

COMPARISON OF DISTRIBUTION SYSTEMS POWER FLOW ALGORITHMS FOR VOLTAGE DEPENDENT LOADS

Ulaş Eminoğlu M. Hakan Hocaoglu

E-mail: u.eminoglu@gyte.edu.tr, hocaoglu@gyte.edu.tr

Department of Electronic Engineering, Gebze Institute of Technology, Gebze 41400, Turkey

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ABSTRACT

In this paper, the convergence ability of Distribution Systems Power Flow Algorithms, which are widely used for distribution systems analysis, is compared with different voltage-dependent load models. The convergence ability of methods are also evaluated for different tolerance values, voltage levels, loading conditions and R/X ratios, under the wide range exponents of loads. Results show that Ratio-Flow method is preferable from other methods.

I. INTRODUCTION

In the literature, there are a number of efficient and reliable load flow solution techniques, such as; Gauss-Seidel, Newton-Raphson and Fast Decoupled Load Flow [1-8]. Hitherto they are successfully and widely used for power system operation, control and planning. However, it has repeatedly been shown that these methods may become inefficient in the analysis of distribution systems with high R/X ratios or special network structures [9-11]. Accordingly, a number of methods proposed in the literature [12-29] specially designed for the solution of power flow problem in radial distribution networks.

The methods developed for the solution of ill-conditioned radial distribution systems may be divided into two categories. The first type of methods is utilised by proper modification of existing methods such as, Newton-Raphson [2-8]. On the other hand, the second group of methods is based on forward-backward sweep processes using Kirchoff's Laws or making use of the well-known bi-quadratic equation which, for every branch, relates the voltage magnitude at the receiving end to the voltage at the sending end and the branch power flow for solution of ladder networks [12-29].

Shirmohammadi et al., [12] have presented a compensation-based power flow method for radial distribution networks and/or for weakly meshed structure using a multi-port compensation technique and basic

formulations of Kirchoff's Laws. The radial part is solved by a straightforward two step procedure in which the branch currents are first computed (backward sweep) and then the bus voltages are updated (forward sweep). In the improved version [13], branch power flow was used instead of branch complex currents for weakly meshed transmission and distribution systems by Luo. In [14], Baran and Wu propose a methodology for solving the radial load flow for analysing the optimal capacitor sizing problem. In this method, for each branch of the network three non-linear equations are written in terms of the branch power flows and bus voltages. The number of equations is subsequently reduced by using terminal conditions associated with the main feeder and its laterals, and the Newton-Raphson method is applied to this reduced set. The computational efficiency is improved by making some simplifications in the jacobian. Consequently, numerical properties and convergence rate of this algorithm have been studied using the iterative solution of three fundamental equations representing real power, reactive power and voltage magnitude by Chiang in [15]. In [16], G. Renato Cespedes makes use of well-known bi-quadratic equation which, for every branch, relates the voltage magnitude at the receiving end to the voltage at the sending end and branch power flow. In [16], only voltage magnitudes are computed, bus phase angles do not appear in the formulation which was also used by Das et al., in [17]. Jasmon, in [18], have proposed a load flow technique which, for every branch, leads to a pair of quadratic equations relating power flows at both ends with the voltage magnitude at the sending end for the voltage stability analysis of radial networks. Haque, in [19], have formulated the load flow problem of the distribution system in terms of three sets of recursive equations and analysed load flow results for various voltage dependent load models. The effects of various load models on the convergence pattern of the method are also studied. The effect of voltage-dependency of load on the results and convergence characteristics of power flow solution are also analysed in [20], where the proposed method is also based on Kirchoff's Laws. In [21] authors have proposed Ratio-Flow method which is based on forward-backward

