spectral coefficients of the rate 2/3, 4-state ATCM for AWGN channel are obtained and given in Table 1.

Table-1. The spectral coefficients of the rate 2/3, 4-state symmetric and asymmetric TCM for AWGN channel.

	i	0	1	2	3	4	5
Asym-metric	d_i^2	4,000	4,800	5,600	6,400	7,200	8,000
	a_{d_i}	1	4	20	52	140	384
Symmetric	d_i^2	4,000	4,586	5,172	5,758	6,344	6,930
	a_{d_i}	1	8	16	32	64	128

The free Euclidean distances between signal s_0 and neighbor signals s_1 and s_7 are equal to 0,586 in symmetric signal constellation. At the beginning steps of trellis diagram, the squared free Euclidean distance of the signal s_1, in the $t_{1,5}$ transition of state diagram, dominates the increase in the squared distance of the scheme (d_i^2). When signal points of sub-signal constellation (s_1, s_3, s_5, s_7) rotate by a positive α angle as shown in Fig.1 the squared free Euclidean distance of signal s_1 increases. This also increases the squared distance of the scheme (d_i^2). When the asymmetry angle α is increased further, the squared free Euclidean distance of signal s_7 in the $t_{3,7}$ transition of state transition diagram becomes dominant and causes the squared distance of the scheme (d_i^2) decrease, because the squared free Euclidean distance of s_7 decreases by increasing the asymmetry angle α. As a result the optimum asymmetry angle ($\alpha_{opt.}$) is obtained as $8,13^0$ from Eq.4.

$$3\sin\alpha_{opt.} + \cos\alpha_{opt.} = \sqrt{2} \tag{4}$$

The squared free Euclidean distance of signal s_1 in the $t_{1,5}$ transition and the squared free Euclidean distance of signal s_7 in the $t_{3,7}^2$ transition of state transition diagram become equal to each other ($\delta_1^2 = 2\delta_7^2 = 0,8$) and both dominate the squared distance of the scheme (d_i^2) for this optimum asymmetry angle $\alpha_{opt}=8,13^0$. The squared distance of the scheme (d_i^2) is increased by 0,8 at every step especially at the beginning of the trellis diagram of the code. This is the maximum increase.

III. OPERATION AND THROUGHPUT ANALYSIS OF THE SCHEME

When a k-bit message is ready for transmission, it is first encoded into an n-bit codeword in C_o which is an (n, k) block code for error-detection as in [7]. This n-bit codeword is input to the rate 2/3, ν=2 ATCM encoder. The output of the encoder is a signal sequence $X_1=(x_1, x_2, \ldots\ldots, x_{L+v})$ of the ATCM, where L=n/b is a positive integer and ν is the number of memory elements in the convolutional encoder. Let X_1 be transmitted as a 8-PSK signal sequence over an AWGN channel and $\widetilde{X}_1^{(1)}$ be its received version. $\widetilde{X}_1^{(1)}$ is first decoded by the decoder with soft-decision Viterbi algorithm (VA). If the VA decoder's output is declared error-free by C_o, then the message is recovered and the receiver sends a positive acknowledgement (ACK) signal to the transmitter. If errors are detected, the receiver saves $\widetilde{X}_1^{(1)}$ in a buffer and asks for a retransmission of X_1 by sending a negative acknowledgement (NACK) signal to the transmitter. Let $\widetilde{X}_1^{(2)}$ be the second received version of X_1. Again $\widetilde{X}_1^{(2)}$ is first decoded by the VA decoder. If $\widetilde{X}_1^{(2)}$ is declared error-free, the message is recovered at this time, otherwise; $\widetilde{X}_1^{(1)}$ and $\widetilde{X}_1^{(2)}$ are combined to form $\widetilde{X}_2$ and this sequence is decoded. If decoding is still not successful, both $\widetilde{X}_1^{(1)}$ and $\widetilde{X}_1^{(2)}$ are saved in the buffer and the receiver asks for another retransmission of X_1. With code combining, the decoder operates on a combination of all repeated sequences. That is, the receiver keeps all received sequences $\widetilde{X}_1^{(j)}$, forms $\widetilde{X}_j$ if necessary, and decodes alternatively on $\widetilde{X}_1^{(j)}$, and $\widetilde{X}_j$, until correct data occurs.

