A KALMAN FILTER ALGORITHM FOR GENERALIZED MOVING WINDOWING

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Key words: Kalman Filters, Estimation, Moving Averages

ABSTRACT

Several Kalman filtering algorithms have been implemented in the past years. One of these algorithms uses the moving windowing (MW) approach, which is limited with the use of rectangular windows. In this paper, a generalized moving windowing (GMW) algorithm that utilizes arbitrary shaped windows is proposed. The proposed algorithm has been implemented on a known process and compared with existing algorithms. As a result of this comparison, it is shown that GMW algorithm has a superior performance.

I. INTRODUCTION

Kalman filtering is a relatively recent (1958) development in filtering. Theoretically, its purpose is to estimate the state of a linear dynamic system perturbed by Gaussian white noise. The estimator obtained as a result of Kalman filtering is statistically optimal with respect to any quadratic function of estimation error. Kalman filtering has been applied in several areas such as aerospace, marine navigation, nuclear power plant instrumentation, demographic modeling, manufacturing, etc. [1-3]. Two commonly used estimation algorithms are: moving window (MW) Kalman filtering and least squares mean (LSM) estimation. The moving windowing (MW) approach is limited with the use of rectangular windows. In this paper, we propose a generalized moving windowing (GMW) algorithm that utilizes arbitrary shaped windows.

This paper is organized as follows. Section II presents the mathematical development. In section III, problem formulation and preliminary definitions are presented. Section IV presents the generalized moving windowing Kalman filtering algorithm. In section V, adjustment of the generalized moving window length is discussed. In section VI, the proposed algorithm is compared with MW and LSM algorithms through an application.

II. MATHEMATICAL DEVELOPMENT

Consider a discrete linear system model with a linear observation equation

$$x(k+1) = f(k+1,k)x(k) + \Gamma(k+1)w(k) \qquad (1)$$

$$z(k) = H(k)x(k) + v(k) \qquad (2)$$

where $x(k)$ is an $nx1$ dimensional state vector, $z(k)$ is an $mx1$ dimensional observation vector, $w(k)$ and $v(k)$ are zero-mean white noise vectors whose dimensions are $mx1$ and $rx1$, respectively, $f(k+1,k)$ is nxn dimensional state transition matrix, $H(k)$ is rxm dimensional output matrix, $G(k+1)$ is nxm dimensional probability distribution matrix. The covariance matrices of $w(k)$ and $v(k)$ will be represented by $Q(k)$ and $R(k)$, respectively, where, $Q(k)$ is an mxm dimensional positive semi-definite matrix and $R(k)$ is an rxr dimensional positive definite matrix.

The initial value of $x(k)$, $x(0)$ is a random variable with mean equal to $\bar{x}(0)$ and variance $P(0)$. We will assume that $w(k)$, $v(k)$ and $x(0)$ are uncorrelated with each other. The innovation set will be represented by $\tilde{Z}(k+1) = \{\tilde{z}(1),\ldots,\tilde{z}(k),\tilde{z}(k+1)\}$, where k is the set of non-negative integers. The optimal estimate of $z(k+1)$ given the innovation set $\{\tilde{z}(1),\ldots,\tilde{z}(k),\tilde{z}(k+1)\}$ can be defined to be as follows:

$$\hat{z}(k+1 \mid k) = E\{z(k+1) \mid \tilde{Z}(k)\} \qquad (3)$$

The estimation error, $\tilde{z}(k+1 \mid k)$ can be defined as

$$\tilde{z}(k+1 \mid k) = z(k+1) - \hat{z}(k+1 \mid k) \qquad (4)$$

The expression given in Equation (4) is the difference between the actual and the predicted measurement at $k+1$. Using this expression, the prediction error covariance matrix $P(k+1|k)$ can be obtained as follows:

$$P(k+1 \mid k) = E\{\tilde{x}(k+1 \mid k)\tilde{x}^T(k+1 \mid k)\} \qquad (5)$$

Theorems:

1) Let x and z be jointly distributed, Gaussian random vectors. Then, the conditional distribution of *x* given *y* is normal with mean,

$$E\{x \mid z\} = m_x + P_{xz}P_z^{-1}(z - m_z) \tag{6}$$

and the corresponding estimate can be defined as

$$\hat{x} = E\{x \mid z\}$$

2) The Covariance of the estimation error is given by

$$\begin{aligned} E&\left\{[x - E\{x \mid z\}][x - E\{x \mid z\}]^T\right\} \\ &= P_x - P_{xy}P_y^{-1}P_{xy} \end{aligned} \tag{7}$$

In the above expression, the random variables z and [x-$E\{x|z\}$] are independent.