ladder equation for complex distribution system by using voltage ratio for convergence control. This method were applied with standard Newton-Raphson method for complex distribution systems, which have multiple sources or relatively strong connected loops with extended long radial feeders including laterals, to solve the load flow problem. In [22], R. Ranjan and D. Das have proposed a new method to solve radial distribution networks. They have used simple algebraic recursive expression of voltage magnitude and the proposed algorithm used the basic principle of circuit theory. D. Zimmerman and H. D. Chiang in [23], have formulated load flow problem as a function of the bus voltages and equations are solved by Newton's method. The method has been compared with classical Newton-Raphson and Forward-Backward sweep methods by using a number of test cases. Although required iteration number considerable favoured from classical methods for small tolerances, no results has been provided on the accuracy of the solution in terms of bus voltage magnitudes or angles. The results provided in [23] suggest that undertaken comparisons only cover network structures which are inherently convergent ie. solutions can also be obtained using classical Newton-Raphson method.

In ref. [25], the authors have proposed forward-backward substitution method which is based on the Kirchhoff's Laws. In backward substitution, each branch current is calculated by Kirchhoff's Current Law (KCL). Using these currents, the node voltages are calculated by Kirchhoff's Voltage Law in forward substitution at each iteration. The voltage magnitudes at each bus in an iteration are compared with their values in the previous iteration. If the error is within the tolerance limits, the procedure is stopped. Ladder Network Theory given in ref. [26] is similar to the Forward-Backward Substitution method. In Ladder Network Theory, the currents in each branch are computed by KCL. In addition to the branch currents, the node voltages are also computed by KVL in each iteration. Thus magnitude of the swing bus voltage is also determined. The calculated value of swing bus is compared with its specified value. If the error is within the limit, the procedure is stopped. Otherwise, the forward and backward calculations are repeated as in forward-backward substitution method.

The aim of this paper is to compare the convergence ability of distribution system load flow methods which are widely used for distribution systems analysis. The method, analysed in this paper, are classical Newton-Raphson method [2], Ratio-Flow [21], Forward-Backward Substitution method [25] and Ladder Network Theory [26]. The convergence ability of methods are also evaluated for different tolerance values, different voltage levels, different loading conditions and different R/X ratios, under the wide range exponents of loads. Algorithms are implemented with Matlab codes.

II. DISTRIBUTION SYSTEM LOAD MODELLING

In actual power system operation, different categories and types of loads such as; residential, industrial and commercial loads might be present. The nature of these types of loads is such that their active and reactive power are dependent on the voltage and frequency of the system. Moreover, load characteristics have significant effects on the load flow solutions and convergence ability [19]. Common characteristic of the exponential load models can be given as:

$$P = P_o \left(\frac{V}{V_o} \right)^{np}, Q = Q_o \left(\frac{V}{V_o} \right)^{nq}$$

(1)

Where np and nq stand for load exponents, P_o and Q_o stand for the values of the active and reactive powers at the nominal voltages. V and V_o stand for load bus voltage and load nominal voltage, respectively. Special values of the load exponents can cause specific load types such as 0: constant power, 1: constant current and 2: constant impedance. Common values for the exponents of the model for different load components are given in Table 1 from [27].

Table 1 Common values of exponents for different static load models

Load Component	np	nq
Battery Charge	2.59	4.06
Fluorescent Lamps	2.07	3.21
Constant Impedance	2	2
Air Conditioner	0.5	2.5
Constant Current	1	1
Resistance Space Heater	2	0
Pumps, Fans other Motors	0.08	1.6
Compact Fluorescent Lamps	1	0.35
Small Industrial Motors	0.1	0.6
Constant Power	0	0

III. APPLICATION TO RADIAL DISTRIBUTION NETWORKS

Two test systems (distribution networks with and without laterals) are used to analyse the convergence ability of methods. An 11 kV distribution systems having 12 buses is used to observe the reliability of the method with different static load models.