The throughput is defined as the average number of information bits per transmitted modulation symbol [7]. In the throughput analysis, we assumed that the size of receiver buffer is infinite and the feedback channel is noiseless.

In this scheme, the probability that decoded sequence of X_j contains detectable errors is given by

$$P(D_j) \cong 1 - \left[1 - P_j(E)\right]^L \tag{5}$$

where $P_j(E)$ is the probability of an error event of Viterbi decoding, and upper bounded by the

$$P_j(E) \le \sum_{l=0}^{\infty} a_{d_l}^j \, p(d_l^j) \tag{6}$$

where $\{a_{d_l}^j\}$ is the distance spectra of X_j which can be obtained from the distance spectra of X_1, that is $a_{d_l}^j = a_{d_l}^1$. In Eq.6, $p(d_l^j)$ is the probability that a wrong path at distance d_l^j is selected by the decoder as the transmitted sequence and is given for AWGN channel as [7].

$$p(d_l^j) = Q\left[\sqrt{\frac{d_l^j E_s}{2N_0}}\right] \tag{7}$$

where E_s/N_o is signal-to-noise ratio per transmitted ATCM and d_l^j 's, l=0, 1, ..., are the squared distances.

For every retransmission of X_1, the normalized squared distance of the combined signal sequence increases because the combined signal has more energy, but distance spectra remain unchanged, that is $d_l^j = jd_l^1$ but $a_{d_l}^j = a_{d_l}^1$. The average number of transmission and retransmission is obtained in a similar manner in [7] as

$$1 + P(D_1) + \sum_{j=2}^{\infty} P(D_l)^j \prod_{i=2}^{j} P(D_i) \le E\left[T\right] \le 1 + \sum_{j=1}^{\infty} P(D_j) \tag{8}$$

As a result, the throughput of the proposed scheme is obtained as

$$\eta = \frac{b}{E\left[T\right]} \frac{k/b}{L+\nu} \tag{9}$$

where usually (k/b)/(L+ν)≈1 [7].

Applying distance spectra and the squared distance of the scheme (d_i^2) which are given in Table 1 through Equations (7), (6), (5), (8) and (9), the lower bound on the throughput of the proposed HARQ scheme using a rate 2/3, 4-state ATCM and code combining is obtained as in Fig.3. For comparison, the lower bound on the throughput of the hybrid SR ARQ scheme using the rate 2/3, 4-state TCM and code combining [7] is also shown in Fig.3.

In the calculation of numerical results, we have assumed that the information message length is 200 bits and 16 bits are appended to them for error detection as in [7]. From Fig.3, we observe that the proposed scheme performs much better than the scheme described in [7] at the values of E_s/N_0 between 2-9 dB.

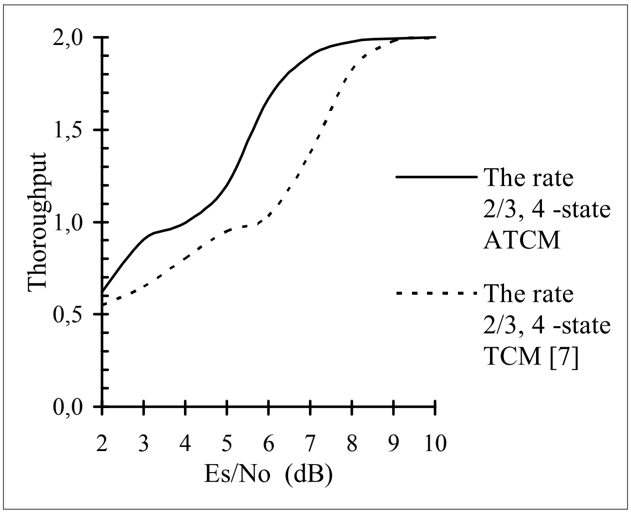

Figure 3. The lower bounds on the throughputs of HARQ using the rate 2/3 4-state asymmetric and symmetric TCM over an AWGN channel.

IV. CONCLUSION

In this paper, the weight spectra of the rate 2/3, 4-state asymmetric trellis coded modulation (ATCM) have been evaluated. Using the weight spectra and the code combining technique, a hybrid SR ARQ scheme is introduced and its throughput analysis is done. Numerical results show that the proposed scheme performs much better than the scheme described in [7] at the values of E_s/N_0 between than 2-9 dB.

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