3) Let, x, u and v be jointly Gaussian random vectors. Then, the random vectors u and v are independent and the relation between the three random vectors is given by the following equation:

$$\mathrm{E}\{x \mid u,v\} = \mathrm{E}\{x \mid v\} + \mathrm{E}\{x \mid u\} - \mathrm{E}\{x\} \tag{8}$$

The proofs of the above theorems can be found in [3].

Next, let us define $\tilde{x}(k+1|k)$, to be the error between the state $x(k+1)$ and its estimate $\hat{x}(k+1|k)$,

$$\tilde{x}(k+1 \mid k) = x(k+1) - \hat{x}(k+1 \mid k) \tag{9}$$

The estimate $\hat{x}(k+1 \mid k)$ can be written as

$$\hat{x}(k+1 \mid k) = \mathrm{E}\{x(k+1) \mid \tilde{Z}(k)\}, \qquad k \geq 0 \tag{10}$$

It can be shown that, $\hat{x}$ and $\tilde{x}$ satisfy the orthogonality principle, i.e.

$$E\{\hat{x}(k+1 \mid k)\tilde{x}(k+1 \mid k)\} = 0 \tag{11}$$

III. PROBLEM FORMULATION AND PRELIMINARY DEFINITIONS

Before defining the Generalized Moving Window (GMW) Kalman filtering algorithm, let us define the parameters that will be used in this process:

f and f^* : nxn State Transition Matrix
G and G^* : nxm Probability Distribution Matrix
$\hat{x}$: $nx1$ Windowed Estimation Vector
$\tilde{x}$: $nx1$ Filtering Error Vector
$\hat{\theta}$: $nx1$ Estimation Vector Without Windowing
θ : $nx1$ State Vector Without Windowing
ξ : nxn Windowing Matrix

The simplest form of a windowing model is shown in Figure 1.

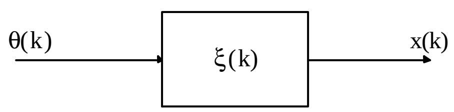

Figure 1. Windowing model.

Hence, the equation for the discrete linear windowing model can be written as follows:

$$x(k) \triangleq x(k) \cdot q(k) \tag{12}$$

Using the above equation, $x(k+1)$ can be written as

$$x(k+1) = x(k+1) \cdot q(k+1) \tag{13}$$

Substituting Equation (12) into Equation (2), we obtain the following observation model

$$z(k) = H(k)x(k)q(k) + v(k) \tag{14}$$

Next, let us obtain the formulation for Kalman filtering Algorithm using Moving Windowing approach according to Equations (1-14). We can write the observation estimate according to Equations (1-3) as follows:

$$\begin{aligned} \hat{z}(k+1 \mid k) &= E\{[H(k+1) \cdot x(k+1) + v(k+1)] \tilde{Z}(k)\} \\ &= E\{H(k+1) \cdot x(k+1) \mid \tilde{Z}(k)\} \\ &\quad + E\{v(k+1) \mid \tilde{Z}(k)\} \\ &= H(k+1) \cdot E\{x(k+1) \mid \tilde{Z}(k)\} \\ &= H(k+1) \cdot E\{x(k+1) \cdot q(k+1) \mid \tilde{Z}(k)\} \\ &= H(k+1) \cdot x(k+1) \cdot E\{q(k+1) \mid \tilde{Z}(k)\} \\ &= H(k+1) \cdot x(k+1) \cdot \hat{q}(k+1 \mid k) \end{aligned} \tag{15}$$