In this paper firstly, convergence characteristic of methods is analysed for seven exponential load models, given in Table 1, in a 12-bus radial system without laterals. The system is taken from [28]. 12-bus system is composed from the 30-bus system omitting laterals and total power of the each laterals are added to its sending end on the feeder. The initial voltage magnitude at all buses is considered to be the same as the source bus. A tolerance of 10^{-4} on voltage magnitude is used for the studies. Table 2 shows the number of iteration by using four methods for

constant power loads and by using three methods for different voltage dependent loads. From Table 2, it is clear that, for different load models, the Ratio-Flow method has fast convergence speed when compared with other methods for the tolerance values of 10^{-4} on the voltage magnitudes. Secondly, the methods are tested on 30-bus radial system with laterals to analyse the performance of methods for different voltage-dependent load models. The convergence speed of methods are compared with a tolerance of 10^{-4} on voltage magnitude. From Table 3, it can be said that the Ratio-Flow method has faster convergence ability than other methods for different types of loads. Moreover, in this case the Newton-Raphson method is not converged. The convergence characteristic of the methods are analysed for various system parameters and loading conditions for different value of the exponents of loads. The value of the n_p and n_q are selected as equal and varied from 0 to 5. The number of iterations required to solve the load flow for different characteristics of the 30-bus distribution system are given in figure 1. Convergence tolerance for all methods are varied from 10^{-2} to 10^{-5} and results depicted in fig 1-a. For all tolerance values, the ratio-flow algorithm needs less number of iteration than the other methods. Sending end source voltage (V_s) are varied from 0.9 pu. to 1.2 pu. by keeping tolerances at 10^{-4} , and results are depicted in fig 1-b. For different source

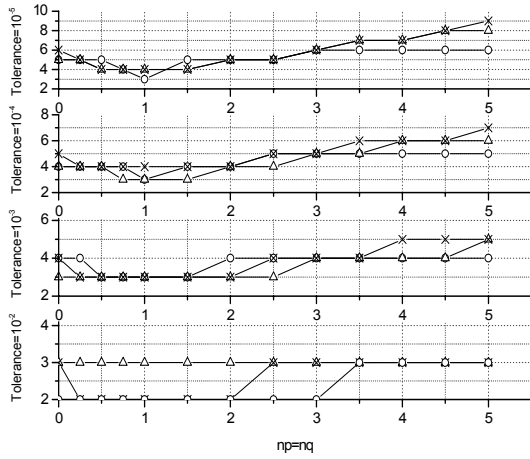
voltage levels, the Ratio-Flow method has fast convergence speed under wide range of load exponents as shown in figure. In addition to that, required iteration number for Ratio-Flow method is less affected by the magnitude of the source voltage. On the other hand, convergence speed of Forward-Backward Substitution method and Ladder Network Theory is more sensitive to the load exponents when source voltage is increased. Different loading conditions are considered by multiplying each nodes power by a load factor (λ) as $S=\lambda*S$ and results depicted in fig 1-c. From the figure, for the heavy loading conditions of the system, maximum number of iteration for the Ratio-Flow method is equal to nine. On the other hand, Forward-Backward Substitution method converges in the 14th iteration and Ladder Network Theory converges in the 13th iteration for the same load. Finally, branch impedances R/X ratios are varied multiplying each branch resistance by a coefficient (k) as $Z=k*R+jX$ and results depicted in fig 1-d. Likewise, it is shown that convergence speed of the Ratio-Flow method is less effected with the increase of the R/X ratios of the distribution system when compared with the Forward-Backward Substitution method and Ladder Network Theory. It is shown that the Forward-Backward Substitution method and Ladder Network Theory have nearly the same convergence characteristics for different parameters of the distribution system.

Table 2 Number of iteration required to solution of the power flow problem of 12-bus system