In the above Equation, $\hat{q}(k+1)$ denotes the estimate of the original system model. It should also be noted that in this equation, $E\{v(k+1) \mid \tilde{Z}(k)\}$ is identically zero. Using the result obtained in Equation (15), we can now derive the innovation expression, $\tilde{z}(k+1)$:

$$\begin{aligned} \tilde{z}(k+1) &= z(k+1) - \hat{z}(k+1 \mid k) \\ &= H(k+1) \cdot x(k+1) \cdot q(k+1) + v(k+1) \\ &\quad - H(k+1) \cdot x(k+1) \cdot \hat{q}(k+1 \mid k) \\ &= H(k+1) \cdot x(k+1) \cdot [q(k+1) - \hat{q}(k+1 \mid k)] \\ &\quad + v(k+1) \\ &= H(k+1) \cdot x(k+1) \cdot \tilde{q}(k+1 \mid k) + v(k+1) \end{aligned} \tag{16}$$

In the above formulation, q and x have the same statistical properties. Let us now derive the new expression for error:

$$\tilde{q}(k+1 \mid k) = q(k+1) - \hat{q}(k+1 \mid k) \tag{17}$$

Using the above equations, let us now derive the expressions that will be used in the algorithm.

Estimation:

$$\begin{aligned} \hat{x}(k+1 \mid k) &= E\{x(k+1) \mid \tilde{Z}(k)\} \\ &= E\{f(k+1,k) \cdot x(k) \cdot q(k) \mid \tilde{Z}(k)\} \\ &\quad + \Gamma(k+1) \cdot E\{w(k) \mid \tilde{Z}(k)\} \\ &= f(k+1,k) \cdot x(k) \cdot \hat{q}(k) \end{aligned} \tag{18}$$

From the above equation, the estimate of the original system model can be written as follows:

$$\hat{q}(k+1\mid k)=E\{q(k+1)\mid \tilde{Z}(k)\} = \left[x(k+1)\right]^{-1}\cdot f(k+1,k)\cdot x(k)\cdot \hat{q}(k) \tag{19}$$

In order to simplify the future derivations, we will use the following notations:

$$H_w(k+1)=H(k+1)x(k+1) \tag{20}$$

$$f_w(k+1,k)=[x(k+1)]^{-1}f(k+1,k)x(k) \tag{21}$$

$$\Gamma_w(k+1)=\left[x(k+1)\right]^{-1}\cdot\Gamma(k+1) \tag{22}$$

Next, let us derive the generalized Kalman filtering algorithm equations by writing down the equations for the new system and observation model:

$$q(k+1)=f_w(k+1,k)\cdot q(k)+\Gamma_w(k+1)\cdot w(k) \tag{23}$$

$$z(k+1)=H_w(k+1)q(k)+v(k) \tag{24}$$

Using the above equations, the prediction error can be written as follows:

$$\begin{aligned}\tilde{x}(k+1\mid k) &= x(k+1)q(k+1)+\Gamma(k+1)w(k+1)\\ &\quad -E\{x(k+1)q(k+1)\mid \tilde{Z}(k)\}\\ &= x(k+1)q(k+1)-x(k+1)\hat{q}(k+1\mid k)\\ &\quad +\Gamma(k+1)w(k+1)\\ &= x(k+1)\left[q(k+1)-\hat{q}(k+1\mid k)\right]\\ &\quad +\Gamma(k+1)w(k+1)\\ &= x(k+1)\tilde{q}(k+1\mid k)+\Gamma(k+1)w(k+1)\end{aligned} \tag{25}$$

In order to obtain the formulation for the Generalized Moving Window (GMW) Kalman filtering algorithm, let us first apply the moving windowing algorithm to the above equations. Let *s(k)* denote the length of the moving window. Then, applying Theorem 3 to the sequences $\{\tilde{z}(k-s(k)),\ldots,\tilde{z}(k)\}$ and *z(k+1)*, we obtain,

$$\begin{aligned}&E\{q(k+1)\mid \tilde{z}(k-s(k)),\tilde{z}(k-s(k)+1),\ldots\tilde{z}(k+1)\}\\ &=E\{q(k+1)\mid \tilde{z}(k-s(k)),\ldots,\tilde{z}(k)\}\\ &\quad +E\{q(k+1)\mid \tilde{z}(k+1)\}-E\{q(k+1)\}\\ &=E\{q(k+1)\mid \tilde{z}(k-s(k))\}\\ &\quad +E\{q(k+1)\mid \tilde{z}(k-s(k)+1),\ldots,\tilde{z}(k+1)\}\\ &\quad -E\{q(k+1)\}\end{aligned} \tag{26}$$

The above equation will be used to derive the GMW Kalman filtering algorithm. Let us write the known parameters into Equation (26) and arrange the terms to obtain the unknown parameters. Hence, we obtain

$$\begin{aligned}\hat{q}(k+1)&=E\{q(k+1)\mid \tilde{z}(k-s(k)+1),\ldots,\tilde{z}(k+1)\}\\ &=E\{q(k+1)\mid \tilde{z}(k-s(k)),\ldots,\tilde{z}(k)\}\\ &\quad +E\{q(k+1)\mid \tilde{z}(k+1)\}-E\{q(k+1)\mid \tilde{z}(k-s(k))\}\\ &=\hat{q}(k+1\mid k)-E\{q(k+1)\mid \tilde{z}(k-s(k))\}\\ &\quad +E\{q(k+1)\mid \tilde{z}(k+1)\}\end{aligned} \tag{27}$$

Applying Theorem 1 to Equation (27), we obtain

$$\begin{aligned}\hat{q}(k+1\mid k)&=E\{q(k+1)\mid \tilde{z}(k-s(k)),\ldots,\tilde{z}(k)\}\\ &=E\{f_w(k+1,k)q(k)\mid \tilde{Z}(k)\}\\ &\quad +E\{\Gamma_w(k+1)w(k)\mid \tilde{Z}(k)\}\\ &=f_w(k+1,k)E\{q(k)\mid \tilde{Z}(k)\}\\ &=f_w(k+1,k)\hat{q}(k)\end{aligned} \tag{28}$$

Applying Theorem 2 and the property $E\{\tilde{z}(k)\}=0$ to the third term in Equation (27), we obtain

$$E\{q(k+1)\mid \tilde{z}(k+1)\}=E\{q(k+1)\}+P_{q\tilde{z}}P_{\tilde{z}}^{-1}\tilde{z}(k+1) \tag{29}$$

Simplifying and rewriting the second terms in Equation (29), we obtain

$$\begin{aligned}P_{q\tilde{z}}&=E\{q(k+1)\tilde{z}^T(k+1)\}\\ &=E\{q(k+1)[H_w(k+1)\tilde{q}(k+1\mid k)+v(k+1)]^T\}\\ &=E\{q(k+1)\tilde{q}^T(k+1\mid k)\}H_w^T(k+1)\\ &=E\{\tilde{q}(k+1\mid k)\tilde{q}^T(k+1\mid k)\}H_w^T(k+1)\\ &=P_q(k+1\mid k)H_w^T(k+1)\end{aligned} \tag{30}$$

$$\begin{aligned}P_{\tilde{z}}&=E\{\tilde{z}(k+1)\tilde{z}^T(k+1)\}\\ &=H_w(k+1)P_q(k+1\mid k)H_w^T(k+1)+R(k+1)\end{aligned} \tag{31}$$

In Equation (30), we have used the orthogonality relation between $\hat{q}(k+1\mid k)$ and $\tilde{q}(k+1\mid k)$.

Now, letting $K(k+1)=P_{q\tilde{z}}\cdot P_{\tilde{z}}^{-1}$ and substituting Equation (30) and (31) into Equation (29), we obtain,

$$\begin{aligned}&E\{q(k+1)\mid \tilde{z}(k+1)\}\\ &=E\{q(k+1)\}+P_q(k+1\mid k)H_w^T(k+1)\\ &\quad\times[H_w(k+1)P_q(k+1\mid k)H_w^T(k+1)+R(k+1)]^{-1}\\ &\quad\times\tilde{z}(k+1)\\ &=E\{q(k+1)\}+K(k+1)\tilde{z}(k+1)\end{aligned} \tag{32}$$