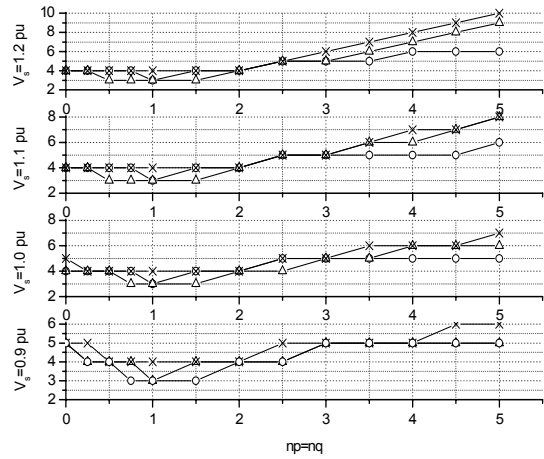
Load Model	Forward-Backward Substitution	Ladder network Theory	Ratio-Flow Method	Newton Raphson
Constant Power	4	4	3	3
Small Industrial Motors	4	4	3	-
Pumps, Fans other Motors	4	4	3	-
Constant Current	3	3	2	-
Air Conditioner	4	4	3	-
Constant Impedance	4	4	3	-
Fluorescent Lamps	4	4	2	-
Battery Charge	4	4	3	-

Table 3 Number of iteration required to solution of the power flow problem of 30-bus system

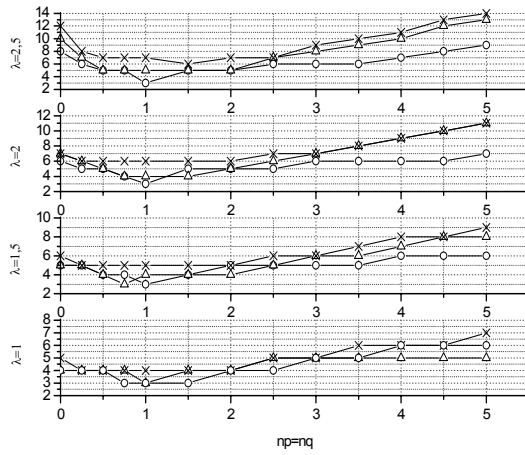
Load Model	Forward-Backward Substitution	Ladder network Theory	Ratio-Flow Method	Newton Raphson
Constant Power	5	4	4	N. A
Small Industrial Motors	4	4	4	-
Pumps, Fans other Motors	4	4	4	-
Constant Current	4	3	3	-
Air Conditioner	4	3	4	-
Constant Impedance	4	4	4	-
Fluorescent Lamps	5	4	4	-
Battery Charge	5	5	5	-



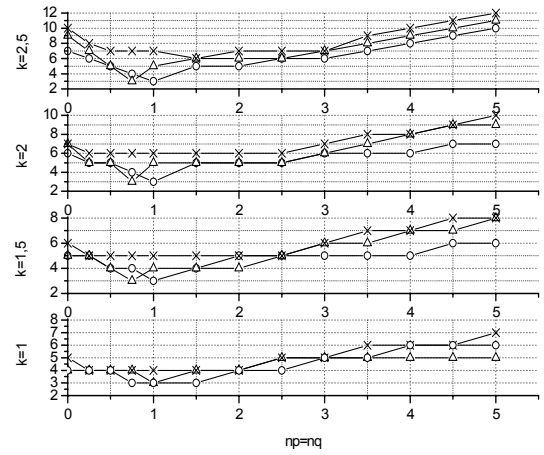
a: Different tolerance values



b: Different source voltage levels



c: Different loading conditions



d: Different R/X ratio

Figure 1 Variation of the number of iteration for different parameters of the system for the Ratio-Flow method (o), Forward-Backward Substitution method (x) and Ladder Network Theory (Δ).

V. CONCLUSIONS

In this paper, convergence ability of distribution system load flow methods which are widely used for distribution systems load flow solution is analysed on ill-conditioned radial systems with different voltage-dependent load models. Four methods has been applied to 12-bus distribution system (without laterals) and 30-bus distribution system (with three laterals). The load flow problem has been successfully solved with different static load types and results obtained by using four methods have been compared. The results have been evaluated under different tolerance values, different voltage levels, different loading conditions and different R/X ratio, under the wide range of the exponents of the loads in the 30-Bus radial distribution systems. The results show that the Ratio-Flow method is simple and has fast convergence ability. Moreover, it needs less number of

iteration and less sensitive to the distribution system parameters than Forward-Backward Substitution method and Ladder Network Theory. Moreover, Classical Newton-Raphson method did not converged for 30-bus distribution system.

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