The term $K(k+1)$ is known as the Kalman gain in literature. Using Theorem 2 and $E\{\tilde{z}(k)\}=0$, the term $E\{q(k+1)\mid \tilde{z}(k-s(k))\}$ from Equation (27) can be written as

$$\begin{aligned}&E\{q(k+1)\mid \tilde{z}(k-s(k))\}\\ &=E\{q(k+1)\}+P_{q\tilde{z}_s}P_{\tilde{z}_s}^{-1}\tilde{z}(k-s(k))\end{aligned} \tag{33}$$

The first factor of the second term in Equation (33) can be written as follows:

$$P_{\tilde{q}\tilde{z}s} = E\{q(k+1)\tilde{z}^T(k-s(k))\}$$
$$= E\{[f_W(k+1,k)q(k)+\Gamma_W(k+1)w(k)]$$
$$\times[H_W(k-s(k))\tilde{q}(k-s(k)\mid k-s(k)-1)+v(k-s(k))]^T\}$$
$$= f_W(k+1,k)E\{q(k)\tilde{q}^T(k-s(k)\mid k-s(k)-1)\}H_W^T(k-s(k))$$
$$= f_W(k+1,k-s(k))P_q(k-s(k)\mid k-s(k)-1)H_W^T(k-s(k)) \quad (34)$$

In Equation (34), we have again used the orthogonality principle. We have also used the following solution of the state equation given in Equation (23),

$$q(k) = f_W(k, k-s(k)) + \sum_{i=k-s(k)}^{k-1} f_W(k, i+1)\Gamma(i)w(i) \quad (35)$$

where, $k-1 \geq k-s(k)$. In Equation (35), the windowed transition function, $f_W(k, k-s(k))$ can be defined as follows:

$$f_W(k, k-s(k)) = f_W(k, k-1)\cdots f_W(k-s(k)-1, k-s(k)) \quad (36)$$

The second factor of the second term in Equation (33) can be written as follows:

$$P_{\tilde{z}s} = E\{\tilde{z}(k-s(k))\tilde{z}^T(k-s(k))\}$$
$$= E\{H_W(k-s(k))\tilde{q}(k-s(k)\mid k-s(k)-1)$$
$$\times\tilde{q}^T(k-s(k)\mid k-s(k)-1)H_W^T(k-s(k))\}$$
$$+ E\{v(k-s(k))v^T(k-s(k))\} \quad (37)$$
$$= H_W(k-s(k))P(k-s(k)\mid k-s(k)-1)$$
$$\times H_W^T(k-s(k)) + R(k-s(k))$$

Substituting the results obtained in Equations (34) and (37) into our main equation given in (32), we obtain

$$E\{q(k+1)\mid\tilde{z}(k-s(k))\} = E\{q(k+1)\} + f_W(k+1, k-s(k)) \times K(k-s(k))\cdot\tilde{z}(k-s(k)) \quad (38)$$

Next, let us write the Kalman gain given by Equation (38) in more detail,

$$K(k-s(k)) = P_q(k-s(k)\mid k-s(k)-1)H_W^T(k-s(k))$$
$$\times[H_W(k-s(k))P_q(k-s(k)\mid k-s(k)-1) \quad (39)$$
$$\times H_W^T(k-s(k)) + R(k-s(k))]^{-1}\tilde{z}(k-s(k))$$

Substituting Equations (23), (32) and (38) into Equation (27) and simplifying, we obtain

$$\hat{q}(k+1) = f_W(k+1,k)\hat{q}(k) + K(k+1)\tilde{z}(k+1) - f_W(k+1, k-s(k))$$
$$\times K(k-s(k))\tilde{z}(k-s(k)) \quad (40)$$

Using Equation (28), we obtain

$$\tilde{q}(k+1\mid k) = q(k+1) - \hat{q}(k+1\mid k)$$
$$= f_W(k+1,k)q(k) + \Gamma_W(k+1)w(k) - f_W(k+1,k)\hat{q}(k)$$
$$= f_W(k+1,k)q(k) + \Gamma_W(k+1)w(k) - f_W(k+1,k)$$
$$\times[f_W(k,k-1)\hat{q}(k-1) - f_W(k, k-s(k)-1)\tilde{z}(k-s(k)-1)$$
$$+K(k)\tilde{z}(k)]$$
$$= f_W(k+1,k)\cdot q(k) + \Gamma_W(k+1)\cdot w(k) - f_W(k+1,k)\cdot\hat{q}(k\mid k-1)$$
$$+K(k)\cdot\tilde{z}(k) - f_W(k+1, k-s(k)-1)\cdot\tilde{z}(k-s(k)-1)$$
$$= f_W(k+1,k)\cdot\tilde{q}(k\mid k-1) + \Gamma_W(k+1)\cdot w(k) + K(k)\cdot\tilde{z}(k)$$
$$-f_W(k+1, k-s(k)-1)\cdot\tilde{z}(k-s(k)-1) \quad (41)$$

We can now derive the covariance matrix $P_q(k+1|k)$ to be as follows:

$$P_q(k+1\mid k) = E\{\tilde{q}(k+1\mid k)\tilde{q}^T(k+1\mid k)\}$$
$$= f_W(k+1,k)P_q(k\mid k-1)f_W^T(k+1,k)$$
$$+\Gamma_W(k+1)Q(k)\Gamma_W^T(k+1) \quad (42)$$
$$-f_W(k+1,k)K(k)H_W(k)P_q(k\mid k-1)f_W^T(k+1,k)$$
$$-f_W(k+1,k)P_q(k\mid k-1)H_W^T(k)K^T(k)f_W^T(k+1,k)$$
$$+f_W(k+1, k-s(k)-1)P_q(k-s(k)-1\mid k-s(k)-2)$$
$$\times H_W^T(k-s(k)-1)K^T(k-s(k)-1)f_W^T(k+1, k-s(k)-1)$$
$$= f_W(k+1,k)P_q(k)f_W^T(k+1,k) + \Gamma_W(k+1)Q(k)\Gamma_W^T(k+1)$$
$$+f_W(k+1, k-s(k)-1)P_q(k-s(k)-1\mid k-s(k)-2)$$
$$\times H_W^T(k-s(k)-1)K^T(k-s(k)-1)f_W^T(k+1, k-s(k)-1)$$

In the above simplification, we have utilized the following relations

$$E\{\tilde{q}(k\mid k-1)\tilde{z}^T(k)\} = P_q(k\mid k-1)H_W^T(k) \quad (43)$$

$$E\{\tilde{z}(k)\tilde{q}^T(k\mid k-1)\} = H_W(k)P_q(k\mid k-1) \quad (44)$$

$$E\{\tilde{z}(k)\tilde{z}^T(k)\} = H_W(k)P_q(k\mid k-1)H_W^T(k) + R(k) \quad (45)$$

The posteriori covariance matrix can now be written as follows:

$$P_q(k+1) = P_q(k+1\mid k) - K(k+1)H_W(k+1) \times P_q(k+1\mid k) \quad (46)$$

IV. GMW KALMAN FILTERING ALGORITHM

From the above derivations, the parameters of the proposed GMW Kalman filtering algorithm can be summarized as follows:

Filtering Algorithm:

$$\hat{q}(k+1) = f_w(k+1,k)\hat{q}(k) + K(k+1)\tilde{z}(k+1) - f_w(k+1,k-s(k))K(k-s(k))\tilde{z}(k-s(k)) \quad (47)$$

Gain Equation:

$$K(k+1) = P_q(k+1 \mid k)H_w^T(k+1) \times [H_w(k+1)P_q(k+1 \mid k)H_w^T(k+1) + R(k+1)]^{-1} \quad (48)$$

A Priori Covariance Algorithm:

$$P_q(k+1 \mid k) = f_w(k+1,k)P_q(k)f_w^T(k+1,k) + \Gamma_w(k+1)Q(k)\Gamma_w^T(k+1) + f_w(k+1,k-s(k)-1)P_q(k-s(k)-1 \mid k-s(k)-2) \times H_w^T(k-s(k)-1)K^T(k-s(k)-1)f_w^T(k+1,k-s(k)-1) \quad (49)$$

A Posteriori Covariance Algorithm:

$$P_q(k+1) = P_q(k+1 \mid k) - K(k+1)H_w(k+1)P_q(k+1 \mid k) \quad (50)$$

V. ADJUSTMENT OF THE GENERALIZED MOVING WINDOW LENGTH

Let $s(k)$ denote the length of the generalized window. The necessary equations for the proposed GMW Kalman filtering algorithm were summarized in Equations (47)-(50). In the windowing process, in order to reduce the undesirable effect of the old data on the new data, we subtract the same number of old data as the new data. With this process, the length of the window stays unchanged. Hence, to accomplish this task let us define two new functions as follows:

$$D(k+1) = f_w(k+1,k-s(k)-1)P_q(k-s(k)-1 \mid k-s(k)-2) \times H_w^T(k-s(k)-1)K^T(k-s(k)-1)f_w^T(k+1,k-s(k)-1) \quad (51)$$

$$B(k+1) = f_w(k+1,k-s(k))K(k-s(k))\tilde{z}(k-s(k)) \quad (52)$$

It can be shown that, if we subtract $D(k+1)$ from the a priori covariance term $P_q(k+1 \mid k)$ and $B(k+1)$ from the estimation term $\hat{x}(k+1 \mid k)$, then the undesirable effect of the old data on the new data is reduced. This effect is further reduced as long as this subtraction process continues. If we set $D(k+1)$ and $B(k+1)$ to zero, we obtain the results of the classical Kalman filtering algorithm.

V. SIMULATION RESULTS AND CONCLUSIONS

In order to demonstrate the proposed GMW Kalman filtering algorithm, the following function was used in the moving windowing process. In this function, a is a constant, x is observation estimate and N is the number of samples.

$$f_p(x) = a \cdot \exp(\log(x^2)/N) \quad (51)$$

The simulation results are shown in Figure 2. In this Figure, OS represents the original signal that is being estimated, GMW represents the estimate using the proposed algorithm, MW represents the estimate obtained using rectangular windowing and LSM represents the estimate using least squares mean estimation technique. Figure 3 shows the estimation error, which has been obtained by subtracting each estimate from the original function. From these figures, it can be observed that the performance of the proposed GMW Kalman filtering algorithm is much superior than the MW Kalman filtering and LSM estimation techniques.

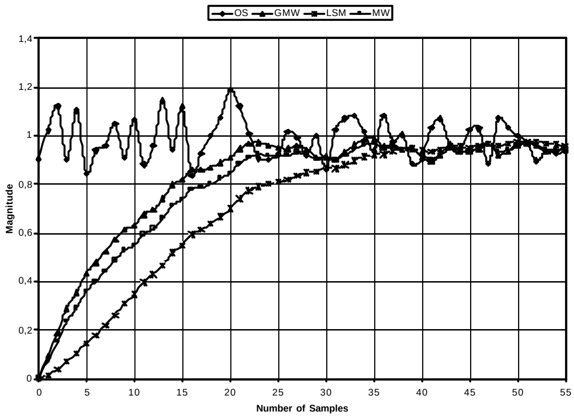

Figure 2. Comparison of the estimates due to different estimation algorithm with respect to the original signal.

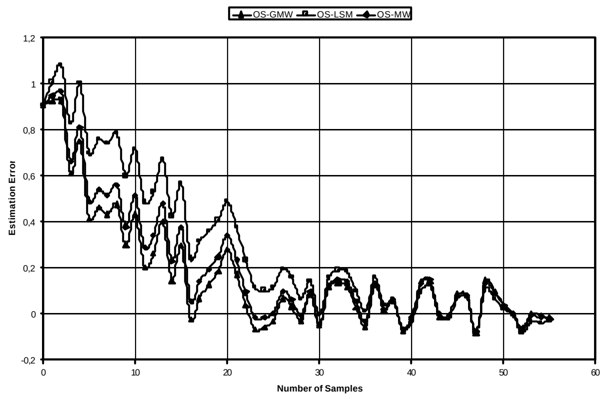

Figure 3. Comparison of the estimation errors of different estimation algorithms.

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